Design of Flat Slabs

Design of one way slabs is like design of parallel 12” beams.
**Slabs - Ribbed Slabs**

**One way joists**

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|                  |
|                  |
+------------------+
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**Details of Construction**

1. Continuous flat surface (2" - 4" thick)
2. Ribs spaced at 20" - 30" (face to face). Usually tapered 1:12 for form removal.
3. Forms (between ribs) either removable boxes or hollowed concrete blocks. (Blocks should be included in weight computation.)
4. Plate reinforcement is calculated for bending or min. reinforcement for shrinkage (0.2% for GR40 or 60) or (0.18% for GR60 >)

(5) Shear design of ribs:
   a. $f_{ce}$ allowable is 10% higher than for regular beams.
   i.e. for short design: $f_{ce} = 1.1 \times 2 \sqrt{f_c} = 2.2 \sqrt{f_c}$
   b. If shear stress exceeds maximum, enlarge width of rib
Details of Construction (2)

(6) Ribs are designed as T sections. Main positive reinforcement includes at least 2 bars.

(7) Tie beams are required as follows:
   (1) One if span \( L \) is: \( 15'0'' \leq L \leq 20'0'' \)
   (2) Two if span \( L \) is: \( 20'0'' \leq L \leq 30'0'' \)
   (3) Three ….

Add Reinforcement:

(8) Height of slab is approximated as:

\[
L \approx \frac{L_{eq}}{20} \quad \text{[or ACI 9.5(a)]}
\]

Equivalent Span \( L_{eq} \)

Note:

\[
\begin{align*}
\text{exterior} & \quad \text{interior} & \quad \text{cantilever} \\
\text{simply supported} & \\
L_{eq} &= \text{span between contraflexure points.}
\end{align*}
\]
Example of Joists Design

Design slab for an interior span of concrete joist floor using 30'' forms; LL = 80 psf; $f' = 3$ ksi; $f_y = 60$ ksi. Use moment coefficients (No redistribution required) (Add 20 psf for ducts, tiles, ceilings)

The design has two steps:
1. Design of the top flat plate supported by the ribs (joists)
2. Design of the joists as T beams (integrimly connected with top flat slab)

Upper Flat Plate Design

(1) Upper Plate Design: (design as unreinforced section)
Assume $t = 3''$ for weight estimate.

\[
M_u = f' \frac{bt^3}{6}
\]

\[
M_d = \phi M_u
\]

\[
M_d = \frac{w_d t^2}{12}
\]

\[
P = 1.4 \times \left( \frac{3.0}{12} \times 0.150 \right) + 1.4 \times 20 + 1.7 \times 80 = 0.22 \text{ ksf}
\]

\[
M_d = 0.22 \times \left( \frac{30}{12} \right)^2 \times \frac{1}{12} = 0.115 \text{ kips/ft}
\]

\[
t = \sqrt{\frac{6M_d}{\phi \beta t}} = \sqrt{\frac{6 \times (0.115 \times 12000)}{0.65 \times 12 \times 5 \times 3000}} = 2.1'' \quad \text{use } t = 3''
\]

\[
A_{env} = A_{min} = \frac{0.18}{100} \times 3 \times 12 = 0.065 \text{ in}^2/\text{ft} \Rightarrow \#3 @ 20''
\]

welded wire mesh \ \Rightarrow 4 \times 12 - W2.5 \times W1.4 from tables \ \Rightarrow (A_v = 0.075)
Design of Joists

By ACI Table 9.5

\[ h_{\text{min}} = \frac{L}{21} = \frac{26 \times 12}{21} = 14.9 \text{ in} \quad \text{say 15\"}} \]

Make joists 12\" and slabs t = 3\" , b = 5\"

(Suggested practical dimensions b = 4\", t = 2\" d = 16\")

* Design One Rib:

Design for Flexure

(d = 14\")

\[ w_{\text{L,rib}} = \frac{1.4}{3} \left[ 1 + \frac{6 \times 12}{35} \times 15 \right] + \frac{1.4 \times 20}{1000} + \frac{1.7 \times 80}{1000} + \frac{35}{12} = 0.744 \text{ k-l/ft} \]

Negative Moment

\[ M_x = \frac{1}{11} w_{\text{L,rib}} L^2 = \frac{1}{11} \times 0.744 \times 24.5^2 = 40.6 \text{ k-l/rib} \]

\[ d = 15\" - \frac{3}{8}\" - \frac{1}{4}\" = 14\" \]

\[ k_{\text{min}} = \frac{200}{6.85f_c} = \frac{200}{0.85 \times 3000} = 0.078 ; \quad k_{\text{max}} = 0.75 \times 0.85 \times \frac{87}{87 + 60} = 0.377 \]

\[ k = 1 - \sqrt{1 - \frac{2 \times 40.6 \times 12}{0.90 \times 0.85 \times 3 \times 5 \times 14^2}} = 0.247 < k_{\text{max}} \quad \text{OK} \]

\[ A_{\text{req}} = \frac{40.6 \times 12}{60 \times 1 \times (0.247)^2} = 0.74 \text{ in}^2 \quad 2\#4 \]

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Verification of Shear

\[ V_d = w_d \left( \frac{L}{2} - d \right) = 0.744 \left( 2.25 - \left( \frac{0.04}{12} \right) \right) = 8.25 \text{ kips} \]

at \( d \) distance away from support

\[ f_c = \frac{8.25}{5 \times 14} = 139 \text{ psi} \]

\[ f_{c,\text{max}} = 2.2 \sqrt{f_c} = 120 \text{ psi} \]

\[ f_c > f_{c,\text{max}} : \begin{cases} (a) \text{ reinforcement required.} \\ (b) \text{ enlarge rib at its end.} \end{cases} \]

Use alternative (b):

\[ b_{\text{req}} = \frac{139}{120} \times 5 = 6.6 \text{ in.} \Rightarrow 6'' \]

Design for Flexure

Positive Moment

\[ M' = \frac{1}{16} w_d L^2 = \frac{1}{16} \times 0.744 \times 24.5 = 27.9 \text{ k-ft} \]

\[ B = \min \begin{cases} 16t + b = 16 \times 3 + 5 = 53 \\ 5 = 35 \leftarrow \text{governs} \\ \frac{L}{4} = 26/12/4 = 78 \end{cases} \]

\[ k = 1 - \frac{2 \times 27.9 \times 12}{0.90 \times 3 \times 35 \times 14^2} = 0.021 < k_{\text{min}} \]

\[ \bar{t} = \frac{3}{14} = 0.21 > 0.021 \] neutral axis in flange.

\[ A_{\text{req}} = \frac{27.9 \times 12}{0.90 \times 60 \times 14 \left( 1 - 0.021^2 \right)} = 0.45 \text{ in}^2/\text{rib} \leftarrow 1 \#4 + 1 \#5 \]

\[ A_{\text{req}} = \frac{200}{f_y} \times \frac{b \times d}{60000} = 0.23 \text{ in}^2/\text{rib} \]

\[ \text{note: smaller b} \ 4 \times 12 \text{ or } W2.5 \times W1.25 \]
Reinforcement

Area of Reinforcing Bars

Table 3-6 - Total Areas of Bars - $A_s (\text{in}^2)$

<table>
<thead>
<tr>
<th>BAR SIZE</th>
<th>BAR DIAMETER</th>
<th>NUMBER OF BARS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td># 3</td>
<td>0.375</td>
<td>0.11</td>
</tr>
<tr>
<td># 4</td>
<td>0.500</td>
<td>0.20</td>
</tr>
<tr>
<td># 5</td>
<td>0.625</td>
<td>0.31</td>
</tr>
<tr>
<td># 6</td>
<td>0.750</td>
<td>0.44</td>
</tr>
<tr>
<td># 7</td>
<td>0.875</td>
<td>0.60</td>
</tr>
<tr>
<td># 8</td>
<td>1.000</td>
<td>0.79</td>
</tr>
<tr>
<td># 9</td>
<td>1.128</td>
<td>1.00</td>
</tr>
<tr>
<td>#10</td>
<td>1.270</td>
<td>1.27</td>
</tr>
<tr>
<td>#11</td>
<td>1.410</td>
<td>1.56</td>
</tr>
</tbody>
</table>