Lecture 18 – PLASTIC ANALYSIS AND DESIGN OF BEAMS AND FRAMES* (2)

**Kinematic Method**

- pick a collapse mechanism and check that $M_p$ are not exceeded anywhere in the structure
- collapse load calculated using virtual work: $\bar{W}_i^* = \bar{W}_e^*$

\[
\bar{W}_i^* = \sum_i M_p \theta_i
\]

\[
\bar{W}_e^* = \sum_j P_j \delta_j + \int_{x=0}^L w(x)f(x)\,dx
\]

<--- Kinematically admissible mechanism

For $u_B = 1^*$, $\theta_A = \frac{1}{2}$, $\theta_C = \frac{1}{5}$, $\theta_B = \left(\frac{1}{2} + \frac{1}{5}\right)$

\[
\begin{align*}
\bar{W}_i^* &= (M_A \theta_A + M_B \theta_B + M_C \theta_C) = M_p \left(\frac{1}{2} + \frac{1}{5} + \left[\frac{1}{2} + \frac{1}{5}\right]\right) \\
&= 1.4 M_p
\end{align*}
\]

\[
\bar{W}_e^* = P \cdot 1^*
\]

\[
\text{P Collapse} = 1.4 \times M_p = 1.4 \times (1263) = 1769 \text{ kN}
\]

\[
\bar{W} = 1769 \times 225 = 202,662 \text{ kNm}
\]

Equilibrium check
What if the incorrect collapse mechanism is selected?

\[ \begin{align*}
\text{Another mechanism} \\
\bar{W}_T^* &= M_p \left( \frac{1}{3.5} + \frac{1}{3.5} + \frac{2}{3.5} \right) = 1.14 M_p \\
W_E^* &= P \left( 1 \times \frac{2}{3.5} \right) = 0.57 P \\
\therefore \text{Collapse: } 2M_p > 2(1.14) = 2.28 M_p > 2.526 \text{ kN}
\end{align*} \]

Now check equilibrium:

\[ \begin{align*}
V_A &= \frac{2M_p + 2(1.5)M_p}{3.5} = 1.43 M_p \\
M_B &= -M_p + 1.43 M_p(2) \\
&= -1.86 M_p > M_p \\
\therefore \text{Equilibrium condition is not satisfied}
\end{align*} \]
Theorems of plastic analysis:

**LOWER BOUND THEOREM**: A collapse load computed on the basis of an assumed moment diagram in which the moments do not exceed the plastic moments is less than or equal to the true collapse load.

**UPPER BOUND THEOREM**: A collapse load computed on the basis of an assumed mechanism will always be greater than or equal to the true collapse load.

**UNIQUENESS THEOREM**: The true collapse load is the load calculated using the upper and lower bound methods.

**Application of the Kinematic Method of Analysis**

The kinematic method is the simplest of the 3 methods presented and is a powerful analysis tool. The key shortcoming is that the collapse load so calculated will likely be unconservative.

The key step in the kinematic method is the definition of basic independent and combined mechanisms. The basic mechanisms are:

1. Beam (member) mechanism
2. Panel (or sidesway) mechanism
3. Gable mechanism
4. Joint mechanism

Each mechanism is shown on the following sheet.
The objective of the kinematic method is to find the lowest possible collapse load by combining mechanisms to reduce the internal work, increase the external work, or both.

Consider now the portal frame below with lateral load $H$ and vertical load $P$. Assume that all members have a plastic moment equal to $M_p$.
For the basic sidesway mechanism
\[ W_e = Hh \theta \]
\[ W_i = 4Mp \theta \]
\[ \therefore H = \frac{4Mp}{h} \]

For the basic beam mechanism
\[ W_e = \frac{PL \theta}{2} \]
\[ W_i = 4Mp \theta \]
\[ \therefore P = \frac{8Mp}{L} \]

Note that the hinge at B rotated \( \theta \) clockwise in both basic mechanisms. By combining these mechanisms, we reduce the internal work at B and increase the internal work at D, namely

\[ W_e = Hh \theta + \frac{PL \theta}{2} \]
\[ W_i = Mp (\theta + 2\theta + \theta + \theta + \theta) \]
\[ = 6Mp \theta \]

For \( h = L \), and \( H = \frac{PL}{2} \), we have

Sway mechanism: \( H = 4Mp/h \)
Beam mechanism: \( H = 4Mp/h \)
Combined mechanism: \( H = 3Mp/h \) \(-\text{true collapse load?}\)
At no location on the frame does \( M \) exceed \( M_p \), and so the solution is unique, and the collapse load is \( 3M_p/h \) in this instance.

Now, how is plastic analysis used in earthquake engineering?

- can be used to estimate the "capacity" of a building frame
- precursor to nonlinear static analysis or pushover analysis
- check the results of more "sophisticated" methods of analysis
  - nonlinear dynamic analysis

A sample five-story, three-bay frame is shown on the following sheet. We will analyze this frame using the kinematic method, but note again that the collapse load calculated using this method is generally unconservative. Two questions (at least) arise

1. What mechanisms should be considered?

2. What lateral loading patterns should be assumed?

For this example, we ignore joint mechanisms, but such mechanisms should be considered for "real world" problems.
For the uniform pattern, and the two mechanisms, we have:

**Mechanism 1**

\[ \theta = \Delta / h, \quad W_I = 3 \Omega \theta \]

\[ W_E = 3P \Delta \left( 1 + \frac{3}{4} + \frac{1}{4} + 0.02 \right) = 0.5 \theta = 45 \Omega h \]

\[ P = 3 \Omega \frac{M_p}{45 h}, \quad \text{and} \quad V_b = 15P = 15 \Omega \frac{M_p}{h} \]

**Mechanism 2**

\[ \theta = \Delta / h, \quad W_I = 8(3\Omega \theta) = 24 \Omega \theta \]

\[ W_E = 3P \Delta \left( 1 + 1 + 1 + 1 \right) = 15 \theta = 15 \Omega h \]

\[ P = 24 \Omega \frac{M_p}{15 h}, \quad \text{and} \quad V_b = 15P = 24 \Omega \frac{M_p}{h} \]

So what have we calculated? Two values of the maximum base shear. Let's look at the other load pattern and other collapse mechanisms.