Lecture #13a

Shear Design of R/C Beams

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Shear Flexure Effects

- Flexure and Shear Diagrams:

- Acting Stresses in Beams

\[ f_x, f_y, f_{xy} \]

different at each individual point in section.
Shear Flexure Effects

- Principal stresses:
  \[ f_{ld} = \frac{f_m}{2} \pm \sqrt{\left(\frac{f_m}{2}\right)^2 + f_v^2} \]

- Typical cracking due to principal tension.

Shear Stresses

(b) Actual shear stresses

Vertical distribution Note: Maximum occurs at the neutral axis, usually in the web of a beam:
Shear Capacity Mechanism

(c) Shear Capacity Mechanism

Failure mechanism:

- $V_c$ carried by compression
- $V_c = \text{concrete resistance}$
- Carried by friction aggregate interlock
- Carried by 'dowel' action
- $V_s = \text{(shear) steel resistance}$

Total resistance: Concrete + Steel Resistance

Mohr Circle - Principal Stresses

Mohr Circle - Principle

$\frac{f_x}{2} + \sqrt{\frac{f_x^2 + f_y^2}{2}} = f_c$

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Concrete Resistance

\[ f_{u,c} = 1.9 \sqrt{f_{c}'} + 2500p\frac{V_u d}{M_u} \leq 3.5 \sqrt{f_{c}'} \]

**Extended Resistance**

Notation as agreed: \( M_u \) = ultimate/nominal moment capacity, \( V_u \) = ultimate shear capacity
Requires iterations since \( V_u \) is not known prior to design.

\[ f_{u,c} = 0.8 + 100p \sqrt{f_{c}'} \leq 2 \sqrt{f_{c}'} \]

**Maximum resistance**

Easy to use. Does not require much iteration from final design.

[Note: Axial loads increase capacity. Extended resistance under influence of axial loads given in tables — see codes.]  
See next page

Concrete Shear Strength

**Table 13.1**  EFFECT OF AXIAL LOAD ON THE SHEAR STRENGTH OF MEMBERS WITHOUT SHEAR REINFORCEMENT—ACI CODE

<table>
<thead>
<tr>
<th>Simplified Method</th>
<th>More Detailed Method</th>
</tr>
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</table>
| **Bending only**  | Formula (11-3), 11.3.1.1  
\( V_e = 15 \sqrt{f_{c}'} d \)  
\( \leq 2.55 \sqrt{f_{c}'} d \)  
\( V_u = \left( 1.9 \sqrt{f_{c}'} + 2500p \frac{V_d d}{M_u} \right) \frac{h_d}{d} \)  
\( \leq 3.5 \sqrt{f_{c}'} d \)  
\( V_d/M_u \) not to exceed unity |

**Bending and axial compression**  
Formula (11-4), 11.3.2.1  
\( V_e = 6 \left( 1 + \frac{N_u}{5000d} \right) \sqrt{f_{c}'} d \)  
\( N_u = \frac{4h - d}{d} \)  
Use \( N_u \) for \( M_u \) in Formula (11-4)  
\( V_d/M_u \) has no limitation  
Formula (11-5), 11.3.2.2  
\( V_e \leq 35 \sqrt{f_{c}'} d \left( 1 + \frac{N_u}{5000d} \right) \)  
\( V_u = \frac{35 \sqrt{f_{c}'} d \left( 1 + \frac{N_u}{5000d} \right)}{N_c} \)  
\( N_c \) is positive for compression and \( N/A_d \) is in psi |

**Bending and axial tension**  
11.3.1.3  
\( V_e = 6 \)  
Design shear reinforcement for total shear  
Formula (11-6), 11.3.2.3  
\( V_e = 2 \left( 1 + \frac{N_u}{5000d} \right) \sqrt{f_{c}'} d \)  
\( N_u \) is negative for tension and \( N/A_d \) is in psi

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Shear Strength of Re-bars

- Number of bars crossing diagonal crack:
  \[ n = \frac{d + \frac{d'}{\tan \alpha}}{s} \]  
  (1)

- Strength of bars crossing crack:
  \[ V_{s}^{\text{ud}} = n \cdot A \cdot f_y \]  
  \( (A_s = \text{cross sectional area of one bar}) \) (2)

Shear Strength of Re-bars

- Vertical strength of reinforcement:
  \[ V_s = V_s^{\text{ud}} \cdot \sin \alpha \]  
  (3)

- Total Shear Resistance [combining (1), (2), and (3)]
  \[ V_s = \frac{A_s \cdot f_y \cdot d}{s} \left( \sin \alpha + \cos \alpha \right) \]
**Total Shear Force Capacity**

\[ V_n = V_u = V_c + V_s \]

- Concrete strength
- Reinforcement strength

\[ V_c = f_{v,c} \cdot b \cdot d \]

\[ V_s = \frac{A_v f_y d}{s} (\sin \alpha + \cos \alpha) \]

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**Total Shear Stress Capacity**

\[ f_v = \frac{V}{bd} \]

\[ f_v = f_{v,c} + f_{v,s} \]

\[ f_{v,s} = \frac{V_s}{bd} = \frac{A_v f_y}{b \cdot s} (\sin \alpha + \cos \alpha) \]

\[ f_{v,c} \approx 0.8 + 100 \rho \sqrt{f' c} \quad \leq 2 \sqrt{f_c} \]
Shear Reinforcement (1)

(a) Stirrups (Economical: Labor – More Materials)

\[ f_{v,s} = \frac{mA_v f_y}{b \cdot s} (\sin \alpha + \cos \alpha) \]

Shear Reinforcement (2)

(b) Bent Bars (Economical: Materials – More Labor)

\[ f_{v,s} = \frac{mA_v f_y}{b \cdot s} (\sin \alpha + \cos \alpha) \]