EXPERIMENTAL VERIFICATIONS OF $H_\infty$ AND SLIDING MODE CONTROL FOR SEISMICALLY EXCITED BUILDINGS

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ABSTRACT: Recently, $H_\infty$ and sliding mode control methods have been investigated for applications to active/hybrid control of seismically excited structures. Numerical simulation results indicate that both control methods are quite promising for civil engineering applications. In the present paper, shaking table experimental tests were conducted to verify the $H_\infty$ and continuous sliding mode control methods. The test structure used was the three-story 1/4-scaled building model, equipped with an active tendon control system, which was used extensively at the State University of New York, Buffalo. Different earthquake records were used as the input excitations, including the El Centro, Pacoima, Hachinohe, and Taft. Both the full-state feedback controllers and the static output feedback controllers were used. Experimental data correlate satisfactorily with numerical simulation results. Extensive experimental results demonstrate that the $H_\infty$ and continuous sliding mode control methods, in particular the static output feedback controllers using only the measured information from a limited number of sensors installed at strategic locations without an observer, are quite promising for practical implementations of active control systems on seismically excited structures.

INTRODUCTION

Recently, advanced control theories have been investigated for applications to control of seismically excited structures. These theories include, for instance, $H_\infty$ [e.g., Spencer et al. (1993, 1994); Dyke et al. (1994, 1995)] $H_\infty$ [e.g., Schmitendorf et al. (1994a, b, c)]; Jabbari et al. (1995)], sliding mode control [e.g., Yang et al. (1993, 1994a, b, c)], and so forth. Numerical simulation results indicated that the $H_\infty$ and sliding mode control (SMC) methods are quite promising for applications to control of civil engineering structures. Likewise, applications of SMC methods to nonlinear or hysteretic civil engineering structures have been explored (Yang et al. 1994b, 1995), and shaking table tests for hybrid control of a sliding-isolated building model have been conducted. Experimental test results demonstrated that the SMC technique is very promising for nonlinear base-isolated buildings (Yang et al. 1995).

Most of the previous active/hybrid control methods for seismic protections were based on either full-state feedback (i.e., the displacement and velocity measurements of all degrees of freedom) [e.g., Soong (1991)] or observer-based output feedback, referred to as dynamic output feedback [e.g., Schmitendorf et al. (1994a); Jabbari et al. (1995); Dyke et al. (1994a)]. From the standpoint of practical implementation of active/hybrid control systems to tall building, it is not possible to install all sensors necessary to measure the full-state vector, because of a large number of degrees of freedom involved. On the other hand, an observer-based controller may require a significant amount of on-line computational effort, due again to the complexity of the structure, thus resulting in system time delays. As a result, static (or direct) output feedback control methods, using only the measured information from a limited number of sensors installed at strategic locations without an observer, are very desirable for practical implementations of control systems to civil structures.

In this paper, control strategies based on (1) the $H_\infty$ control theory presented by Schmitendorf et al. (1994a, b); and (2) the continuous sliding mode control (CSMC) presented by Yang et al. (1994b) for seismically excited structures are verified experimentally using shaking table tests. Emphasis is placed on the static output feedback control methods using only the measured responses from a few sensors for practical implementations. The test structure was the 1/4-scaled model of a three-story linear elastic building previously used by Chung et al. (1989) in full-state feedback experiments. An active tendon controller was installed in the first-story unit, as shown in Fig. 1. Satisfactory correlations were achieved between experimental data and numerical simulation results. Experimental results demonstrate that the performance of both $H_\infty$ and CSMC control strategies is quite remarkable.

FIG. 1. Three Degree of Freedom Test Structure
CONTROL METHODS

The method of CSMC and $H_s$ are described briefly in the following. Only the required equations and formulas relevant to the experimental tests are given. For detailed derivations, the reader is referred to Yang et al. (1994b) and Schmitendorf et al. (1994a, b). The equation of motion for a lumped-mass linear elastic building subjected to a one-dimensional earthquake ground acceleration $\ddot{x}_d(t)$ can be expressed as

$$MX(t) + CX(t) + KX(t) = HU(t) + \eta \ddot{x}_d(t)$$  (1)

where $X(t) = [x_1, x_2, \ldots, x_n]'$ is an $n$ vector denoting the interstory drift; a prime indicates the transpose of either a vector or a matrix; $M$, $C$, and $K$ = $(n \times n)$ mass, damping, and stiffness matrices; $U(t) = [u_1, u_2, \ldots, u_r]'$ is a $r$-control vector with $u_1$ being the control force from the ith controller; $H = (n \times r)$ matrix denoting the location of $r$ controllers; and $\eta$ = an $n$-influence vector. The rotation of each floor is assumed to be very small so that only the translational degrees of freedom are accounted for in (1). However, the influence of the rotations on the stiffness matrix $K$ is taken into account as is described later for the test structure.

In the state space, (1) can be written as

$$Z(t) = AZ + BU(t) + E \ddot{x}_d(t)$$  (2)

where $Z(t)$ is a $2n$ state vector; $A$ = a $(2n \times 2n)$ system matrix; $B$ = a $(2n \times r)$ location matrix; and $E$ = a $2n$ influence vector for the earthquake excitations

$$Z(t) = \begin{bmatrix} X(t) \\ \dot{X}(t) \end{bmatrix}; E = \begin{bmatrix} 0 \\ M^{-1} \eta \end{bmatrix}; B = \begin{bmatrix} 0 \\ M^{-1} H \end{bmatrix};$$

$$A = \begin{bmatrix} 0 & 1 \\ -M^{-1} K & -M^{-1} C \end{bmatrix}$$  (3)

Continuous Sliding Mode Control

The theory of SMC is to design controllers to drive the response trajectory into the sliding surface (or switching surface), whereas the system response on the sliding surface is stable. For a linear structure, the sliding surface $S = 0$ can be expressed as

$$S = PZ$$  (4)

where $S = [s_1, s_2, \ldots, s_r]'$ is an $r$-vector with $s_i$ being the ith sliding variable and $r$ the total number of controllers; $P$ = a $(r \times 2n)$ matrix to be determined such that the motion on the sliding surface $S = 0$ is stable. $P$ can be determined using either the method of linear quadratic regulator (LQR) or pole assignment as described in detail in [e.g., Utkin (1992); Yang et al. (1994b)].

For the design of controllers, a Lyapunov function $V = 0.5S'S$ is considered and the control vector $U$ is determined such that $V \leq 0$. Taking the derivative of $V$ and using both (4) and the state equation, (2), one obtains

$$\dot{V} = \sum_{i=1}^{r} \lambda_i(u_i - G_i)$$  (5)

where $\lambda_i$ and $G_i$ are the $i$th elements of the row vector $\lambda$ and column vector $G$, respectively, given by

$$\lambda = S'PB; \quad G = -(PB)'P(AB + E \ddot{x}_d)$$

A continuous sliding mode controller, which does not have undesirable chattering effect, can be obtained as (Yang et al. 1994b)

$$u_i = G_i - \lambda_i \delta_i$$  (7)

where $\delta_i \geq 0$ is the sliding margin. Substitution of (7) into (5) leads to $V = -\sum \lambda_i^2 \delta_i \leq 0$.

Civil engineering structures are designed to be stable without any control system. It has been shown in Yang et al. (1994b) that the following controller is stable as long as the structure without control (open-loop system) is stable

$$u_i = \alpha_i G_i - \lambda_i \delta_i$$  (8)

where $0 \leq \alpha_i \leq 1$.

The method of static output feedback using only the measurements from a limited number of sensors without an observer is described in detail in Yang et al. (1994b). For the specific test structure, Fig. 1, with one tendon controller installed in the first-story unit, we only measure the drift $x_1$ and velocity $\dot{x}_1$ of the first-story unit, and these quantities are used directly for the design of the sliding surface and controller. In this case, $r = 1$, and $S$ and $U$ are scalars, that is, $S = x_1$, and $U = u_1$. Hence, the sliding surface becomes

$$S = s = p_i x_1 + \dot{x}_1$$  (9)

where $p_i > 0$ to guarantee the stability of the sliding surface $S = 0$.

Let $Z_m$ be a $2n$ observation vector that is obtained from the state vector $Z$ by setting zeros for those state variables that are not measured: $Z_m = [x_1, 0, \ldots, 0, \dot{x}_1, 0, \ldots, 0]'$. Then, the controller can be obtained from (8) and (6) by replacing $Z$ by $Z_m$ and using (9) as follows (Yang et al. 1994b):

$$U = u_i = \alpha_i(-k_{11}x_1 - c_{11}\dot{x}_1 + m_1 \dot{x}_1 p_i) + \delta_i(p_i x_1 + \dot{x}_1)m_i$$  (10)

where $k_{11}$ and $c_{11}$ are the stiffness and damping coefficients of the first-story unit, respectively; and $m_1$ = mass of the first floor. Both $k_{11}$ and $c_{11}$ are the (1, 1) elements of the $K$ and $C$ matrices, respectively. In (10), the feedforward part is neglected for experimental convenience. By substituting (10) into (1), it can be shown easily that the controlled structure is stable. The static output feedback controller given by (10) is used in the experimental tests. The methods of static output feedback for CSMC were described in detail in Yang et al. (1994b) for linear and nonlinear structures.

$H_s$ Control Theory

For $H_s$ control, the state equation given by (2) holds, except that the state variable $x_i$ in (1) denotes the relative displacement of the $i$th floor with respect to the ground, rather than the interstory drift used in CSMC. Hence, matrices $M$, $C$, $K$, $H$, and $\eta$ in (1) should be defined appropriately. The “controlled output” defined in the following:

$$i = C_i Z$$  (11)

is used to identify response quantities that should be reduced, for example, interstory drifts and floor accelerations. In the design of full-state controllers, we are interested in control laws of the form

$$U = KZ$$  (12)

that provide desirable performance; that is, we seek controllers such that the $L_2$ gain from disturbance (i.e., $\ddot{x}_d$) to the controller output (i.e., $\bar{z}$) is less than a specified value $\gamma$. From standard results, such a controller can be calculated by finding positive definite matrices $P$ and $Q$ and a positive scalar $\epsilon$ such that the algebraic Riccati equation

$$PA + A'P + P(\gamma^2 \epsilon E^2 - \epsilon^{-1}BB)'P + C'C + Q = 0$$  (13)

is satisfied. Then, a stabilizing controller satisfying the $H_s$ performance criterion is given by
U = \ddot{\mathbf{K}}Z = -\frac{1}{2\mathbf{e}} \mathbf{B}'\mathbf{PZ} \quad (14)

To avoid the need for measurements of all of the states, recent advances in design of static output feedback \( H_s \) control laws can be used. For the static output feedback case, we seek controllers of the form

\[ \mathbf{U} = \ddot{\mathbf{K}}Y = \dddot{\mathbf{K}}C_z \quad (15) \]

that provide desirable performance bounds, where \( Y = \mathbf{C}_z \mathbf{Z} \) is the \( p \)-dimensional measured output and \( \mathbf{C}_z \) is a \( p \times 2n \) observation matrix. The approach used here follows the work of Iwasaki and Skelton (1993) and Stoustrup and Niemann (1993). First we modify the controlled output variable used in the full-state feedback case slightly to

\[ 2 = \mathbf{C}_z \mathbf{Z} + \mathbf{D}_z \mathbf{U} \quad (16) \]

where the term \( \mathbf{D}_z \mathbf{U} \) is added to provide a penalty term on excessive control force. In general, the static output feedback approach is considerably more complicated than that of the full-state feedback design. We seek a positive definite matrix \( \mathbf{P} \) that satisfies the following two matrix inequalities simultaneously:

\[ \mathbf{N}_1 = \mathbf{P}A + \mathbf{A}'\mathbf{P} + \mathbf{C}_z\mathbf{C}_z + \gamma^{-2}\mathbf{P}\mathbf{EE}'\mathbf{P} \]

\[ -\left( \mathbf{PB} + \mathbf{C}_z\mathbf{D}_z \right) \mathbf{R}^{-1} \left( \mathbf{PB} + \mathbf{C}_z\mathbf{D}_z \right)' < 0 \quad (17) \]

\[ \mathbf{V}_z\left( \mathbf{PA} + \mathbf{A}'\mathbf{P} + \mathbf{C}_z\mathbf{C}_z + \gamma^{-2}\mathbf{P}\mathbf{E}\mathbf{E}'\mathbf{P} \right) \mathbf{V}_z < 0 \quad (18) \]

where

\[ \mathbf{R} = \mathbf{D}_z\mathbf{D}_z' \quad (19) \]

In (18), the \( 2n \times (2n-q) \) matrix \( \mathbf{V}_z \) is defined as follows. Let \( \mathbf{V} \) be the eigenvector matrix of \( \mathbf{C}_z\mathbf{C}_z' \), where \( \mathbf{C}_z \) is the \( p \times 2n \) observation matrix defined previously. Then, the \( \mathbf{V} \) matrix is partitioned as \( \mathbf{V} = [\mathbf{V}_1, \mathbf{V}_2] \), where \( \mathbf{V}_1 \) is a \( 2n \times q \) matrix consisting of eigenvectors corresponding to \( q \) nonzero eigenvalues (\( \leq p \)) and \( \mathbf{V}_2 \) is a \( 2n \times (2n-q) \) matrix consisting of eigenvectors corresponding to zero eigenvalues. Both \( \mathbf{V}_1 \) and \( \mathbf{V}_2 \) can be obtained easily through the use of popular software packages, such as MATLAB and MATLAB-X.

Finding a matrix that satisfies the preceding requirements leads to a nonconvex optimization problem. For the results presented here, the MATLAB Optimization Toolbox was used to seek elements of an upper triangular matrix \( \mathbf{P} \), such that for \( \mathbf{P} = \mathbf{PP}_t \), the maximum eigenvalues of the left-hand sides of the two inequalities are negative. Once an appropriate positive definite \( \mathbf{P} \) has been found, the controller gain \( \mathbf{K} \) can be found from \( \mathbf{P} \) and the system matrices through a routine procedure described in Appendix I [see Schmitendorf et al. 1994b and Iwasaki and Skelton (1993) for details on calculating \( \mathbf{K} \)].

**Experimental Setup and Results**

Experimental verifications of \( H_s \) and CSMC were performed on the shaking table at the National Center for Earthquake Engineering Research (NCEER) at the State University of New York, Buffalo. The test structure was the 1/4-scaled model of a three-story building previously used by Chung et al. (1989) and Dyke et al. (1994) for linear structure, and Yang et al. (1995) for hybrid control of sliding-isolated building. A hydraulic actuator is connected to four prestressed tendons attached in the first-story unit. The arrangement of the controller and instrumentation were similar to those of Chung et al. (1989), as shown in Fig. 1.

Prior to experimental tests, system identifications were performed to obtain three natural frequencies and mode shapes from which the stiffness and damping matrices were determined. All the matrices appearing in the equation of motion, (1), are given as follows: \( \mathbf{H} = [1, 0, 0]' \); \( \mathbf{m} = [-m_1, m_2, m_3]' \); where \( m_i \) is the mass of the \( i \)-th floor with \( m_1 = m_2 = m_3 = 5.6 \text{ lb} \cdot \text{s}^2/\text{in} \) (981.4 kg).

\[
\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 \\ m_2 & m_0 & 0 \\ m_3 & m_0 & m_3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 2.8567 & -0.8577 & 0.1902 \\ 1.3837 & 2.4316 & -0.8502 \\ 1.5515 & 1.3613 & 2.2115 \end{bmatrix}
\]

\[
\mathbf{K} = \begin{bmatrix} 13383.2 & -8920.8 & 217.42 \\ -1893.4 & 9201.6 & -8707.4 \\ 216.9 & -1957.3 & 6750.1 \end{bmatrix}
\]

(20)

For the stiffness and damping coefficients, the units are lb/ in. (= 175.165 N/m) and lb·s/ in. (= 175.165 N·s/m), respectively. It should be mentioned that all the experimental and simulation results presented in this paper were obtained in English units, that is, mass in lb·s²/ft, length in inches and force in pounds. These results were converted later into SI units, as shown in the tables and figures. Hence, the control parameters are associated with the English units.

The structural system was tested using four earthquake excitations: (7) the S00E component of the El Centro record; (2) the N21E component of the Taft record; (3) the N-S component of the Pacoima Dam record; and (4) the E-W component of the Hachinohe record. These four excitations were scaled to peak ground accelerations of 0.089g, 0.0831g, 0.0736g, and 0.1021g, respectively. The same phase compensation technique used in Chung et al. (1989) was adopted to compensate the system time delay.

Although many different designs for \( H_s \) and CSMC were implemented and tested, only the test results for three designs for each control method are presented in Tables 1–4, corresponding to different seismic excitations. The values presented in these tables are the maximum peak recordings for each of the experiments, where \( x_i \) is the interstory drift in mm, \( f_0 \) is the floor acceleration in g, and \( u \) is the horizontal control force in terms of the percentage of the building weight. The peak response quantities without control are shown in column (2) of the tables. Also shown in the parentheses of Tables 1–4 are the percent reductions of response quantities relative to the uncontrolled case in column (2). Finally, the simulated peak control forces are shown in the brackets in the last rows of Tables 1–4. These simulated values are discussed later.

The performance of the control laws obtained by the \( H_s \) techniques are presented in cases A, B, and C, columns (3)–(5), with case A corresponding to a full-state feedback control law and cases B and C corresponding to two static output

**TABLE 1. Experimental Peak Responses to 0.089g (PGA) El Centro Earthquake**

<table>
<thead>
<tr>
<th>Response</th>
<th>No control (2)</th>
<th>( H_s )</th>
<th>CSMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_i ) (mm)</td>
<td>5.055</td>
<td>1.219</td>
<td>1.168</td>
</tr>
<tr>
<td>( f_0 )</td>
<td>7.188</td>
<td>3.150</td>
<td>2.819</td>
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<tr>
<td>( u ) (%)</td>
<td>5.613</td>
<td>2.921</td>
<td>2.565</td>
</tr>
<tr>
<td>( x_{sa} ) (g)</td>
<td>0.253</td>
<td>0.131</td>
<td>0.072</td>
</tr>
<tr>
<td>( f_{sa} )</td>
<td>0.358</td>
<td>0.145</td>
<td>0.148</td>
</tr>
<tr>
<td>( x_{sa} ) (g)</td>
<td>0.449</td>
<td>0.194</td>
<td>0.213</td>
</tr>
<tr>
<td>( f_{sa} )</td>
<td>0.949</td>
<td>0.842</td>
<td>0.714</td>
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</table>

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Table 2. Experimental Peak Responses to 0.0831g (PGA) Taft Earthquake

<table>
<thead>
<tr>
<th>Response (1)</th>
<th>No control</th>
<th>A (2)</th>
<th>B (3)</th>
<th>C (4)</th>
<th>D (5)</th>
<th>E (6)</th>
<th>F (7)</th>
<th>CSMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1 (mm)</td>
<td>4.369</td>
<td>0.889</td>
<td>1.067</td>
<td>0.965</td>
<td>1.092</td>
<td>1.295</td>
<td>1.956</td>
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<tr>
<td>x2 (mm)</td>
<td>6.731</td>
<td>2.972</td>
<td>3.297</td>
<td>2.632</td>
<td>2.903</td>
<td>1.803</td>
<td>1.499</td>
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<tr>
<td>x3 (mm)</td>
<td>5.415</td>
<td>2.656</td>
<td>2.540</td>
<td>2.219</td>
<td>2.311</td>
<td>1.499</td>
<td>1.041</td>
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<tr>
<td>x4 (g)</td>
<td>0.236</td>
<td>0.146</td>
<td>0.080</td>
<td>0.074</td>
<td>0.079</td>
<td>0.064</td>
<td>0.060</td>
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<tr>
<td>x5 (g)</td>
<td>0.291</td>
<td>0.199</td>
<td>0.155</td>
<td>0.140</td>
<td>0.156</td>
<td>0.111</td>
<td>0.078</td>
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<tr>
<td>x6 (g)</td>
<td>0.414</td>
<td>0.210</td>
<td>0.206</td>
<td>0.173</td>
<td>0.185</td>
<td>0.116</td>
<td>0.086</td>
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<td>u (%)</td>
<td>—</td>
<td>10.45</td>
<td>10.59</td>
<td>8.97</td>
<td>9.81</td>
<td>6.79</td>
<td>10.13</td>
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Table 3. Experimental Peak Responses to 0.0736g (PGA) Pa-coima Earthquake

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<th>Response (1)</th>
<th>No control</th>
<th>A (2)</th>
<th>B (3)</th>
<th>C (4)</th>
<th>D (5)</th>
<th>E (6)</th>
<th>F (7)</th>
<th>CSMC</th>
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<tr>
<td>x1 (mm)</td>
<td>2.591</td>
<td>0.610</td>
<td>0.553</td>
<td>0.559</td>
<td>0.635</td>
<td>0.914</td>
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<td>x2 (mm)</td>
<td>3.937</td>
<td>1.626</td>
<td>1.270</td>
<td>1.219</td>
<td>1.118</td>
<td>1.168</td>
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<tr>
<td>x3 (mm)</td>
<td>3.327</td>
<td>1.676</td>
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<td>1.092</td>
<td>1.067</td>
<td>1.143</td>
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<tr>
<td>x4 (g)</td>
<td>0.185</td>
<td>0.124</td>
<td>0.065</td>
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<td>x5 (g)</td>
<td>0.198</td>
<td>0.118</td>
<td>0.063</td>
<td>0.070</td>
<td>0.057</td>
<td>0.052</td>
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<tr>
<td>x6 (g)</td>
<td>0.272</td>
<td>0.159</td>
<td>0.087</td>
<td>0.095</td>
<td>0.089</td>
<td>0.099</td>
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<tr>
<td>u (%)</td>
<td>—</td>
<td>5.53</td>
<td>4.14</td>
<td>4.06</td>
<td>4.33</td>
<td>4.90</td>
<td>6.82</td>
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Table 4. Experimental Peak Responses to 0.1021g (PGA) Hachinohe Earthquake

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<th>Response (1)</th>
<th>No control</th>
<th>A (2)</th>
<th>B (3)</th>
<th>C (4)</th>
<th>D (5)</th>
<th>E (6)</th>
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<td>x1 (mm)</td>
<td>4.699</td>
<td>1.854</td>
<td>1.854</td>
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<td>2.057</td>
<td>2.337</td>
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<tr>
<td>x2 (mm)</td>
<td>6.004</td>
<td>2.921</td>
<td>3.023</td>
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<td>3.124</td>
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<tr>
<td>x3 (mm)</td>
<td>4.547</td>
<td>2.565</td>
<td>2.311</td>
<td>2.159</td>
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<td>1.753</td>
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<td>x4 (g)</td>
<td>0.204</td>
<td>0.190</td>
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<td>x5 (g)</td>
<td>0.288</td>
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<td>0.166</td>
<td>0.169</td>
<td>0.130</td>
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<tr>
<td>x6 (g)</td>
<td>0.337</td>
<td>0.239</td>
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<td>0.180</td>
<td>0.137</td>
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<tr>
<td>u (%)</td>
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<td>11.98</td>
<td>11.42</td>
<td>11.26</td>
<td>10.59</td>
<td>11.88</td>
<td>13.38</td>
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Typically, the control gain K, (14), can be replaced by βK, for a range of β, without affecting the stability of the closed-loop system and/or the guaranteed performance bound y. In other words, if K is a stabilizing control, then so is βK for any β > 0 and β is a tuning parameter chosen by the designer (see Schmitendorf et al. [1994a] for detail discussion). The results in case A correspond to β = 1.6, that is, U = βK. Alternatively, the controller for case A could be obtained directly from (13) and (14), with appropriate choices of ε and Q. For the sake of simplicity, details are omitted.

Cases B and C correspond to static output feedback controllers. For both cases, γ = 0.5 and

\[ C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 \\ 10^{-7} \end{bmatrix} \]

(22)

The only measurements are the displacement and velocity of the first floor (with respect to the ground)

\[ C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \]

(23)

In case B, through the use of different initialization in the optimization subroutine, the following P matrix was obtained

\[ P = 215.0 \begin{bmatrix} 2.703 & 0.239 & 2.317 & 2.696 & 2.699 \\ 2.703 & 87.78 & 2.900 & 0.063 & 0.046 & 0.015 \end{bmatrix} \]

\[ 0.239 & -2.900 & 86.34 & 0.037 & 0.070 & 0.079 \\ 2.317 & 0.063 & 0.037 & 0.044 & 0.052 & 0.052 \\ 2.696 & 0.046 & 0.070 & 0.052 & 0.135 & 0.155 \\ 2.699 & 0.015 & 0.079 & 0.052 & 0.155 & 0.255 \end{bmatrix} \]

(24)

which in turn results in \[ \hat{K} = [156.65 \quad 291.90] \].

Case C corresponds to calculations that were performed similar to case B for the scaled model without pretension tendons. The structural properties of the model used are approximately 20% different from those of the actual model.

This controller was used to test the robustness of the controller to parametric uncertainty. The controller gain obtained for this case was \[ \hat{K} = [-4.284 \quad 286.86] \].

For CSMC, the static output feedback control technique, (9) and (10), was used, in which only \( x_1 \) and \( \dot{x}_1 \) were measured. The ground acceleration \( x_0 \) was not used and the sliding margin was \( \delta_1 = 0 \). The parameters used are as follows: (1) \( \alpha_1 = 0.0167 \) and \( p_1 = 3.000 \) for case A in column (6) of Tables 1 and 4; (2) \( \alpha_1 = 0.167 \) and \( p_1 = 300 \) for case B in column (7) of Tables 1 and 4; and (3) \( \alpha_1 = 0.333 \) and \( p_1 = 150 \) for case C in column (8) of Tables 1 and 4.

With the El Centro Earthquake input, time histories of the interstory drifts are presented in Figs. 2–4. In these figures, the dashed curve denotes the uncontrolled case in column (2) of Table 1, whereas the solid curve indicates the controlled response. Fig. 2 displays the results of case A in Table 1 for \( H_c \) (full-state feedback), whereas Fig. 3 shows the results of case B in Table 1 for \( H_e \) (static output feedback). Fig. 4 presents the result of case A in Table 1 for CSMC (static output feedback).

Tables 1–4 and Figs. 2–4 demonstrate that the performance of both \( H_c \) and CSMC is quite impressive. The \( H_c \) and CSMC achieved 47–79% and 44–79% reductions, respectively, for peak interstory drifts. In general, CSMC performed slightly better than \( H_c \) for the acceleration reduction; of course the trend may be different should different designs be tested. However, the reductions for floor acceleration responses were also quite remarkable.

For every test series conducted, numerical simulations of the response quantities were performed. Time histories for
the interstory drifts under the El Centro earthquake based on numerical simulations are presented as dashed curves in Figs. 5–7, and the experimental records are shown by solid curves for comparison. Figs. 5 and 6 show the results of cases A and B for $H_c$, respectively, and Fig. 7 presents the results of case A for CSMC. In other words, solid curves in Figs. 5–7 are identical to the corresponding solid curves in Figs. 2–4. Finally, the peak response quantities in 20 s of the earthquake episode are summarized in Table 5 for comparison between the experimental records and simulation results.

As observed from Fig. 5, experimental records for the time histories of the response quantities track the simulated time histories very well for the full-state feedback controller. For the static output feedback controllers (for both $H_c$ and CSMC), deviations for some peak values were observed in Figs. 6 and 7. However, the frequencies of the response time histories for the experimental records correlate very well with those of the simulated results, as shown by Figs. 6 and 7.

Because actuator dynamics and actuator-structure interaction are not accounted for in the simulation, differences between experimental data and simulation results are expected. In general, the correlations for all test series are quite satisfactory for the peak response quantities, as typically shown by Table 5, except peak control forces $u(\%)$ indicated in the
<table>
<thead>
<tr>
<th></th>
<th>H₂</th>
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<tbody>
<tr>
<td></td>
<td>A</td>
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<td>1.219</td>
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<td>1.626</td>
<td>1.219</td>
<td>1.600</td>
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<td>1.651</td>
<td>1.524</td>
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<td>1.930</td>
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<td>2.819</td>
<td>2.870</td>
<td>2.616</td>
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<td>2.565</td>
<td>2.794</td>
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<td>0.072</td>
<td>0.082</td>
<td>0.075</td>
<td>0.078</td>
<td>0.073</td>
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<td>0.071</td>
<td>0.078</td>
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<tr>
<td>x₅ (g)</td>
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<td>0.134</td>
<td>0.148</td>
<td>0.131</td>
<td>0.140</td>
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<td>0.195</td>
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<td>0.196</td>
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<tr>
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<td>5.96</td>
<td>7.03</td>
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</table>

Simulated peak control forces are also shown in brackets in the bottom row of Tables 1-4. As observed from Tables 1-5, mismatches between the calculated and measured peak control forces are noticeable. The mismatch appeared only on several short-duration peaks in the measured data. This can be attributed to effects of actuator’s inertia at large force rates (not considered in the simulations). However, since the “overshoots” had a very short duration, these did not affect the measured performance of the controller when compared to the simulated one.

CONCLUSION

Active control methods based on $H_2$ and CSMC theories were successfully implemented and verified on a multi-degree-of-freedom scaled building model at NCEER. In particular, the proposed static output feedback controllers, which can be used for practical implementations of control systems on full-scale buildings, were experimentally demonstrated to have outstanding performance. Depending on the particular design of controllers, the reduction percentage for the peak response quantities can be as high as 79%. For the peak response quantities, simulation results correlated reasonably well with experimental data. The correlation for the response time histories can be expected to improve if the actuator dynamics and actuator-structure interaction are taken into account in the simulations [e.g., Dyke et al. (1995)].

Different static output feedback controllers can be designed for $H_2$ and CSMC methods, and their robustness was demonstrated by extensive simulation results [e.g., Schmitendorf et al. (1994c); Yang et al. (1994b)]. Hence, these control strategies are very promising for practical applications to full-scale buildings. For control of linear civil engineering structures, another well-known static output feedback controller is based on the classical LQR approach. However, all the static output controllers are not aiming at the minimization of the maximum (or peak) response, which is perhaps the most important performance criterion. As a result, it is highly desirable to explore as many feasible static output controllers as possible. The static output controllers based on $H_2$ and CSMC, as verified in this paper, provide valuable alternatives to that of LQR for practical applications.

ACKNOWLEDGMENTS

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APPENDIX I. DETERMINATION OF GAIN MATRIX $\mathbf{K}$

Once a $P$ matrix satisfying both (17) and (18) is determined, the gain matrix $\mathbf{K}$ can be obtained through the following procedure [as discussed in Schmitendorf et al. (1994b) and Iwasaki and Skelton (1993)]. The gain matrix used in this paper is given by

$$\mathbf{K} = [\mathbf{LGR}^{-\frac{1}{2}} - (\mathbf{PB} + \mathbf{C} \mathbf{D}_i \mathbf{R}^{-\frac{1}{2}})] \mathbf{C}_2$$

where $\mathbf{R} = \mathbf{D}_i \mathbf{D}_i$ is defined in (19) and $\mathbf{R}^{-\frac{1}{2}}$ denotes the inverse of the square root of $\mathbf{R}$. In the present paper, $\mathbf{R}$ is a diagonal matrix. For such a diagonal matrix, $\mathbf{R}^{-\frac{1}{2}}$ is also a diagonal matrix whose entries are the square roots of the corresponding entries of $\mathbf{R}$. Matrix $\mathbf{C}_2$ is the Moore-Penrose (pseudo) inverse of $\mathbf{C}_2$ calculated as

$$\mathbf{C}_2 = \mathbf{V}_2 \mathbf{S}^{-1} \mathbf{W}_1$$

where $\mathbf{V}_1 = 2n \times q$ matrix consisting of eigenvectors of $\mathbf{C}_2 \mathbf{C}_2^T$ corresponding to nonzero eigenvalues and $\mathbf{W}_1$ is the $p \times q$ matrix consisting of those eigenvectors of $\mathbf{C}_2 \mathbf{C}_2^T$ that correspond to nonzero eigenvalues. Recall that $\mathbf{C}_2 \mathbf{C}_2^T$ and $\mathbf{C}_2 \mathbf{C}_2^T$ have the same nonzero eigenvalues. The diagonal $q \times q$ matrix $\mathbf{S}$ has the square root of these nonzero eigenvalues as its diagonal entries.

Matrices $\mathbf{L}$ and $\mathbf{G}$ in (25) are computed as follows. First, $\mathbf{V}^T \mathbf{N} \mathbf{V}$ is partitioned as

$$\mathbf{V}^T \mathbf{N} = \begin{bmatrix} \mathbf{V}_1^T & \mathbf{V}_2^T \end{bmatrix} \begin{bmatrix} \mathbf{N}_1 & \mathbf{N}_{12} \\ \mathbf{N}_{12}^T & \mathbf{N}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q}_{12} \\ \mathbf{Q}_{12}^T & \mathbf{Q}_2 \end{bmatrix}$$

in which $\mathbf{N}_1$ is defined in (18) and $\mathbf{V} = \begin{bmatrix} \mathbf{V}_1, \mathbf{V}_2 \end{bmatrix}$ has been described in the text. The dimension of $\mathbf{Q}_{11}$ is $q \times q$ while $\mathbf{Q}_{22}$ is $(2n-q) \times (2n-q)$. Next, we define matrices $\mathbf{T}$ and $\mathbf{T}'$, with dimensions $(2n-q) \times r$ and $r \times r$, respectively, where $r$ is the total number of controllers.

$$\mathbf{T} = \mathbf{V}_1 (\mathbf{PB} + \mathbf{C}_2 \mathbf{D}_i \mathbf{R}^{-\frac{1}{2}})$$

$$\mathbf{T}' = \mathbf{Q}_{12} (\mathbf{D}_i + \mathbf{T}^{-1} (\mathbf{Q}_{22} + \mathbf{T}^{-1} \mathbf{T}))^{-1}$$

The $2n \times r$ matrix $\mathbf{L}$ in (25) is given by

$$\mathbf{L} = \mathbf{V} \begin{bmatrix} \mathbf{T} \\ \mathbf{T}' \end{bmatrix}$$

Finally, the $(r \times r)$ matrix $\mathbf{G}$ in (25) is given by

$$\mathbf{G} = \mathbf{V}_a \mathbf{V}_b^T$$

where $\mathbf{V}_a$ is the eigenvector matrix for the $r \times r$ matrix $\mathbf{L}' \mathbf{V}_2 \mathbf{V}_2' \mathbf{L}$ and $\mathbf{V}_b$ is the eigenvector matrix for the $r \times r$ matrix $\mathbf{T} \mathbf{T}'$.

APPENDIX II. REFERENCES


