Design of Amplified Structural Damping Using Optimal Considerations

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Abstract: A method is suggested to reduce the structural seismic response using viscous dampers with their effect magnified with mechanical levers (lever arms). The lever arms are used to magnify the drift and the drift velocities transferred from the structure to the dampers thus producing larger energy dissipation in smaller devices. The increase of energy dissipation in each unit results in a reduction of the number of units necessary to achieve the same reduction in structural response, or it results in a reduction of the size of dampers required for the same purpose. The use of the proposed technique permits one to reduce the number of frame bays obstructed by diagonal or Chevron bracing elements. An optimal design procedure, based on the linear quadratic regulator (LQR) technique, is proposed in this study to obtain the initial properties of the viscous dampers. The study presents the amplification and the contribution of the construction details to such amplification. A seven-story structure model with the proposed system was simulated numerically in order to check the damping efficiency. The numerical example shows that using the lever arms for dampers’ connection, it yields significant reduction in the structural response, very close to that designed according to the LQR strategy.

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Introduction

Structures designed for seismic regions should be able to absorb and dissipate a large amount of seismic energy in order to maintain the response within acceptable limits. In current practice it is expected that the dissipation of energy occurs in structural elements experiencing inelastic behavior, with associated structural damage. Recent efforts were dedicated to finding methods to enhance the building energy absorbing capacity while avoiding damage in the structural components. Extensive information on such methods is available in state-of-the-art publications (Hanson et al. 1993; Soong and Dargush 1997; Constantinou et al. 1998).

Adding fluid viscous dampers in structures is becoming a well-accepted practice in design or retrofit of earthquake resistant structures. Constantinou and Symans (1992) and Reinhorn et al. (1995) studied analytically and experimentally the influence of supplemental fluid viscous dampers on structural seismic response of elastic and inelastic structures. They concluded that damping is beneficial to reduce deformations and, in elastic structures, reduce also forces and accelerations. Aiken and Kelly (1996) presented an overview of an extensive dynamic testing program of fluid viscous dampers that was conducted as part of the seismic retrofit of the Golden Gate Bridge. Whittaker and Constantinou (2000) discussed new procedures for the design of buildings incorporating fluid viscous dampers and demonstrated that addition of viscous dampers reduces the spectral acceleration and consequently the target displacement in elastic systems.

Gluck et al. (1996) suggested and adapted the optimal control theory using a linear quadratic regulator (LQR) to design of linear passive viscous and viscoelastic devices dependent on their deformation and velocity. This methodology can be used for braced structures, which are implemented in new construction, or in rehabilitation and retrofit.

Amplification of Damping

It is generally recognized that rigid structures have small deformations during a seismic event. These small deflections and the associated velocities, also small, may render the damping devices ineffective. Amplifying the deformations and velocities in the damping devices, it can increase the efficiency of such damping devices. Taylor (2000) and Constantinou et al. (2001) suggested and investigated the combination of fluid dampers with a mechanical toggle brace assembly magnifying the small structural displacements to obtain more efficient damping. It was shown through simulated earthquake testing, analytical studies, and simplified design procedures, that the toggle-brace damper amplifies damping force and increase energy dissipation. An alternative amplifying technique using a “scissor-jack” configuration was also developed recently (Constantinou et al. 2000). It was demonstrated that the scissor-jack configuration magnifies the damper displacement and reduces the required damper force while still producing the desired damping effect.

Gluck (1996) proposed the use of levers to change the damping characteristics in a structure while increasing their efficiency by magnifying the drifts and drift velocities that are transferred...
from the structure to dampers. Ribakov et al. (2000) developed a procedure for design of passively controlled structures using optimal control and viscous dampers with the dampers connected to the structure through lever arms. The “equivalent” lever arm approach was then used to change the effect of standard, off-the-shelf devices in order to achieve an optimal configuration. Other amplifying configurations were proposed for connecting semi-active and active dampers in order to reduce the control forces and the energy required for control (Gluck and Ribakov 2000; Ribakov 2000).

The amplification of damping suggested herein consists of a lever arm system shown in Fig. 1. It consists of a Chevron brace (1), a lever arm (2) connected by means of hinges (3a) and (3b) to the story diaphragm (4), and the Chevron brace, respectively. A plate (5) is used to connect the two elements of the Chevron brace and the lever arm. When the structure develops a horizontal drift at the top story (4), the lever arm turns around hinge (3b), hence a magnified displacement drift and drift velocity are transferred to the damper at the bottom of the lever.

However, the efficiency of amplifiers can be substantially reduced due to the deformations of connecting elements. The objective of this study is to develop a design procedure for structures with amplifiers of damping using lever arms while compensating for the bending deformations of structural levers. A numerical example demonstrates the proposed design technique and shows its efficiency.

**Optimal Design of Dampers**

In order to determine the required damping and its distribution through the structure an optimal control strategy is adopted. In current practice dampers are placed at floor level of the structure (Fig. 2) and are connected between the floor diaphragms with a Chevron [Fig. 2(b)] or with diagonal braces [Fig. 2(c)]. The optimal dampers’ properties can be determined by applying a “passive” control strategy based on the active control method of LQR (Gluck et al. 1996), aimed to obtain the viscous or viscoelastic damping coefficients of the physical dampers. The optimal design method of Gluck et al. (1996) is adopted in this paper and it is summarized for sake of completion. An alternative control design technique can be used; however, the LQR technique offers a suitable solution and a way of integrating more complex techniques in the design.

The response of a structure provided with supplemental dissipating devices can be described by the following dynamic equation of equilibrium (Soong 1990):

\[
M \ddot{u}(t) + C \dot{u}(t) + Ku(t) = Lf_c(t) + Df_c(t)
\]

where \(M\), \(C\), and \(K\) = mass, damping, and stiffness matrices, respectively; \(u(t)\), \(\dot{u}(t)\), and \(\ddot{u}(t)\) = displacement, velocity, and acceleration vectors, respectively; \(f_c\) = vector of forces in the

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**Fig. 1.** Lever arm: (a) installation and (b), (c) connection details

**Fig. 2.** Frame structure (a) uncontrolled, (b) with viscous dampers connected to Chevron braces, (c) with viscous dampers connected to diagonal braces
supplemental devices; \( f_e \) = external excitation vector; and \( D \) and \( L \) = control and excitation forces-location matrices, respectively.

The system of second order differential Eq. (1) may be simplified by a transformation into a state space form

\[
\dot{z}(t) = Az(t) + Bf_e(t) + Hf_s(t)
\]

where \( z(t) = [\ddot{u}(t), \dot{\ddot{u}}(t)]^T = \) vector of the displacements and velocities at each degree of freedom of the structure; \( A = \) system’s matrix; \( B \) and \( D \) = location matrices specifying, respectively, the locations of dampers and external excitations

\[
A_{2n \times 2n} = \begin{bmatrix} 0 & 1 \\ -M^{-1}K & -M^{-1}C \end{bmatrix}
\]

\[
B_{2n \times m} = \begin{bmatrix} 0 \\ M^{-1}D \end{bmatrix}
\]

\[
H_{2n \times r} = \begin{bmatrix} 0 \\ M^{-1}L \end{bmatrix}
\]

For the case of linear control forces they can be written as follows:

\[
f_e(t) = Gz(t) = [G_u; G_\dot{u}]z(t) = G_u\ddot{u}(t) + G_\dot{u}\dot{\ddot{u}}(t)
\]

where \( G = \) gain matrix. Using Eq. (4), the equation of motion Eq. (2) reduces to

\[
\dot{z}(t) = Az(t) + Hf_s(t)
\]

where the matrix of the controlled system, \( A_c = A + BG \). \( G = \) gain matrix, which can be determined from the minimization of the performance index, defined as

\[
J = \int_0^{t_f} [z^T(t)Qz(t) + f_e^T(t)RF_e(t)]dt
\]

where \( t_f = \) total time of the considered dynamic event; \( Q_{2n \times 2n} = \) positive semidefinite matrix; and \( R_{m \times m} = \) positive definite matrix. \( Q \) and \( R \) = weighting matrices, representing the relative importance of the state variables and of the control forces in the minimization procedure (Soong 1990). These matrices can be expressed in a parametric form (Gluck et al. 1996) using a diagonal formulation

\[
R = 10^{-p}I_{n \times n}; \quad Q = 10^{-p}I_{2n \times 2n}
\]

where \( I \) = unit diagonal matrix; and \( p \) = parameter, which is used to adjust the solution’s weights to lie in the practical range. An alternative formulation of \( R \) can be a tridiagonal matrix with positive units on the diagonal and negative units on the off-diagonal. Such matrix may lead to a more economical scheme of dampers.

The gain matrix \( G \) takes the following form:

\[
G = -0.5R^{-1}BTP
\]

where \( P = \) matrix solution of the Ricatti algebraic equation (assuming in the optimization that \( P \) is time invariant and independent of the excitation, see Soong (1990))

\[
A^TP + PA - 0.5PBR^{-1}B^TP + 2Q = 0
\]

The solution is readily available from mathematical libraries such as MATLAB (1993). Although the above solution is optimal only for white noise excitations, it was demonstrated that for practical purposes the solution is close to optimal for earthquake excitations (Soong 1990).

The gain matrix \( G \) contains the requirements for control coefficients. Some of the coefficients can be implemented in passive devices, while others cannot. Procedures to extract properties of passive dampers through: (1) spectral approach, through a (2) dominant mode approach, and through a (3) truncation approach were suggested by Gluck et al. (1996). Using the dominant mode approach, the properties of the damping devices, i.e., the stiffness and viscous damping properties, can be approximately assumed independent, and in a structure with dominant mode, \( i \), the damping coefficients of the supplemental devices can be obtained as follows:

\[
\Delta c_i = \sum_j g_{ij,d} \Phi_{pm} / \Phi_{im}
\]

where \( \Phi_{im} \) = element of the “drift” eigenvector corresponding to mode \( m \) and degree of freedom \( i \) and \( g_{ij,d} \) = gain coefficients in the terms of drift velocities. According to Eq. (10), the damping coefficients are independent of the earthquake history, and dependent only on the characteristics of the structure. The same method yields requirements for stiffness enhancements, which can be determined by a relation similar to Eq. (10) replacing the \( \Delta c \) with \( \Delta k \) and the gain coefficients in terms of displacement shifts (Gluck et al. 1996). Gluck et al. (1996) showed also that the “dominant mode approach” is as efficient as the spectral approach and almost as efficient as a pure actively controlled system with the same objectives.

Amplification of Damping Using Levers—Design Procedure

For the suggested system shown in Fig. 1, the values of the displacement and of the velocity transferred to the damper can be determined by

\[
d_{d,i} = AR_i d_i, \quad \dot{d}_{d,i} = AR_i \dot{d}_i
\]

where \( AR_i = \) effective amplifying ratio at the \( i \)th floor; \( d_i, \) and \( \dot{d}_i \) = drift and the drift velocity at the \( i \)th floor; and \( d_{d,i} \) and \( \dot{d}_{d,i} \) = displacement and the velocity that are transferred to the damper at the \( i \)th floor.

Increasing the drift displacements and velocities in the damper results in increased damping forces and dissipated energy, making the system more effective. It can be shown that the effective interstory damping force is proportional to the square of the amplification factor (AR) (Constantinou et al. 2001). This increased efficiency enables one to reduce the number of damping devices and eliminate the obstructions in many open frames, or use the same number of damping units but smaller in size.

The effectiveness of the lever arm system increases for larger amplifying ratios, which depends on the geometry of the system and on the stiffness of the lever arm. The deflection of the lever arm decreases the amplifying ratio and the effectiveness of the system. A simple technique for selection of dampers is developed herein. When the desired amplification was determined from other considerations, the height of the Chevron brace \( I_2 \) (Fig. 3) can be obtained as

\[
l_2 = H_{d}AL / (1 + AL)
\]

where \( H_{d} = \) story height; and \( AL = \) amplifying lever ratio for the case of a rigid lever arm

\[
AL = l_2 / l_1
\]
\[
\Delta = \frac{F_d l_2^2}{3EI_{LA}}(l_1 + l_2) = \frac{F_d l_1^2}{3EI_{LA}}H_n
\]

where \(F_d\) and \(I_{LA}\) = damping force and the lever arm inertia moment, respectively. The effect of the mass of the lever arm is neglected here for sake of simplicity.

The deflection of the lever arm, \(\Delta\), should be limited by an allowable value, \(\Delta_{all}\), say a proportion of the whole drift:

\[
\Delta \leq \Delta_{all} = k \cdot d
\]

where \(k = \) factor smaller than 1.

For use of linear viscous dampers, the damping force can be calculated as

\[
F_d = C_d d \cdot AR
\]

where \(C_d\) = viscous damping coefficient of the device; and \(AR\) = effective amplifying ratio of the lever arm system considering the losses due to bending and other effects. It takes the form

\[
AR = \frac{AL \cdot d - \Delta}{d} = AL - k
\]

\[K = \begin{bmatrix}
29.28 & -14.64 & -14.64 & -14.64 & -14.64 & -14.64 & 0 \\
-14.64 & 31.59 & -16.95 & 30.96 & -14.01 & 28.02 & -14.01 \\
-16.95 & 30.96 & -14.01 & 28.02 & 25.13 & 11.12 & -14.01 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \times 10^7 \text{ (N/m)}
\]

where \(M\) and \(K\) = mass and the stiffness matrices of the structure. An initial damping ratio of 2\% was assumed for the first vibration mode of the uncontrolled structure.

The following four seismic excitations were used as input in order to examine the structural behavior: Kobe (1995), Loma Prieta (1989), Northridge (1994), and Tabas (1978).

First the response of an uncontrolled structure was obtained. Then the optimal control theory (Gluck et al. 1996) was used in order to obtain the viscous damping coefficients of the dampers connected to the Chevron braces. It yields the following characteristics of dampers

\[
\Delta C_k = \begin{bmatrix}
17.2 & 17.2 & 19.9 \\
16.5 & 16.5 & 13.1 \\
0 & 0 & 0
\end{bmatrix} \frac{kN \times \text{sec}}{\text{cm}}
\]

The results of the optimal solution were truncated to the practical values of viscous coefficients of 4.0 \(kN \times \text{sec/cm}\) per unit. The number of units used at each floor of the structure was assumed as follows:

\[
N_{0_{\text{units}}} = \begin{bmatrix}
4 & 4 & 4 \\
5 & 4 & 4 \\
3 & 3 & 3
\end{bmatrix}
\]

The effect of the deflection on the lever arm system effectiveness was then studied for the flexible lever arm (\(I_{LA}\))
The results of the study are presented in Fig. 4. Numerical simulation shows that using larger amplifying ratios may yield improved structural response like in the case of the Kobe and Loma Prieta earthquakes [see Figs. 4(a and b)]. However, the effect of the flexibility of the arm may cancel the effects of amplification, such as for Northridge and Tabas earthquakes [see Figs. 4(c and d)].

A structure with amplified damping systems (using lever arms) was designed according to the proposed technique. It was assumed that only one damping device with lever arm system is placed at each floor. The design was performed for AL = 6 and $k = 1$ [Eq. (18)], yielding the lever arms inertia moment $I_{LA} \geq 1,230,367$ cm$^4$.

The response of the structure with the lever arms with the above properties was simulated in order to examine the proposed design technique, using subroutines written in MATLAB (MATLAB 1993).

Fig. 5 presents the maximum displacements in the structure with different control systems. It shows that implementing the optimally designed viscous dampers connected to Chevron braces yields a reduction of 40–75% in the maximal displacements, compared to the uncontrolled structure. It was no significant change in the base shear forces of the structure. The number of dampers required varies from three to five at each story [see Eq. (22)].

Same effect and reduction can be achieved by connecting the dampers through the lever arm systems designed according the proposed technique. The number of dampers, however, would be a single unit at each floor thus leaving most of the story bays free of structural obstructions. Such system should be attractive to either new construction or retrofit.

Conclusions

The paper presents a study aimed to improve the damping efficiency in storied buildings subjected to earthquakes by connecting viscous dampers to structure through lever systems. A procedure for design of the structural damping system is proposed. The design method is based on an optimal strategy used in actively controlled systems, and it considers the efficiency losses from the physical implementation of devices and systems.

A numerical simulation of a seven-story steel structure with the proposed system was carried out. Implementation of the proposed system improves the behavior of the structure during seismic excitations. Reduction in peak displacements ranging between 40 and 75% compared to the uncontrolled structure is obtained with proposed system. This is obtained with no significant change in the base shear forces of the structure.

Connecting the viscous dampers to the lever arm systems yields much higher energy dissipation in the structural system with smaller devices. If larger dampers are used then less damping units are required in order to achieve enhanced structural behavior, minimizing the frame bay obstructions and leaving more flexibility for architectural design of the structure. Moreover, the large damping without stiffness changes in the structure may be extremely valuable for retrofit of existing structures. Such solution may not require changes in the load path system or changes in the structure’s foundations.

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References


Fig. 5. Maximum structure displacements with different damping systems