Optimal Design of Supplemental Dampers for Control of Structures

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ABSTRACT: A suggested method for design of supplemental dampers in multistory structures is presented. Active optimal control theory is adapted to design linear passive viscous or viscoelastic devices dependent on their deformation and velocity (best represented by Kelvin model). The theory using a linear quadratic regulator (LQR) is used to exemplify the procedure. The design is aimed at minimizing a performance cost function, which produces a most suitable minimal configuration of devices while maximizing their effect. The method is fully effective using full-state static feedback. Since the active feedback action require a linear combination of all states and passive devices cannot supply it, the paper introduces a methodology to eliminate the off-diagonal interactions between states using various engineering ways. The paper shows the development for velocity feedback only, for the sake of simplicity. However, the full-state formulation can be manipulated similarly to obtain a combined position-velocity feedback design. The paper shows a numerical implementation of the design methodology for a structural model prepared for further experimental considerations.

1. INTRODUCTION

Active control theory provides a suitable framework for design of control systems in which forces are introduced in structures to reduce the unwanted effects of vibrations. The control theories assume that each force-generating device has the capability to process information from all observable sensors simultaneously and generate compatible forces. This control can be obtained using either active or semi-active operating systems. Passive devices (Hanson et al., 1993; Constantinou et al., 1994a) produce forces depending either on their elongation or internal velocity, or both, dictated by the structure movement. The parameters that govern such behavior are fixed by design. For example, viscoelastic type damping devices develop forces, which can be approximated by:

\[ F_d = k_d x(t) + c_d \dot{x}(t) \]  

in which \( k_d \) and \( c_d \) are constant parameters for unique frequency input. The optimal linear control approach is used in this paper to determine the constant coefficients for the damping devices. Inaudi et al., (1993) used a stochastic linearization along with a similar optimization procedure to determine initial design values for damping devices. The design process developed in this paper is deterministic and can be used for structures using damping braces, which can be easily implemented in new construction or in rehabilitation and retrofit (Hanson et al., 1993; Fierro et al., 1993; Constantinou et al., 1994b; Reinhorn et al., 1995). The procedure can be used with some approximation for design of other force delivery devices with passive characteristics such as friction or hysteretic devices (Constantinou et al., 1994a). The process is illustrated by a design example for a small model structure.

2. OPTIMAL CONTROL THEORY

For a frame structure braced by devices that control it’s vibration the equation of motion may be written as:
\[ M \ddot{x}(t) + C \dot{x}(t) + Kx(t) = E f(t) + Du(t) \]  
\[ \dot{z}(t) = Az(t) + Bu(t) + Hf(t) \]

in which, matrices \( M, C, K \) characterize mass, structural damping and stiffness related to the deformations \( x(t) \) at various degrees of freedom. The brace forces are included in the system as control forces \( u(t) \) at locations indicated by matrix \( D \) designed to reduce the response due to excitation forces, \( f(t) \) at locations indicated by \( E \). The equation of motion can be easily compacted to a state space formulation:

\[ A = \begin{bmatrix} 1 & 0 \\ -M^T K & -M^T C \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1} D \end{bmatrix}, \quad H = \begin{bmatrix} 0 \\ M^{-1} E \end{bmatrix} \]

Assuming that the control forces are of linear form, for sake of simplicity:

\[ u(t) = G z(t) = [G_x : G_{\dot{x}}] z(t) = G_x x(t) + G_{\dot{x}} \dot{x}(t) \]

in which, the gain matrix, \( G \) includes the constant coefficients, \( G_x, G_{\dot{x}} \), for the structural control devices. The gain matrix \( G \) is obtained from the minimization of a performance index (Gluck, Reinhorn, Gluck, and Levy, 1996) as:

\[ G = -1/2R^{-1}B^T P \]

in which, \( P \) is the solution of Ricatti equation:

\[ A^T P + PA - 1/2B R^{-1} B^T P + 2Q = 0 \]

The control forces are obtained therefore:

\[ u(t) = G_x x(t) + G_{\dot{x}} \dot{x}(t) \]

or explicitly:

\[
\begin{bmatrix}
  u_1 \\
  u_2 \\
  \vdots \\
  u_n 
\end{bmatrix} =
\begin{bmatrix}
  g_{11,x} & g_{12,x} & \cdots & g_{1n,x} \\
  \vdots & \vdots & \ddots & \vdots \\
  g_{n1,x} & g_{n2,x} & \cdots & g_{nn,x} 
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_n 
\end{bmatrix} +
\begin{bmatrix}
  g_{11,\dot{x}} & g_{12,\dot{x}} & \cdots & g_{1n,\dot{x}} \\
  \vdots & \vdots & \ddots & \vdots \\
  g_{n1,\dot{x}} & g_{n2,\dot{x}} & \cdots & g_{nn,\dot{x}} 
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2 \\
  \vdots \\
  \dot{x}_n 
\end{bmatrix}
\]

(7b)

If passive diagonal braces in a structure (see Fig. 1) supply the control forces, then these forces are dependent on their constant stiffness and damping coefficient as follows:

\[ u^{*}(t) = K_x x(t) + C_{\dot{x}} \dot{x}(t) \]

or explicitly:

\[
\begin{bmatrix}
  u_1^{*} \\
  u_2^{*} \\
  \vdots \\
  u_n^{*} 
\end{bmatrix} =
\begin{bmatrix}
  k_1 & -k_2 & & & \\
  -k_2 & k_1 + k_2 & -k_3 & & \\
  & \ddots & \ddots & \ddots & \ddots \\
  & & -k_n & k_n & \\
  & & & & -k_n & k_n 
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_n 
\end{bmatrix} +
\begin{bmatrix}
  c_1 & -c_2 & & & \\
  -c_2 & c_1 + c_2 & -c_3 & & \\
  & \ddots & \ddots & \ddots & \ddots \\
  & & -c_n & c_n & \\
  & & & & -c_n & c_n 
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2 \\
  \vdots \\
  \dot{x}_n 
\end{bmatrix}
\]

(8b)

The coefficients of the passive formulations, \( k_{ij}, c_{ij} \), in Eq. (8) are derived from the gain coefficients \( g_{ij,x}, g_{ij,\dot{x}} \) in Eq.(7) using several approximations which are described below. For simplicity of further derivations Eqs. (7) and (8) can be transformed using story-drift formulation (see Fig. 1) obtained from the linear transformations for deformations \( x(t) \) and for the diagonal braces forces \( v(t) \):
Figure 1  Structural Frame with Diagonal Damping Braces        Figure 2. Model Structure For Numerical Example.

\[ x(t) = T_d(t) \quad v(t) = T^T u(t) \]  

(9)

where \( T \) is a matrix of units on the upper right triangle and the rest zeros (Gluck et al, 1996) and where \( v(t) \) are the control forces in terms of story drifts and drift velocities obtained from Eq. (7) are given as:

\[ v(t) = G_d d(t) + G_{\dot{d}} \dot{d}(t) \]  

(10)

in which:

\[ G_d = T^T G_x T \quad G_{\dot{d}} = T^T G_{\dot{x}} T \]  

(11)

Using the same transformation the brace forces of Eq. (8) can be written as:

\[ \mathbf{v}^* (t) = K_d d(t) + C_d \dot{d}(t) \]  

(12)

in which:

\[ K_d = T^T K_x T = \text{diag} (\Delta k_i) \quad \text{and} \quad C_d = T^T C_x T = \text{diag} (\Delta c_i) \]  

(13)

where \( \Delta k_i \), \( \Delta c_i \) are supplemental stiffness properties and damping from each brace in the structure at level \( i \).

To determine the individual components of matrices \( K_d \) and \( C_d \) in Eq. (13), a least squares approach is considered. Since the stiffness \( K_d \) and damping \( C_d \) can be assumed independent, the least squares will be applied separately for \( K_d \) and \( C_d \). Only the damping coefficients of \( C_d \) are determined below. However, the same types of formulations can be applied to determine the coefficients of \( K_d \).

Applying the least square approximation to the difference between the formulations of Eqs. (10) and (12) using the notation of Eqs. (11) and (13) results in explicit form:

\[ \frac{d}{dd_k(t)} \left\{ \sum_{j=0}^{T} g_{k,j,d} \dot{d}_j(t) - D c_k \dot{d}_k(t) \right\}^2 dt = 0 \]  

(14)

\[ \Delta c_k = \sum_{j=0}^{T} g_{k,j,d} \dot{d}_j(t) dt / \int_{0}^{T} \dot{d}_k(t) dt \]  

(15)
where, $T$ is the total time for the event considered.

The above coefficients can be determined using further simplifications as outlined in the following:

**Response spectrum approach - “Peak fit”**: Assume that in the time interval $T$, the velocity can be obtained from a modal spectrum approach using the square root of sum of squares (SRSS) superposition:

$$
\dot{d}_{ji} = \left[ \sum_i \left( \phi_{ji} P_i S_{vi} \right)^2 \right]^{1/2}
$$

where, $\dot{d}_{ji}$ is the velocity in mode $i$ at degree of freedom (d.o.f.) $j$, $\phi_{ji}$ the mass normalized shapes, $P_i$ is the participation factor ($= \sum_j m_j \phi_{ji}$), and $S_{vi}$ the spectral velocity of mode $i$. Damping

$$
\Delta c_k = \frac{\sum_j g_{kj,d} \sum_i \left( \phi_{ji} P_i S_{vi} \right)^2}{\sum_i \left( \sum_j \phi_{ki} P_i S_{vi} \right)^2} \left[ \sum_i \left( \phi_{ki} P_i S_{vi} \right)^2 \right]^{1/2}
$$

**Single mode approach**: In applications involving building structures in earthquakes, most often only one mode of vibration is relevant. If mode $i$ is retained in Eq. (17), i.e., $i = m$, then:

$$
\Delta c_k = \frac{\sum_j g_{kj,d} \phi_{jm}}{\phi_{km}} = \sum_j g_{kj,d} \phi_{jm}^k
$$

where, $\phi_{jm} = \phi_{jm} / \phi_{km}$ is the modal shape normalized to unit at degree-of-freedom $k$. In this case the damping coefficients are not anymore dependent on the history of the event.

**Truncation approach**: If only a single gain factor is considered, i.e., the one corresponding to the degree-of-freedom $k$. In such case, $j = k$ in formulation of Eq. (18) and:

$$
\Delta c_k = g_{kk,d}
$$

This simplified formulation can be obtained directly from Eq. (10), “truncating” all the off-diagonal terms of matrix $G_d$.

3. IMPLEMENTATION OF PASSIVE VISCOS OR VISCOELASTIC DAMPERS

As indicated above, the optimal solution given by Eq. (10) can be implemented only by active means, which can provide the combination of information from all degrees-of-freedom through the gain matrix, $G$, which is fully populated (Reinhorn et al., 1993). However, using passive braces with viscous, viscoelastic, added damping and stiffness (ADAS), or friction dampers, which can be modeled by Eq. (12) (Reinhorn et al., 1995), the structure can be protected close to the optimum as derived above. For example, fluid viscous or viscoelastic dampers can be modeled using an equivalent Kelvin model:

$$
f_d(t) = k(\omega) d(t) + c(\omega) \dot{d}(t)
$$

in which, the storage stiffness, $k(\omega)$, and the damping coefficient, $c(\omega)$, (derived from the loss stiffness divided by the circular frequency, $\omega$), can be well approximated as constants in narrow frequency bands of structural response (Reinhorn et al., 1995). To determine the desired optimal, or near optimal, coefficients $k_e$ and $c_e$, expressions from Eqs. (17), (18), or (19) can be used (similarly for $k$ as for $c$).
4. NUMERICAL EXAMPLES

To illustrate the procedures outlined above and the performance of optimally, or near optimally designed dampers, a 1:5 scale three-story model structure made of two flexible shear frames with rigid floors is considered (see Fig. 2). An artificial mass simulation was used to retain a frequency scaling in a constant accelerated field (Bracci et al., 1993). The complete mass, stiffness and structural damping of the structure without dampers, as obtained from structural identification are:

\[
\begin{bmatrix}
200.40 & 0 & 0 \\
0 & 200.40 & 0 \\
\text{symm} & 178.00 & \text{symm}
\end{bmatrix} \quad \text{[kg]}
\]

\[
\begin{bmatrix}
238,932 & -119,466 & 0.0 \\
238,932 & -119,466 & \text{symm}
\end{bmatrix} \quad \text{[N/m]}
\]

\[
\begin{bmatrix}
26499 & -78.09 & -160.8 \\
24689 & -92.15 & \text{symm}
\end{bmatrix} \quad \text{[N–sec/m]}
\]

The natural frequencies of the structure are 1.78 Hz, 4.96 Hz, and 7.05 Hz in the first three modes, respectively. Fluid viscous dampers [Taylor Devices Co. (Constantinou et al., 1992)] were considered in this study. These dampers can be constructed as linear or nonlinear functions of velocity. In this study, the configuration studied by Constantinou et al., (1992) was considered. These are dampers which can be represented by a linear relation within the expected range of operations (Eq. (1)), without stiffening characteristics \( (k_d = 0) \). The optimal design of dampers was developed using the procedure outlined above for weighting matrices, \( R \) and \( Q \) (in Eq. (6)):

\[
R = 10^p I_{3 \times 3}; \quad Q = I_{6 \times 6}
\] (22)

in which, \( I_{nxn} \) indicate a unit diagonal matrix of size \( n \times n \), and \( p \) is a variable parameter used to adjust the solution’s weights toward the practical range. The optimal solution was obtained by solving the algebraic Ricatti equation (Eq. (6)) using MATLAB package (Gluck et al. 1996)

The gain matrices were obtained from a parametric analysis, varying \( p \) in Eq. (22) between 2 and 9. By increasing the parameter \( p \), the demand for damping increases and the response decreases. Therefore increasing \( p \) one can increase the size of dampers up to the limit of “shelf” availability. The results are listed in Table 1 using the form:

\[
G = g_o \begin{bmatrix}
1 & \varepsilon_{12} & \varepsilon_{13} \\
\varepsilon_{21} & n_1 & \varepsilon_{23} \\
\varepsilon_{31} & \varepsilon_{33} & n_2
\end{bmatrix}
\] (23)

The gain matrix \( G_d \) (Eq. (10)) in Table 1 was obtained by direct solution of the Ricatti equation with the matrices \( A \) and \( B \) transformed into coordinates suitable with the transformation in Eq. (9). The results obtained for these matrices are different than those from the transformation in Eq. (11), due to the different influence of weighting matrices, \( R \) and \( Q \). The results obtained for the \( G_d \) (Eq. (10)) are most representative to the solution, therefore, are used further to determine the damper sizes as follows:

It can be noted that off-diagonal terms, \( \varepsilon_{ij} \), in the gain matrix \( G_d \) shown in Table 1 are substantially different than zero. However, the damping matrix \( [C_d, \text{Eq. (13)}] \) for the damping braces in Fig. 2 takes a diagonal form. Therefore, the damping coefficients \( \Delta c_k \) need to be determined according to the procedure outlined in the previous sections: (i) by “truncation” using Eq. (19); (ii) by “single mode approach”; or (iii) by “engineering” round off of the solution determined in (ii) such that will fit “off-the-shelf” devices.

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procedure outlined in the previous sections: (i) by “truncation” using Eq. (19); (ii) by “single mode approach”; or (iii) by “engineering” round off of the solution determined in (ii) such that will fit “off-the-shelf” devices.

Table 1 - Variation of gain matrices for the 3-story building (notation in Eq. (23))

<table>
<thead>
<tr>
<th>p</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{g}$, Eq. (11)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_0$</td>
<td>N-sec/m</td>
<td>2.31</td>
<td>21.28</td>
<td>156.16</td>
</tr>
<tr>
<td>$n_1$</td>
<td>--</td>
<td>1.35</td>
<td>1.26</td>
<td>1.09</td>
</tr>
<tr>
<td>$n_2$</td>
<td>--</td>
<td>1.68</td>
<td>1.54</td>
<td>1.26</td>
</tr>
<tr>
<td>$E_{ij}$</td>
<td>--</td>
<td>0.82</td>
<td>0.66</td>
<td>0.32</td>
</tr>
<tr>
<td>$E_{ij}^{max}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{d}$, Eq. (11)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_0$</td>
<td>N-sec/m</td>
<td>2.20</td>
<td>20.79</td>
<td>151.00</td>
</tr>
<tr>
<td>$n_1$</td>
<td>--</td>
<td>1.40</td>
<td>1.40</td>
<td>1.40</td>
</tr>
<tr>
<td>$n_2$</td>
<td>--</td>
<td>1.89</td>
<td>1.89</td>
<td>1.88</td>
</tr>
<tr>
<td>$E_{ij}$</td>
<td>--</td>
<td>0.84</td>
<td>0.84</td>
<td>0.83</td>
</tr>
<tr>
<td>$E_{ij}^{max}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{d}$, Eq. (10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_0$</td>
<td>N-sec/m</td>
<td>3.87</td>
<td>32.81</td>
<td>196.83</td>
</tr>
<tr>
<td>$n_1$</td>
<td>--</td>
<td>2.73</td>
<td>2.63</td>
<td>2.35</td>
</tr>
<tr>
<td>$n_2$</td>
<td>--</td>
<td>4.26</td>
<td>4.09</td>
<td>3.62</td>
</tr>
<tr>
<td>$E_{ij}$</td>
<td>--</td>
<td>3.21</td>
<td>3.04</td>
<td>2.59</td>
</tr>
<tr>
<td>$E_{ij}^{max}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results for p=6 leading to acceptable size dampers, are listed in Table 2. The results are presented using a notation similar to Eq. (23) in which $g_{0}$ is replaced by $\Delta c_{d}$ for the damping matrix $C_{d}$.

The optimal gain matrix is listed in column (6) of Table 2 for comparison. If the design is done by truncation large dampers are required for the higher stories of the structure. For the single mode approach the design leads to larger size dampers overall, but all of same approximate size. The engineering round off leads to identical dampers that can be found “off-the-shelf.”

Table 2 - Properties for dampers in structure in Fig. 2 (notation in Eq. (21))

<table>
<thead>
<tr>
<th>$\Delta c_{d}$ or $g_{0}$</th>
<th>N-sec/m</th>
<th>“Truncation”</th>
<th>Single-Mode “Estimator”</th>
<th>“Engineering” Round Off</th>
<th>Gain Matrix $G_{d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_1$</td>
<td>--</td>
<td>2.13</td>
<td>0.95</td>
<td>1.00</td>
<td>2.13</td>
</tr>
<tr>
<td>$n_2$</td>
<td>--</td>
<td>3.23</td>
<td>0.93</td>
<td>1.00</td>
<td>3.23</td>
</tr>
<tr>
<td>$E_{ij}^{max}$</td>
<td>--</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>2.22</td>
</tr>
</tbody>
</table>

The design obtained above was evaluated using several earthquake excitations. The structure was subjected to (i) El Centro N-S 1940 accelogram with peak ground acceleration, (PGA) of 0.34 g; (ii) Mexico City SCT 1985 accelerogram (PGA 0.20g); (iii) Hachinohe 1968 accelogram (PGA 0.20g). The displacements at the third and first floors of the structure are shown in Fig. 3. The peak response is shown in Table 3. The structure without dampers has the largest response (“uncontrolled”), while the response using the design based on single-mode approach (estimator) is almost identical to that for the exact optimal solution (which can be obtained only by active means). The design using the “truncation” approach produces also substantial reduction, but not as efficient as the single mode (“estimator”) approaches. The engineering round-off selection has the same performance as the single-mode approach. The single-mode approach used the first mode of the structure, which is dominant in earthquake response in tall structures. If there is uncertainty of which modes might be dominant, then the response spectrum design (“peak fit”) can be used as outlined in the previous section.
Table 3 - Peak Response of Top Floor Subjected to Ground Motions

<table>
<thead>
<tr>
<th></th>
<th>Displacement [mm]</th>
<th>Velocity [m/s]</th>
<th>Acceleration [m/s²]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>El Centro 1940</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncontrolled</td>
<td>22.5</td>
<td>0.25</td>
<td>3.10</td>
</tr>
<tr>
<td>Optimal Design (Active Control)</td>
<td>9.2</td>
<td>0.10</td>
<td>1.55</td>
</tr>
<tr>
<td>“Estimator” Design</td>
<td>9.3</td>
<td>0.10</td>
<td>1.52</td>
</tr>
<tr>
<td>“Truncation” Design</td>
<td>13.5</td>
<td>0.15</td>
<td>2.11</td>
</tr>
<tr>
<td><strong>Mexico 1985</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncontrolled</td>
<td>18.0</td>
<td>0.13</td>
<td>1.52</td>
</tr>
<tr>
<td>Optimal Design (Active Control)</td>
<td>10.1</td>
<td>0.06</td>
<td>0.48</td>
</tr>
<tr>
<td>“Estimator” Design</td>
<td>10.1</td>
<td>0.06</td>
<td>0.50</td>
</tr>
<tr>
<td>“Truncation” Design</td>
<td>12.0</td>
<td>0.07</td>
<td>0.60</td>
</tr>
<tr>
<td><strong>Hachinohe 1968</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncontrolled</td>
<td>59.1</td>
<td>0.63</td>
<td>7.52</td>
</tr>
<tr>
<td>Optimal Design (Active Control)</td>
<td>25.2</td>
<td>0.24</td>
<td>3.05</td>
</tr>
<tr>
<td>“Estimator” Design</td>
<td>25.0</td>
<td>0.24</td>
<td>3.06</td>
</tr>
<tr>
<td>“Truncation” Design</td>
<td>32.2</td>
<td>0.32</td>
<td>4.20</td>
</tr>
</tbody>
</table>

Figure 3. Structural Response at First and Third Story for Various Ground Motions

5. REMARKS AND CONCLUSIONS

The paper presents a method of design of supplemental passive damping devices based on optimal linear control theory. Other control schemes can be also applied if the gain matrices are properly handled as shown in here. The design presented can be used to size viscous, viscoelastic or added damping and stiffness devices (ADAS) dampers, (with some additional approximations), if their location was established. It can also help to optimize their location, if an iterative process is followed adjusting the terms in the optimization weighting matrices \( Q \) and \( R \) and the coefficients in the location matrix \( B \).

The design considered herein is based on the control gains, which can be implemented only by an active system. However, the comparison in the numerical example shows that same equivalent effect can
be obtained using passive devices only. The above remark is valid for structures dominated by a single mode of vibration. If more modes are contributing to the response, a single passive damper cannot provide the same effect. In such a case a fully active system is more efficient.

The design solution for the passive devices is always stable, as provided implicitly by the formulation based on the Riccati equation. The design using passive devices is free of time delays (implicitly avoided by all the reactive passive systems) and is reliable in its operations.

The procedure outlined in the paper can also be used to size also friction dampers for which the reaction force is dependent on an adjustable normal force (Gluck, Reinhorn, Gluck and Levy, 1996).

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REFERENCES


