GLOBAL SPECTRAL EVALUATION OF SEISMIC FRAGILITY OF STRUCTURES

Andrei M. Reinhorn¹, Raul. Barron-Corverra², A. Gustavo Ayala³

ABSTRACT

An approach for assessing the global seismic fragility of inelastic structures is presented. The fragility is obtained from the distribution of structural response, which is evaluated from response spectra with an associated probability distribution, compared with the performance limits states. Formulas that provide the probability distribution of spectral ordinates are developed based on the crossing theory of random functions and the equivalent linearization method. The ground motion is modeled as a stationary zero-mean Gaussian process with a Fourier spectrum typical of far-field earthquakes. The theoretical results are compared within a numerical simulation study. It is shown that a Gaussian distribution can be assumed for the distribution of response of hysteretic systems under far-field excitations. It is shown also that a white noise representation may lead to large inelastic displacements that are not likely to not occur under earthquake excitations. An application example is presented to illustrate the use of this approach.

Introduction

Current simplified procedures for seismic fragility evaluation such as those proposed and used in HAZUS (NIBS 1999) and others represent fragility by lognormal cumulative probability functions. In these procedures, the variability in seismic demand (i.e. the expected inelastic response of the structure), which is the main contributor to fragility, is provided without explicit consideration of the influence of the structural parameters such as yield strength, damping, initial period, thus this variability is assumed as constant through all the spectrum. Certainly this is not a correct assumption as studies by Pekcan (1999) on elastic oscillators showed that the variability of spectral demand is period and damping dependent and that it may vary in the range of 0.18 to 0.58. Studies by Barron-Corvera (2000) on inelastic models showed a considerable influence of these parameters on the statistics of the response as shown further in this paper. Therefore, the applicability of above simplified procedures for evaluating seismic fragility is restricted.

Suggested approach for seismic fragility evaluation

A schematic illustration of the approach suggested for seismic fragility evaluation is presented in Figure 1. In this approach, constant yield-reduction-factor inelastic spectra (Reinhorn 1997) are used for the evaluation of inelastic response. This approach considers the nonlinear capacity

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curve of the structure with variations due to uncertainties and compares it with the inelastic
demand spectrum with its own uncertainties. Figure 1 presents the spectral capacity (dotted
lines) and the demand spectrum (continuous lines) with their corresponding uncertainty “bands”.
The intersection area of these two bands represents the zone where the response of the structure
is expected to be. This area is defined by the mean value in these functions with one standard
deviation in each side. The probabilistic combination of these two functions defines a unit
volume over the area of intersection of these two bands.

![Figure 1. Seismic demand variability](image)

The fragility is obtained by comparing the distribution with a predetermined performance limit.
Mathematically, the seismic fragility, which represents the conditional probability that the
maximum response, $R$, exceeds a performance limit state, $r_{\text{lim}}$, conditional on the intensity, $I$, of
ground motion, can be estimated by using the following formulation:

$$\text{Fragility} = P[R \geq r_{\text{lim}} | I] = \sum_{j} P[R \geq r_{\text{lim}} | I, C] \cdot P(C = c_j) \quad (1)$$

In evaluating the probability of exceeding the limit state, $r_{\text{lim}}$, from the unity volume of Figure 1,
three capacity curves are used (e.g. the mean and the mean plus and minus one standard
deviation), where $P(C = c_j)$ represents the probability that capacity $c_j$ occurs. The probability of $R > r_{\text{lim}}$ conditional on $I$ and $C$ can be determined from the probability distribution function (PDF)
characteristic to the probabilistic inelastic response spectrum.

**Maximum displacement PDF of hysteretic systems**

In a continuous valued random process $X(t)$, a peak occurs whenever $\dot{X}(t) = 0$ and $\ddot{X}(t) < 0$; this suggest that the probabilistic behavior of the peaks can be obtained from the join distribution
of $X(t)$, $\dot{X}(t)$, and $\ddot{X}(t)$. Rice (1944, 1945) derived the distribution function, $F_Z(z)$, of the peaks
(local maxima, $Z = X_{\text{max}}$) for a zero-mean stationary Gaussian process. The probabilistic
behavior of the maximum value among the peaks and valleys is obtained by assuming that $N$
independent maxima and their corresponding $N$ minima are observed in the interval $(0, t_d)$ each
having the same PDF:
where the number of independent peaks (or valleys), $N$, is related to the mean rate of zero-crossings and the duration time $t_d$; $\Phi(\cdot)$ represents the standard Gaussian distribution; and $\sigma_x, \sigma_x, \sigma_{\dot{x}}$ are the root mean square (RMS) values of the displacement, velocity, and acceleration, respectively.

The hereditary behavior of the hysteretic force makes an analytical solution for RMS extremely difficult; hence, approximations are almost always necessary. To model the seismic hysteretic behavior of a single degree of freedom oscillator the following equation of motion is used:

$$m\ddot{X}(t) + c\dot{X}(t) + akX(t) + (1 - \alpha)kZ(t) = mA_g(t), \quad 0 \leq \alpha \leq 1$$

where $m$, $c$ and $k$ are the corresponding mass, damping and stiffness of the oscillator, $A_g(t)$ is the random ground acceleration and the variable $Z$ is used to model the hysteretic force. This defined in a rate form given by a nonlinear equation. The model proposed by Suzuki & Minai (1987) is used:

$$\ddot{Z}(t) = X(t)[1-U[X(t)]] \cdot U[Z(t) - 1] - U[-X(t)] \cdot U[-Z(t) - 1]$$

where $U[\cdot]$ is the unit step function. An equivalent linearization method (Barron-Corvera, 2000) is used to linearize Eq. (5), which can be written as:

$$\ddot{Z}(t) \approx a_0 + a_1X(t) + \dot{X}(t) + a_3Z(t)$$

where $a_i$ are unknown parameters that can be conveniently determined by minimizing the mean square error, which is represented by the difference between Eq. (5) and (6). A numerical simulation study showed that the response of hysteretic systems does not depart significantly from being Gaussian when they are excited by far field earthquakes (Barron-Corvera, 2000).

**RMS of motion parameters and distribution of displacement maxima**

Barron Corvera (2000) showed also that the RMS displacement predicted by the linearized models is very similar, though somewhat lower than those obtained from simulations. The discrepancies are larger for yield reduction factors of 2 and 4 and that the simulation and theoretical solutions get closer for higher values of this factor (i.e. $r_y > 10$). The RMS values of velocity and acceleration are surprisingly well predicted by the equivalent linearization method.

An excellent agreement was obtained in the inelastic responses for the oscillators with 5% damping ratio, however, for the 20% damping oscillator the spread of the response and the mean values are larger than the predicted values. These differences can be attributed to the larger differences in the RMS displacement obtained from simulations and the equivalent linearization method. An improved approximation for the PDF of maximum response could be obtained if the RMS values from direct Monte Carlo simulations are used; however, the computationally intensive process is not justified for design purposes. Therefore, for the sake of simplicity in the
design process a lower accuracy might be acceptable and approximation might be suitable.

**Approximated distribution of maximum displacement**

The evaluation of the PDF of maximum displacement in Eq.(2) requires the calculation of the RMS displacement, $\sigma_x$, the bandwidth parameter, $\eta$, and the number of zero up-crossings, $N$. All these parameters can be obtained from the equivalent linearization method. This requires a statistical representation of ground motion, which is usually given in terms of its PSD function. However, such an approach requires considerable expertise and is unlikely to be suitable for routine design process. Therefore simplified approximated formulas were derived as shown in Table 1 (see Barron-Corvera 2000). Since Eq. (2) is applicable to the elastic and inelastic case but with different values for the parameters $\sigma_x^E$, $\eta$, and $N$, distinction is made between these two cases by using the following notation: $\sigma_x^{E}$, $\eta_E$, and $N_E$ for the elastic case, $\sigma_x^{I}$, $\eta_I$, and $N_I$ for the inelastic case and are shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Distribution parameters $\sigma_x$, $\eta$, and $N$</th>
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<tr>
<td><strong>Elastic</strong></td>
</tr>
<tr>
<td>$\sigma_x^E(\xi) = \sigma_x^E(5%) \cdot 10^{-a}$</td>
</tr>
<tr>
<td>where $a = 0.5 \cdot [\log(\xi) - \log(5)]$</td>
</tr>
<tr>
<td>$\eta_E = (1 - 1.2\xi) \cdot \eta_0$</td>
</tr>
<tr>
<td>$N_E = t_d / T_0$</td>
</tr>
<tr>
<td>(7a)</td>
</tr>
</tbody>
</table>

In Table 1, $\sigma_x^E(5\%)$ is the 5%-damping RMS displacement, $\xi$ is the damping ratio, $t_d$ is the strong motion duration, $T_1$ is the corner period that divides the short-period region from the medium-period region, and $\eta_0$ is a normalized bandwidth function (Barron-Corvera, 2000). Three regions define this function: 1) the short-period or acceleration-sensitive region ($T_0 < T_1$), where $\eta_0$ reaches its maximum value, 2) the medium-period or velocity-sensitive region ($T_1 < T_0 < 4T_1$) where $\eta_0$ changes linearly, and 3) the long-period region ($T_0 > 4T_1$) where $\eta_0$ reaches its minimum value. Values of this function for the long-period region and different damping ratios are given in Table 2.

<table>
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<tr>
<th>Table 2. Bandwidth parameter $\eta_0$</th>
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Example of seismic fragility evaluation

To illustrate the application of the suggested procedure for estimating the global seismic fragility, a numerical example is presented. The building selected is a typical lecture hall constructed in mid region of US; Figure 2 shows a plan view of the typical floor framing.

The building is a four-story structure with reinforced concrete frames and shear walls. The exterior frames in the transverse (E-W) direction are shown in Figure 3.

The seismic hazard at the site is characterized by a standard design spectra as suggested in the NEHRP-97 Guidelines (ATC 1997) as shown in Fig 4.

The capacity of structure to resist lateral loads in the transverse (E-W) direction is evaluated by a nonlinear static analysis “pushover” using the computer program IDARC2D (Valles et al. 1996), see also http://civil.eng.buffalo.edu/idarc2d50/. The capacity curve, $Q-u$, which relates the base shear versus the lateral displacement at the selected floor, is shown in Figure 5 (a). The capacity curve, $Q-u$, is transformed to the spectral capacity (Fig 5b) $Q^*-u^*$, by using the mass-normalized mode shapes, $\phi$, and the participation factors, $\Gamma$, as indicated in Eq. (10) (Reinhorn, 1997).
\[ Q^* = \frac{Q}{\Gamma_j^2 g} \quad \text{and} \quad u^*_N = \frac{u_N}{\phi_N \Gamma_j} \]  

(10)

The performance limit states used by Hwang & Huo (1994) on a similar study for the evaluation of seismic fragility of buildings with RC shear walls are used to evaluate the fragility of this case study. (see Table 3).

![Figure 5. Capacity curves, (a) floor capacity curve, (b) spectral capacity curve](image)

<table>
<thead>
<tr>
<th>Limit State</th>
<th>Drift Limit Ratio (%)</th>
<th>Drift Limit (mm)</th>
<th>Spectral Drift Limit (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incipient Damage</td>
<td>0.2 - 0.5</td>
<td>6.5</td>
<td>15.0</td>
</tr>
<tr>
<td>Moderate Damage</td>
<td>0.5 - 1.0</td>
<td>16.0</td>
<td>35.0</td>
</tr>
<tr>
<td>Heavy Damage</td>
<td>1.0 &lt;</td>
<td>32.0</td>
<td>70.0</td>
</tr>
</tbody>
</table>

**Evaluation of the probability distribution function of maximum response**

The probability of exceeding each performance limit state is evaluated from the PDF of maximum displacement, \( F_{z_{\max}}(z) \), given in Equation 2. The elastic number of zero up-crossings, \( N_E = 30 \) for a strong motion duration \( t_d = 15 \text{ sec} \). The calculated values is \( \eta_E = 0.93 \). The RMS displacement for the 5% damping ratio, \( \sigma^E_X(5\%) = 13.96 \text{ was calculated using Equation 7a with the values: } \bar{Z}_{\max} = 3.008, S_d^E(5\%) = 42 \text{ mm. The inelastic parameters } \sigma^I_X = 20.0 \text{ mm, } \eta_I = 0.434, \text{ and } N_I = 12.1 \text{ were calculated from Equations (7b), (8b), and (9b), respectively, where a yield reduction factor of } r_y = 3.45 \text{ was used. This factor is determined from the ratio of elastic spectral acceleration, } S_{0y} = 0.69, \text{ divided by the yield spectral acceleration, } S_{0y} = 0.2, \text{ - see Figure 6.}

Since the uncertainty in seismic demand is by far greater than the uncertainty in structural capacity, for simplicity, in this study, the probabilities of exceeding limit-states (i.e., fragility) are evaluated using only the mean capacity. The fragility is obtained from the following equation:
Fragility = \( P[R \geq r_{\text{lim}} \mid I] = 1 - \Phi \left( \frac{r_{\text{lim}}}{\sigma_X \sqrt{1 - \eta^2}} \right) - \eta \Phi \left( \frac{\eta r_{\text{lim}}}{\sigma_X \sqrt{1 - \eta^2}} \right) \exp \left( - \frac{r_{\text{lim}}^2}{2(\sigma_X^2)^2} \right) \)^{-2/NI} \) \( (11) \)

Figure 7 presents the fragility curves obtained by fitting a lognormal cumulative probability function to the set of probability-data points obtained with the procedure described above.

Influence of structural parameters on fragility

Fragility studies can be used among other things to develop strategies to strengthen or retrofit structures to reduce their vulnerability. The effect of the structural parameters on fragility; for example, the influence of strength, stiffness, and damping may be used in decision on use of various retrofit techniques. A substantial move of a given fragility curve toward the right would indicate reduced probability of response exceeding a given limit state in case of a specified event. Several results of influence of structural parameters on global fragility are noteworthy:

(i) Figure 8 presents the influence of variation of stiffness on fragility. Increasing stiffness reduces fragility, however the changes are not significant. (ii) Figure 9 presents the fragility curves for three values of strength. Increasing the strength does not produce significant changes in fragility. This is due to the fact that the inelastic displacement is not affected substantially by small changes in strength. (iii) Damping has a considerable effect in reducing displacements; therefore it clearly produces changes in fragility even if the changes in damping are small. Figure
10 presents the influence of small variations of damping in fragility.

However, the effect of retrofitting structures by adding substantial damping using mechanical devices can be easily emphasized by use of the fragility curves. Figure 11 presents the fragility curves for the Moderate Damage State for three damping ratios the 5%, 15%, and 25%. It can be seen that a considerable reduction of fragility is obtained when damping is added from 5% to 15% and further to 25%. It can be shown that the addition of damping to low damped structures is more beneficial than addition of same amount of damping to an already highly damped structure – see Fig. 11.

![Figure 10. Influence of small variations of damping](image1)

![Figure 11. Influence of increased damping](image2)

**Conclusions**

This paper introduced a method for the practical determination of the seismic fragility of a structure by establishing a reliable simplified approach in which fragility is evaluated from the spectral capacity curve and the seismic demand spectrum. The fragility is obtained by calculating the probability of exceeding the response parameters associated to the considered performance limit states.

I can be remarked that: (i) The validity of this approach relies on the proper consideration of the inelastic behavior of the structures i.e. using a nonlinear representation of capacity and corresponding nonlinear demand spectra. (ii) The probabilistic behavior of the inelastic response is greatly influenced by structural parameters such as, damping ratio, yield strength level, initial period, and post-yielding stiffness ratio. (iii) Uncertainties in seismic demand are explicitly considered in the PDF of spectral ordinates allowing a direct calculation of the probability of exceeding a given limit-state. (iv) A Gaussian distribution can be effectively assumed for the response of hysteretic systems when they are driven by far field ground motions.

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