Inelastic Analysis Techniques in Seismic Evaluations

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ABSTRACT: The analytical methods available to the design engineer today are either dynamic time history analyses, or monotonic static nonlinear analyses, or equivalent static analyses with simulated inelastic influences. This paper suggests in addition to the inelastic time history analyses, some simplified analyses techniques based on equating the seismic demand expressed in terms of response spectra with the inelastic capacity described in terms of force-deformation characteristics. The inelastic response in terms of accelerations and maximum displacements can be evaluated accurately, for a single-degree-of-freedom (s.d.o.f.) system, or approximately for a multi-degrees-of-freedom (m.d.o.f.) system. Such method is similar with the proposed technique for evaluation of response by NEHRP, 1996 to be released in the future codes.

1. INTRODUCTION

In an effort to develop design methods based on performance it is clear that the evaluation of the inelastic response is required. The methods available to the design engineer today are either dynamic time history analyses, or monotonic static nonlinear analyses, or equivalent static analyses with simulated inelastic influences. Although the inelastic time history analyses (ITHA) are becoming more cost effective, the static monotonic nonlinear analyses (push-over type) provide sufficient insight in the expected behavior for design purposes. The simplified techniques currently available seem to produce rational results, without an apparent theory behind. This paper suggests in addition to the inelastic time history analyses, some simplified analyses techniques based on equating the seismic demand expressed in terms of response spectra with the inelastic capacity described in terms of force-deformation characteristics. The response demands obtained from the proposed method can be used along with a composite energy-displacement spectrum to determine the seismic performance of a structure in terms of local, or global, damage levels. The response spectrum analysis is a well recognized method to evaluate the seismic response of structures. Traditionally the inelastic response spectra are obtained either from inelastic analysis using equal ductility demands or from the elastic response using ductility based adjustments (Newmark and Hall, 1982) Such spectra are usually concerned with the acceleration response, or force demands, only. The inelastic deformations seem to more valuable in evaluating the inelastic response. A simultaneous quantification of deformation and force responses are valuable for interpreting the response and for the design process.

Recently methodologies based on so called capacity spectrum were developed (Dierlein et al, 1991, Freeman, 1994). The response is estimated using the elastic response spectrum presented in terms of acceleration spectrum in conjunction with a curve representing the monotonic nonlinear resistance of a structure. The inelastic capacity function (or the monotonic force and displacement resistance envelope mentioned above) was used by Freeman (1994) in conjunction with a combined elastic acceleration and displacement response spectra to determine response both accelerationand displacement demands. The inelastic hysteretic effects were considered through an increased period.

This paper surveys several techniques for the evaluation of inelastic response and suggests to find simultaneously the inelastic deformation and acceleration demands for simple structures defined by single (s.d.o.f.) and by multiple-degree-of-freedom (m.d.o.f.) models through a spectral approach.
2. INELASTIC TIME HISTORY ANALYSIS

A simple dynamic system will respond to a support motion \((\ddot{u}_g)\) depending on its dynamic properties linked to its mass \((m)\), structural restoring force \((Q(u))\) and the internal structural damping \((c)\) according to:

\[ m\ddot{u}(t) + c\dot{u}(t) + Q(u(t)) = -m\ddot{u}_g(t) \]  

where \(u(t)\) is the displacement response, and the overdot indicates the time derivative operand. For an elastic structure the restoring force follows a linear relation:

\[ Q(u) = k u(t) \]  

(2)

In such case, the structure is characterized by its circular frequency, \(\omega^2_0 = (k / m)\), and its critical damping ratio, \(\xi = c / 2m\omega_0\), and the dynamic equation (Eq. (1)) reduces to its acceleration form:

\[ \ddot{u} + 2\xi\omega_0\dot{u} + \omega^2_0u = -\ddot{u}_g \]  

(3)

However, for an inelastic structure the solution of Eq.(1) can be obtained only by direct numerical integration (Newmark, Wilson, Runge-Kutta, or other numerical schemes). The most common approach is to describe the structural member capacity using resistance functions, \(Q(u)\), represented by a bi-linear model with an initial stiffness, \(k_0\), with a yield strength limit (or just strength), \(Q_y\), and with a hardening characteristic, \(a\). The solution is solved step-by-step, solving at each step a linear incremental equation:

\[ \Delta\ddot{u} + 2\xi\omega_{oi}\Delta\dot{u} + \omega^2_{oi}\Delta u = -\Delta\ddot{u}_g \]  

(4)

in which \(\omega_{oi}\) and \(\xi_{oi}\) are the instantaneous circular frequency and damping ratios, respectively, similarly with the described above. These properties depend on the restoring force function \(Q(u)\) and it’s rate of change. The response acceleration and displacement obtained form such solution characterize the inertial forces and the deformations of the structure.

Approximated solutions to the inelastic can be obtained using a linearized equation of the type of Eq.(3) with equivalent damping ratios \(\xi_{eq}\) and frequency \(\omega_{eq}\) (Iwan and Gates, 1979). This method, although approximated, yields satisfactory results for a wide range of structures (Valles, Reinhorn et al., 1996b).

It should be understood that during the dynamic response the response function is dependent on the response itself and on the history of its deformations. If one defines the function \(Q(u)\) as the strength capacity of the structure during the dynamic response, then from Eq (1) yields:

\[ Q(u(t)) = m\ddot{u}(t) + m\ddot{u}_g(t) + c\dot{u}(t) \]  

(5)

This force is dependent on the ground and response accelerations and on the damping of the system, implicitly.

The inelastic time history analysis requires a good knowledge of the expected site specific ground motions. Such ground motions are difficult to define and the results of the inelastic time history analyses may lack the necessary reliability as required for design. Analysis methods described below, which include monotonic inelastic analyses, or inelastic spectral methods can be used, alternatively. Using either spectral approach, or just engineering judgment, such methods may provide sufficient information for design.

3. MONOTONIC NONLINEAR ANALYSIS - CAPACITY OF INELASTIC STRUCTURES

Let's define the capacity of a structure as the maximum force, \(Q_{max}(u)\), and the associated deformation, \(u_{max}\), which a structure might exhibit during a series of seismic events with continuously growing intensity. It can be shown that for a bi-linear structure, the locus of all force maxima and their associated displacements are coincidental with the restoring force function \(Q(u)\) having bi-linear characteristics. Therefore, in the assumption that the locus of the maxima follows the bilinear function, then the capacity function can be calculated by increasing monotonically the inertial forces (see Eq.(5)) and determine the associated deformation. Such analysis known as the nonlinear monotonic - pushover - analysis, (or collapse mode analysis), is a simple and efficient technique to study the strength-deformation capacity of a building under expected inertial force distributions. Therefore, solution of the equation of motion is carried out at each load increment, similarly with the step-by-step analysis, but the number of analysis steps are considerably less than the ones involved in an inelastic time-history analysis. Although for a different hysteretic behavior of structures and components the above assumption is not accurate, the locus of maxima is close to the monotonic envelope according to a study underway by the author)

**Strength-deformation capacity curves:** The sequence of component yielding, and the history of deformations and shear forces in the structure can be traced, as the lateral loads are monotonically increased. Often the results are presented in graphs that describe the variation of the story shear capacity versus story drift, or base shear capacity versus top displacement, for a global description. Along the response curve, critical stages in the response can be identified, such as first cracking or yielding in structural elements. Furthermore, strength and service limit states, such as the failure of an element, the
formation of a collapse mechanism, etc., can be marked, as shown in Fig.1.

The strength-deformation capacity curve determined from a pushover analysis of a multi degree of freedom (MDOF) structure depends on the lateral force distribution used to load it (see Fig. 2). The distribution must follow the description in Eq (5), adjusted for the MDOF response:

\[
q(u)_i = m_i \phi_i \Gamma S_a (\omega_i \xi_i) \quad \text{or} \quad q(u)_i = m_i \phi_i (B_S / \Gamma_j)
\]

in which \( q, m, \phi, \Gamma, \) and \( S_a \) are the lateral force, the story mass, the modal shape (mass normalized, such that \( \phi_j^T M \phi_j = 1 \)), where \( \Gamma \) represents transposition and \( M \) is the mass matrix), the participation factor (\( \Gamma_j = \phi_j^T M \phi_j \) is the modal participation factor) and \( r \) is a vector of units (\( r^T = \{1,1,\ldots,1\} \)) and the acceleration modal spectral response for mode \( j \) at degree of freedom \( i \), respectively; It should be noted that the influence of damping required by Eq.(2) is implicitly included through the acceleration spectrum; \( BS \) is the base modal reaction, or the base shear, defined as:

\[
BS_j = Q_{BJ} = \Gamma_j^2 S_a (\omega_j \xi_j)
\]

If several modes are considered, then the lateral forces can be expressed as (similarly with the formulation for harmonic loadings (Chopra,1995)):

\[
q(u)_i = m_i \phi_i \Gamma_i S_a (\omega_i \xi_i) \bullet \text{srss}(f_j \gamma_j s_{aj})
\]

where \( \text{srss} \) is the short description of the square root of the sum of the squares superposition defined as:

\[
\text{srss}(x_j) = (\sum x_j^2)^{1/2}
\]

The nondimensional coefficients in Eq (8) are defined as ratios to the properties of the first mode, i.e. (i) the modal ratios: \( \gamma_j = \Gamma_j / \Gamma_1 \) and \( f_j = \phi_j / \phi_1 \), and (ii) the spectral ratios for various modes, defined as \( s_{aj} \)

\[
= S_a (\omega_j \xi_j) / S_a (\omega_1 \xi_1) \quad \text{and} \quad s_{aj} = S_d (\omega_j \xi_j) / S_d (\omega_1 \xi_1)
\]

for accelerations and displacements, respectively. The ratios \( s_{aj} \) and \( s_{dj} \) are dependent on the modal frequencies and can be approximated for simplicity by various functions. A suggested approximation is given by Valles, Reinhorn et al. (1996b) and shown in the Appendix II, for sake of completion.

If the base shear is expressed also in terms of the first mode characteristics:

\[
BS = \gamma_j^2 S_a (\omega_j \xi_j) \bullet \text{srss}(f_j \gamma_j s_{aj})
\]

then the force distribution can be expressed as:

\[
q(u)_i = m_i \phi_i \Gamma_1 S_a (\omega_1 \xi_1) \bullet \text{srss}(f_j \gamma_j s_{aj})
\]

or

\[
q(u)_i = BS \frac{m_i \phi_i \Gamma_1}{\Gamma_1} \frac{\text{srss}(f_j \gamma_j s_{aj})}{\text{srss}(f_1 \gamma_1 s_{aj})}
\]

Since the structure is inelastic, at each incremental load after the yielding occurred, the modal characteristics \( \phi_j, \omega_j, \Gamma_j, \) and \( S_{aj} \), change instantaneously and, therefore, all terms in Eq. (11) change as a function of displacement, \( u \). Therefore, if a monotonic analysis is done, then the dynamic characteristics at each step need to be adjusted suitably. The above analysis was developed by the author (Bracci et al, 1997) and incorporated in a computer platform, IDARC2D Ver.4.0. (Valles et al.,1996a). Eq. (11) is the general force distribution, probably the most accurate, but also the most laborious. Three alternatives can be derived from this formulation of forces for monotonic nonlinear analyses:

(a) The full formulation as shown by Eq (11), including all modes of vibration.

(b) Neglecting the higher modes in Eq (11), with adaptable first mode only. (see also Eq. (6) for \( j=1 \)).

(c) Assuming similarly to Eq (11), an engineering variation of the mode shapes as described by the

Fig.1 - Capacity curve from nonlinear analysis

Fig.2 - Loads for monotonic nonlinear analysis
seismic codes, i.e.: the force at floor “i”, located at an elevation $h_i$, is calculated according to:

$$q_i = BS\frac{W_i h_i^k}{\sum_{l=1}^{N} W_l h_l^k}$$  \hspace{1cm} (12)$$

where $k$ is the parameter that controls the shape of the force distribution. The recommended value for $k$ may be calculated as a function of the fundamental period of the structure ($T$): $1.0 < k = 1.0 + (T-0.5)/2 < 2.0$. Nevertheless, any value of $k$ can be used to consider different acceleration profiles. Note that $k = 0$ produces a constant variation of the acceleration (uniform load distribution, while $k = 1$ produces a linear variation (inverted triangular distribution), and $k = 2$ yields a parabolic distribution for the story accelerations.

Various options of these analyses were performed and evaluated for typical concrete building in New Madrid area (Valles et all, 1996b) and showed that for a regular building the resistance capacity function, $Q(u)$, resulting from the above monotonic analysis is only slightly sensitive to the methods shown in (a) through (c) above. The resulting strength-deformation capacity curves can be defined for important local characteristics of a structure, such as story shear versus story drift, or for global characteristics such a base shear versus top building displacement.

**Approximation of Capacity Curves**

The capacity diagram, which may have a curved shape, can be approximated by a set of bilinear curves (see Figs. 3) for an even more simplified analysis according to:

$$Q(u) = Q_x \left\{ u / u_y - (1 - \alpha) \left( u / u_y - 1 \right) \right\} U\left[ u / u_y - 1 \right]$$  \hspace{1cm} (13)a$$

in which, $Q_y$ and $u_y$ are the yielding strength and displacement, respectively, $\alpha = K_y / K_o$ is the post yield (hardening) stiffness ratio stiffness; while $U[u/u_y - 1]$ is a step function [equals 0 for $u/u_y < 1$ or equals 1 for $u/u_y > 1$]. The same relation can be described in terms of the ductility ratio, $\mu = u / u_y$ as:

$$Q(\mu) = Q_y \left\{ \mu - (1 - \alpha)(\mu - 1) \right\} U[\mu - 1]$$  \hspace{1cm} (13)b$$

The bilinear capacity curves are determined considering the same post-yielding stiffness, and equal energy to failure. Equating the monotonic energy to failure, the capacity curve can be expressed as:

$$A_m = \frac{1}{2} Q_y u_y + \frac{1}{2} (Q_u + Q_y)(u_y - u_y)$$  \hspace{1cm} (14)$$

where $A_m$ is the work done to monotonic failure; and $Q_y$, $u_y$ and $u_u$ are the yield force, the yield deformation, and the ultimate deformation of the equivalent bilinear capacity curve. The yield force can be determined in terms of the yield displacement $u_y$, and the post-yielding stiffness $K_y$ ($= \alpha K_o$).

$$u_y = \frac{2Q_u u_o - K_y u_o^2 - 2A_m}{Q_u - K_y u_o}$$  \hspace{1cm} (15)$$

The initial stiffness, $k_o$, or the frequency, $\omega_o = (k/m)^{1/2}$, or period, $T_o = (1/2\pi\omega_o)$, the yielding level, $Q_y$, and the post-yielding hardening coefficient, $\alpha$, are the important parameters that describe the inelastic behavior, along with the ultimate deformation capacity ($u_o$).

4. SEISMIC DEMAND FROM INELASTIC COMPOSITE RESPONSE SPECTRAS

**Seismic demand from elastic spectra.** The maximum seismic response for a single structure is dependent on its strength capacity envelope and on its energy dissipation through the hysteretic mechanism. The spectral methods offer a simple way to find the maximum response, if an appropriate spectrum is available. Attempts are done to prepare such spectra to a well defined ground motion $\ddot{u}_g(t)$ for a family of single-degree-of-freedom (s.d.o.f.) systems characterized by an initial stiffness (or period), an yielding strength, and a stiffness hardening coefficient. The response spectrum can be obtained for displacements, $S_d(\omega, \xi) = max \left| u \right|$, for absolute accelerations, $S_a(\omega, \xi) = max \left| \ddot{u} + \dddot{u} \right|$, or for any other desired response quantities, such as the acceleration at maximum displacement, identified as the pseudo-acceleration. For practical purposes the absolute acceleration response is used to determine the force acting on the support system:

$$F(\omega, \xi) = m S_a(\omega, \xi)$$  \hspace{1cm} (16)$$

Fig.3 - Equivalent bi-linear model for strength-deformation capacity

$$A_m = \frac{1}{2} Q_y u_y + \frac{1}{2} (Q_u + Q_y)(u_y - u_y)$$  \hspace{1cm} (14)$$
A typical set of response spectra are shown in Fig. 4(a) and (b). A combination of the response acceleration and displacement spectra for the same system characteristics ($\omega_o$, $\xi_o$) will form a new function defined here as the “composite spectrum,” (Freeman, 1993) as shown in Fig. 4(a) and (b) for an elastic system and for an inelastic system, respectively. For harmonic excitations, the acceleration spectrum and the displacement spectrum are approximately correlated:

$$S_a(\omega_o, \xi_o) \cong \omega_o^2 S_d(\omega_o, \xi_o)$$  

(17)

This relation is preserved (approximately) for random ground excitations. Therefore, the square of the frequency, $\omega_o^2$, can be viewed as the slope of a line crossing the composite spectral function at point $S^E(\omega_o, \xi_o)$ (see Fig. 4(a)). For a s.d.o.f. system the composite spectra provide simultaneous information on response displacement, $S_d$, and response acceleration, $S_a$. With a simple dimensional transformation the composite spectrum may be adjusted using to provide a direct relation between acceleration and force spectra for an s.d.o.f.

$$S_a(\omega_o, \xi_o) / g = Q(u) / W = Q^*(u)$$  

(18)

or for an m.d.o.f., for the base shear, (from Eq.(10)):

$$S_d(\omega_o, \xi_o) / g = \frac{Q(u)}{W} / \Gamma_i^2 \cdot \text{srss}(\gamma_j^2 s_j)$$

$$= Q^*(u)$$  

(19)

where $Q^*(u)$ will be defined as the spectral capacity and $S_a$ will be defined as the spectral demand.

The displacement spectrum is directly related to the deformation of an SDOF ($S_d(\omega_o, \xi_o)$ = $u_{\text{max}}$), however, for an m.d.o.f. the displacement spectrum is related to the story deformation:

$$S_d(\omega_o, \xi_o) = u_j / \phi_{ij} \Gamma_i \cdot \text{srss}(f_j^2 \gamma_j s_{ij}) = u^*$$  

(20)

where $u^*$ will be defined as the spectral capacity and $S_d$ will be defined as the spectral demand.

For an elastic linear s.d.o.f. system one can find the elastic response at the intersection of the elastic composite spectrum with the line sloped according to the natural period ($\omega_o = 2 \pi / T_o$) of the structure. For an m.d.o.f. system the response is found at an intersection of the line with the slope as above, however corrected as follows:

$$(S_d(\omega_o, \xi_o) / g) / S_d(\omega_o, \xi_o) = \omega_o^2 \cdot \{\phi_{ii} \text{srss}(f_{ij} \gamma_j^2 s_{ij}) / \Gamma_1 \text{srss}(\gamma_j^2 s_{ij})\}$$  

(21)

Therefore for an elastic structure in which the slope described above is the stiffness of the capacity curve (with suitable adjustment) the response can be obtained graphically at the intersection of the capacity curve with the composite spectrum. (see Fig 4(a)).

**Rigorous inelastic spectra** The equation of motion, Eq. (1) and (4) can be solved using time history analysis to obtain the maximum deformation for a variety of structures having an initial frequency of $\omega_o$, a yield force resistance, $Q_y$ and a post yielding stiffness ratio of $\alpha$. The maximum inelastic deformation, $u_{\text{max}}$, and the maximum force, $Q_y$, will define the inelastic displacement and force spectra as a function of the yielding force, $Q_y$, and the initial frequency, $\omega_o$, (or the yield displacement $u_y = Q_y / \omega_o^2$ m). The inelastic spectra of interest can be derived for selected values of the yielding strength, $Q_y$, derived from the elastic force response divided by an arbitrary factor, $R_\mu$:

$$Q_y = S^E(\omega_o, \xi_o) / R_\mu = F^E_m / R_\mu$$  

(22)

where $R_\mu$ is a constant coefficient which can be defined as a “strength response reduction factor,” and $F^E_m$ (or $S^E = S^E_a W / g$), is the elastic force response. In such case, it is possible to derive the response...
spectra for all oscillators, which have various initial frequencies, $\omega_o$, and with an yield strength defined from the elastic response devided by a certain $R_\mu$ factor ($Q_y = F_m^E(\omega_o) / R_\mu$). Spectra can be generated for a specific value of $R_\mu$. A typical composite inelastic response spectra is shown in Fig.4(b) along with a generalized strength-deformation capacity envelope of structure, previously defined in the paper.

It should be noted that the composite spectra so obtained are functions of the initial frequency, $\omega_o$, (or period $T_o$) and of the “reduction” factor, $R_\mu$, when the yield level is obtained from the elastic response spectrum. A typical family of inelastic composite response spectra are shown in Fig.5(a) linking the force response with the displacement spectra.

**Approximated inelastic spectra** The spectral curves described above are result of rigorous inelastic time history analyses based on an assumed hysteretic model (bilinear for this case), and for known accelerograms. However, for the practicing engineer a simplified inelastic spectral representation, which does not require an initial time history analysis is desired. Approximations of composite spectra can be obtained from an extensive statistical analysis of time history analyses (in progress). However, some approximated curves can be derived from currently available deformation relations developed by Krawinkler and Nasser (1992), Vidic Fajfar and Fischinger (1994), or Chang and Mander (1994) of the type:

$$u_m = u_y \left\{1 + \frac{1}{c_1} (R^e - 1)\right\}^{1/c_3}$$

(23)

where $c_1$, $c_2$, and $c_3$ are constants dependent on the ground motion frequency content defined by the corner period, $T_g$, the type of hysteretic rules, and other factors. For this paper the relation suggested by Krawinkler and Nasser (1992) for which $c_1 = c_2 = c; c_3 = 1$, is adopted, in which:

$$c = \frac{T_o^a + b}{1 + T_o^a}$$

(24)

where, $a$ and $b$ are factors defined in Appendix I. Replacing the deformation in Eq. (23) by the spectral values a relation between an approximated inelastic displacement spectrum, $S_d^I$, and a given elastic displacement spectrum, $S_d^E$, can be defined as follows:

$$S_d^I = \frac{S_d^E}{R_\mu} \left\{1 + \frac{1}{c} (R_\mu - 1)\right\} \geq \frac{S_d^E}{R_\mu}$$

(25)

where, the reduction factor is defined as:

$$R_\mu = \frac{S_A^E W}{Q_y g}$$

The inelastic acceleration spectrum, $S_a^I$, derived for a bilinear system described in Eq.13 is given by:

$$S_a^I = \frac{S_a^E}{R_\mu} \left[1 + \alpha \left(\frac{S_d^I}{u_y} - 1\right)\right]$$

(26)

where, $u_y = S_d^E / R_\mu$.

The approximated composite inelastic spectra derived from Eqs. (25) and (26) can link the elastic composite spectra $S_d^E - S_d^I$ with the reduction factor $R_\mu$, and the function of the structure strength, $Q_y$, with the post-elastic hardening coefficient $\alpha$. Composite spectra obtained from Eqs. (25) and (26) are shown in Fig. 5(a). It should be noted that the reduction factor, $R_\mu$, in Eqs. (25) and (26) is dependent on the elastic acceleration response and is not a constant for all structures. It should be noted also that the above approximated and simplified composite spectra can be used as substitute for the ones generated rigorously by time history analyses. Only the rigorous inelastic spectra are used further in paper.

Fig.5 - Comparison of approximate and time history based inelastic spectra
Seismic demand from inelastic spectra. The response of an inelastic system can be derived similarly as the elastic response from the intersection of it’s spectral capacity diagram, $Q^*(u^*)$, and the composite response spectra, if proper adjustments were made. The displacement and force response of an inelastic system for which the capacity diagram, $Q(u)$, is described by a bilinear model, i.e., by the initial natural frequency $\omega_0$ (or the initial stiffness $k_0$) along with the yielding level $Q_y$ and the post-yielding stiffness ratio, $\alpha$, can be evaluated following four steps:

(a) The “elastic force response”, $F_m^E$, is determined first from the rigorous elastic composite spectra for the initial properties;

(b) A “reduction factor”, $R_\mu$, is calculated from the ratio $F_m^E / Q_y$.

(c) Then the structure’s inelastic composite spectrum is derived by interpolation for the same $R_\mu$ determined above;

(d) The inelastic response ($F_d$, or $Q_d$, and $u_d$) is found at the intersection of the capacity diagram, $Q(u)$, and the rigorous (or the approximated) composite spectra curve for $R = R_\mu$.

The response obtained as outlined above indicates the seismic force (or acceleration) demand and the seismic displacement demand, assuming that the ultimate deformation capacity, $u_u$, at which failure occurs, is larger than this demand. If the ultimate deformation capacity is smaller than the demand, the response will lead to failure of the structure before reaching $u_u$. Notably, the demand is influenced by the cyclic response that leads to deterioration of both strength and deformation capacity, $Q(u)$. In such case, the seismic response demand can be obtained using a deteriorated capacity curve, $Q^{det}(u)$, instead of the initial capacity.

For an m.d.o.f. system the same procedure should be followed using however the spectral capacity curve, $Q^*(u^*)$, as defined by Eqs. (19) and (20) to obtain a compatible relation with the composite spectral demand. The response obtained is the “spectral response”, $Q_d^*$ and $u^*$, while the physical response $Q_d$ and $u_d$, can then be obtained by an inverse application of Eqs (19) and (20).

The inelastic demand can be obtained using either single mode or multiple modes considerations using the above procedures. It should be noted that the forcing function for the calculation of the capacity of the structure is dependent on the spectral values at each step of the analysis. However, from numerical studies of regular structures, it can be concluded that only the first mode characteristics and spectral ratios seem to be important (Valles, Reinhorn et al., 1996b). Therefore, an approximation of these spectral ratios can be made successfully using a building code approach, as indicated in Appendix II, simplifying the computations. Moreover, same numerical studies of regular buildings indicate that the single mode approach based on the first mode produces almost identical results as the multi-mode approach. For inelastic structures with uniform characteristics (stiffness and strength), i.e., “regular buildings” per NEHRP and UBC recommendations, the modal shapes $\phi_j$ do not change substantially with the loading increments. In such case, a constant force distribution (not adaptable) can be used to determine the desired capacity diagrams via the monotonic nonlinear analysis (push-over). This is the current procedure proposed by the FEMA/NEHRP 293, 1996. recommendation for future code provisions. This is also the recommended approach based on this work.

In summary, the capacity diagram of a structure, independently of the accuracy with which it was obtained, can be used to obtain an estimate of the seis-
mic demand when used with the inelastic composite spectra. For practical purposes this suggested method can use predetermined or standardized inelastic composite spectra and a simplified capacity diagram to obtain an approximation of the inelastic response without executing a dynamic time history analysis. 

**Seismic demand from elastic composite spectra using equivalent properties.** An alternative to the inelastic spectrum is the elastic spectrum for approximate dynamic characteristics of the structure, which simulate the inelastic hysteretic behavior. According to this method it is not necessary to develop inelastic spectra and the traditional elastic spectra calculated for various damping values seem to be sufficient. However, this method considers the use of an equivalent linear system to estimate the nonlinear response (see Fig. 7).

![Graphical evaluation from elastic spectra](image)

Fig. 7 Graphical evaluation from elastic spectra

Such equivalent system is defined by an increased damping depending on the maximum inelastic excursion. A summary of different methods to determine the equivalent damping ratio is presented by Iwan and Gates (1979). The average stiffness and energy method seems to give the smallest percentage of error for various ductility ratios. For this method, critical damping ratios are defined according to:

$$\zeta_{eq} = \frac{3}{2\pi^2} \left[ \frac{1-\alpha}{(1+\ln\mu) + \alpha\mu} \right]$$

for $\mu > 1$ \hspace{1cm} (27)

The idealized bilinear capacity curve obtained from the monotonic nonlinear analysis (push-over) is superimposed with the elastic spectral response curves similarly with the method using the inelastic spectra. At the intersection of the spectral capacity curve with the composite demand spectrum for the appropriate damping ratio will show the displacement demand. The response ductility obtained from the above analysis must satisfy the Eq. (27). If not, several iterations might be required. The elastic spectrum may be relabeled using ductility ratios (based on Eq. 27) instead of damping ratios. The point where the ductility along the capacity curve coincides with the equivalent ductility of the intersecting spectral demand curves, yields an estimate of the inelastic response. This method was used successfully to evaluate the contribution of various types of damping devices, viscous, viscoelastic, friction, etc to the retrofit of damaged concrete frames (Reinhorn et al., 1995). A comparison done by Kunnath et al (1996) shows that using the elastic spectra with equivalent damping produces almost identical results as the method described above using inelastic spectra for small and medium ductilities. However, at large deformations this method produces less accurate results when compared to the inelastic time history analyses.

5. EXAMPLE OF RESPONSE EVALUATION USING INELASTIC COMPOSITE SPECTRA

The above approach was used by the authors successfully for the evaluation of several concrete and steel buildings (Valles et al,1996b, Bracci et al,1997, Naeim et al, 1996). For sake of illustration, a building used for a retrofit case study, modeled after a structure in southern California, retrofitted using viscoelastic dampers (Lysiak, 1996) is presented herein. The building is a three-story reinforced concrete moment resisting frame structure with shear walls. The overall footprint of the building is 54.9m x 36.6m consisting of nine 6.1m bays in the longitudinal direction and six 6.1m bays in the transverse direction. The overall height of the structure is 12.2m, with the story heights all equal to 3.6m.

The building was modeled and analyzed using IDARC-2D Ver. 4.0 (Valles et al, 1996a), using both monotonic capacity analysis (pushover), the inelastic spectral evaluation and inelastic time history analysis. The “pushover” analysis provided the strength-deformation capacity envelope. The composite spectra for a family of simulated ground motions compatible to a prescribed site spectra of Southern California was calculated for various $R_\mu$ factors. The capacity and the composite spectra are used to determine the inelastic response (see Fig. 8(a)).
The physical capacity is first adjusted to the spectral capacity according to Eqs. 19 and 20. The initial slope, $k_0$, and the yield level $Q_y$ is determined using the secant approach ($Q_y^*/W = 0.27$) and the elastic response is obtained from the graph ($F_E^*/W = 0.94$). The force ratio $R_\mu$ is determined as $R_\mu = 0.94/0.27 = 3.5$. The spectral response is obtained by interpolation as $Q^*/W = 0.30$; $u_d^*/H = 0.0075 (=0.75\%)$. The actual response is obtained by inverse application of Eqs. 19 and 20 and is estimated as $Q/W = 0.24$, $u/H = 0.010 (=1.0\%)$.

The response obtained from the time history analysis for the same ground motion is used as a verification of the simplified procedure (see Fig. 8). The time history response indicates $Q/W = 0.268$ and the drift ratio 0.0112 (1.12%). Therefore, the inelastic spectral procedure produces sufficiently accurate results.

If the structure is retrofitted using dampers, then the response is modified due to the changes of the composite spectrum. If the damping can be represented by a pure viscous model, then the strength-deformation capacity diagram will not change (Reinhorn et al., 1995) and the response can be determined from the modified composite spectrum, as shown in Fig. 8(b). The adjusted composite spectrum is determined first for the increased damping, $S'_{\xi} (\xi_o + \Delta \xi)$; then is reduced by the corresponding reduction factor $R_\mu$ as previously described to obtain the inelastic spectra $S'(R_\mu, \xi_o + \Delta \xi)$ - see Fig. 8(b). This is intersected with the modified capacity diagram to produce the inelastic response quantities. It should be noted that if only damping is added to the structure (such as with fluid type devices (Reinhorn et al., 1995)), the capacity diagram remains unchanged after retrofit as in Fig. 8(a). However, where the source of damping provides also additional stiffness,
as described by Kelvin or Maxwell models (Reinhorn et al., 1995), the capacity diagram \( Q(u) \) will be also modified due to the increase of stiffness (see Fig. 8(b)). The response can be obtained for the inelastic composite spectra for the enhanced viscous contribution, i.e. \( S'(R, \xi + \Delta \xi) \) in Fig.8(b), intersected with a the modified capacity diagram adjusted for increased stiffness (see also Fig.8(b)). Using this technique it is possible to observe that the damping addition affects the forces and deformations differently. Additional damping always reduces deformations. Increase in stiffness further reduces deformations, but may increase overall forces (Reinhorn, 1995).

6. DAMAGE EVALUATION

The inelastic response evaluation is not complete when the seismic demand is determined. The evaluation needs to consider the seismic demand in respect to the seismic capacity. Since the inelastic analysis follows closely the strength capacity diagram, the strength demand and capacity are automatically satisfied by the analysis. The quantity which needs further evaluation is the deformation and the changes in the capacity during the response. The evaluation of damage proposed herein is based on a generalized index for which the renown Park-Ang damage model is a particular case. Derived from fatigue characteristics, the damage model compares the maximum achieved permanent deformation with an ultimate capacity to deform before breaking, which is continuously reduced in correlation with the amount of energy dissipated. Such index is 0 if the deformation is less than the yield and 1.0 in case of collapse. The values in between indicate the performance of the structure after the event if the indices are correlated with a performance scale (Reinhorn et al., 1992) in terms of spectral response the damage index can be evaluated by:

\[
DI = \left[ S_d(Q_y,T_o)/u_y - 1 \right]/\left[ u_y/u_y - 1 \right]^{*} \left[ 1 - [S_E(Q_y,T_o)/u_y]/[4Q_y(u_y/u_y - 1)] \right]^{28}
\]

where \( S_E \) is the inelastic energy dissipation spectrum associate with the round motion and can be obtained from prior analysis, while all the other quantities were defined earlier. The inelastic displacement spectrum can be also approximated from the elastic acceleration spectrum based on Eq. (25).

7. REMARKS AND CONCLUSIONS

This paper presents methods to determine building response using inelastic time history analyses or inelastic spectra evaluated for various strength reduction factors from a selected ground motion or from a given elastic spectrum. (Such a spectrum may be site-specific on one developed by averaging spectral demands for various ground motions). The maximum displacement and force responses are simultaneously obtained depending on the capacity envelope used to describe the inelastic characteristics of the structure. The comparison of time history analysis results and those of the simplified spectral method indicates good agreement for both single and multi-degree-of-freedom structures.

8. REFERENCES


Dampers.” MS Thesis, State Univ. of New York at Buffalo, N.Y.


8. APPENDIX I - COEFFICIENTS FOR APPROXIMATED INELASTIC SPECTRA

The coefficients determined by Krawinkler and Nasser (1992) are given in Table AI-1 below:

<table>
<thead>
<tr>
<th>Table AI-1 - Coefficients for approximations</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>0%</td>
</tr>
<tr>
<td>2%</td>
</tr>
<tr>
<td>10%</td>
</tr>
</tbody>
</table>

α - postyielding stiffness (hardening) ratio

9. APPENDIX II - SPECTRAL RATIOS

The spectral ratios can be approximated by:

\[
s_{a j 1} = \begin{cases} 
1 & \text{for } T_i \leq T_s \\
T_i / c & \text{for } T_i > T_s \text{ and } T_j < T_s \\
(T_i / T_j)^r & \text{for } T_j \geq T_s 
\end{cases}
\]

where \( T_s \) is the cut-off period for velocity dependent range and, \( r \) and \( c \) the spectral constants, \( r=2/3 \) in NEHRP (1995), or \( r=1 \) in UBC (1995) and NEHRP (1997), and \( c = A_s \) in all standards.

\[
s_{a j 1} = \begin{cases} 
(T_i / T_j)^2 & \text{for } T_i \leq T_s \\
(T_i / T_j)^2 * T_i^r / c & \text{for } T_i > T_s \text{ and } T_j < T_s \\
(T_i / T_j)^{2+r} & \text{for } T_j \geq T_s 
\end{cases}
\]