Study of wire rope systems for seismic protection of equipment in buildings

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Wire rope isolators have found numerous applications in the shock and vibration isolation of military hardware and industrial machinery. In this study, the usefulness of these devices for the seismic protection of equipment in buildings is investigated. Installation methods of entirely supporting equipment on wire rope isolators and of combining them with locked casters are studied experimentally and analytically. It is found that the use of wire rope isolators in stiff configurations may substantially improve the seismic response of equipment in comparison to other installation methods. Mathematical models for describing the hysteretic behaviour of wire rope isolators are developed and experimentally calibrated and verified. Analytical predictions of seismic response are shown to agree well with experimental results.

Keywords: equipment, seismic, isolation, wire rope, buildings

The principle of seismic isolation is to introduce an interface at the base of a structure that can attenuate the magnitude of the horizontal movement of the ground transmitted to the structure during an earthquake. This results in a significant reduction in floor accelerations, storey shears and inereory drifts, thus providing protection to the structure itself as well as to all items and equipment mounted on the structure. The reduction of the seismic forces imparted to the structural system is achieved by introducing flexibility and energy absorption capability in the isolation system. The introduction of flexibility increases the fundamental period of the isolated structure to values well above the predominant period of the earthquake excitation so that the isolation effect is primarily produced by the absorption of the earthquake energy. This desirable effect is, however, produced at the expense of large isolation system displacements which are in the range of 200–500 mm for strong earthquake excitation. While the displacements appear to be large, they are in reality small in comparison with the building dimensions and can be accommodated by the isolation system, usually without instability problems.

The same principle may be used to isolate and directly protect sensitive equipment housed mainly in conventionally constructed buildings where the high floor accelerations during an earthquake can be catastrophic for them.

However, earthquake motions, when transmitted through conventionally constructed buildings, which in strong excitation respond elastically, reach the upper floors amplified and with their frequency content spread over a wide range of frequencies. Isolation in this case becomes difficult. To achieve effective isolation, it is necessary to increase the period of the isolated equipment to large values which typically are larger than those required for effective isolation of buildings. This results in displacements which are unacceptable for single items of equipment. Furthermore, the construction of very flexible isolation systems for single items of equipment is impractical because such systems are usually not capable of carrying the weight of the supported equipment.

To counteract these problems, the Japanese construction industry developed elaborate isolation systems for computer floors which support many items of equipment. These systems utilise either low friction sliding bearings, or multistage rubber bearings, or pneumatic isolators.

The seismic protection of a single piece of equipment may be also achieved not by lengthening their period, and thus delaying the earthquake energy, but by absorbing the earthquake energy through a stiff and highly energy-dispersive system. Such a system may provide a degree of protection while allowing relatively small displacements. Makris and reported experimental results on a system consisting of helical steel springs immersed in highly viscous fluid for the seismic protection of equipment. The system was used to support a slender equipment cabinet which was subjected to strong floor seismic motions. The system, which resulted in a frequency of 3.5 Hz in the isolated equipment, was
capable of reducing accelerations by a factor of 2 in comparison to the nonisolated equipment, while allowing displacements at the isolation level which did not exceed 10 mm. This spring- viscous damper system evolved from a widely used vibration isolation system. Other systems which are widely used in shock and vibration isolation of equipment are investigated in this paper for use in seismic protection systems. Wire rope isolators are mounting assemblies made of stranded wire rope which is wound on a bobbin and held in place by metal retainers. In a further development, arch wire rope isolators are formed by two groups of oppositely inclined, arch-like, open-loop wire rope elements which are clamped between retainer bars. Both helical and arch wire rope isolators consist of twisted stainless steel cable. They have flexibility in all three directions, large displacement capacity and inherent self centering due to friction between the intertwined wires. Their ability to absorb energy is the same in all three directions. These isolators have found numerous applications in the shock and vibration isolation of industrial and defense equipment, electronic systems, critical machinery and other sensitive equipment.

This paper reports the results of an investigation on the use of wire rope isolators as a means of providing seismic protection to single slender items of equipment which are attached to vibrating floors. Helical and arch type wire rope isolators were used to support a slender equipment cabinet in various configurations. Furthermore, helical wire rope isolators were used to provide only restoring force in a computer equipment supported by casters. The isolated equipment was subjected to a shake table to floor excitation which was determined by filtering recorded earthquake motions through an actual seven-storey building. Experimental results were also obtained for the equipment being either fixed to the floor or supported to the floor by other commonly used means. It was found that for certain configurations of wire rope, it was possible to achieve substantial reduction of the acceleration transmission to the isolated equipment in comparison to other conventional means of supporting the equipment. This paper reports results on three of these configurations. Further results are presented in a report by Demetriades et al. The dynamic behaviour of wire rope isolators are also developed, calibrated and presented. The models are capable of describing, with good accuracy, the observed dynamic response of the tested equipment.

Table 1 Geometrical characteristics of tested wire rope isolators

<table>
<thead>
<tr>
<th>Designation</th>
<th>Type</th>
<th>Testing direction</th>
<th>Number of coils</th>
<th>Diameter of rope (mm)</th>
<th>L (mm)</th>
<th>W (mm)</th>
<th>H (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isolator 1</td>
<td>Arch</td>
<td>Comp/Comp</td>
<td>2</td>
<td>12.700</td>
<td>285.75</td>
<td>116.84</td>
<td>120.65</td>
</tr>
<tr>
<td>Isolator 4</td>
<td>Helical</td>
<td>Comp/Comp</td>
<td>8</td>
<td>15.875</td>
<td>246.70</td>
<td>152.60</td>
<td>119.38</td>
</tr>
<tr>
<td>Isolator 5</td>
<td>helical</td>
<td>Single</td>
<td>8</td>
<td>9.525</td>
<td>215.90</td>
<td>104.90</td>
<td>76.20</td>
</tr>
</tbody>
</table>

Modelling of wire rope isolators

Wire rope isolators have different response characteristics depending on the diameter of the wire rope, the number of strands, the cable length, the cable twist, the number of cables per section and the direction of the applied force.

To determine their dynamic characteristics in the horizontal and vertical directions, a series of dynamic tests was conducted on a number of isolators by imposing cyclic sinusoidal motion of specified amplitude, frequency and initial force. Five wire rope isolators were selected for testing. Results are presented for three of these isolators. Their geometrical characteristics are presented in Table 1 with reference to Figure 1. The first two of these isolators were used in the isolation of an equipment cabinet in which the isolators supported the weight. During shake table testing, these isolators were subjected to simultaneous compression/tension and full rotations (see Figure 1). Accordingly, component testing was restricted to only two directions. The third isolator (No. 5 in Table 1) was used in the isolation of a computer equipment which was supported by casters. In this application, the isolators were only subjected to shearing motion (see Figure 1) without allowance for vertical motion. They were, accordingly, only tested in that direction.

Wire rope isolators exhibit nonlinear hysteretic behaviour. In applications in which the isolators carry the weight of the isolated equipment, they are subjected to simultaneous motions in all three directions. It is, thus, expected that the forces which develop in these three directions exhibit interaction. In modelling the behaviour of the isolators, it was assumed that this interaction is not important and that each isolator can be modelled by three hysteretic, noninteracting spring elements placed along the three principal directions (vertical, roll and shear).

Testing and modeling in horizontal direction

Isolators 1 and 4 (Table 1) were tested in the roll direction by an arrangement which could impose motion in the roll direction while allowing for simultaneous full rotation in the vertical direction. This restrained the behaviour of the isolator in its actual use in which they are allowed to reduce in height during horizontal deformation. However, the arrangement could not precisely simulate the actual conditions, so some stiffening of the isolators was observed at large horizontal displacements. This stiffening was disregarded in the modelling.

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Tests were conducted by imposing five cycles of motion at frequencies of 0.1, 1, 2 and 5 Hz and amplitude of 6.4, 12.7 and 19.1 mm. Recorded force—displacement loops showed stable hysteretic behavior for all five cycles, symmetry and independence to frequency.

The recorded behavior represented classical hysteretic behavior and could be easily modelled by the smooth bilinear hysteretic model of Boss[2]. The model, in its more general form by Wen[6] is

$$ F = \alpha \frac{Z}{Y} U + (1 - \alpha) F Z $$  \hspace{1cm} (1)

where $F$ = force, $U$ = displacement and $Z$ is a hysteretic dimensionless quantity given by the following differential equation:

$$ YZ + \gamma UZ^{p-1} + \beta UZ^p - AU = 0 $$  \hspace{1cm} (2)

In the above equations $\alpha$, $\beta$, $\gamma$, $A$ and $n$ are dimensionless quantities that control the shape of the hysteretic loop, and $F$, $Y$, and $Z$ are the yield force and yield displacement, respectively. A dot denotes differentiation with respect to time.

The authors[2] obtained closed-form solutions of equation (2) and showed that parameters $A$, $\beta$ and $\gamma$ should satisfy the condition $A = \beta + \gamma$. Furthermore, Constantinou et al.[20] have shown that for $A = 1$, $\alpha$ represents the ratio of postyielding to preyielding stiffness. Under these conditions, the model collapses to a model of viscoelasticity.

For the analytical modelling of wire rope isolators, the values $\beta = 0.1$, $\gamma = 0.9$, $A = 1$ and $n = 1$ were used. The other parameters of the model are listed in Table 2. This table includes two more quantities of interest. They are the postyielding stiffness, $K$, and the characteristic strength, $Q$. Comparisons of experimental and analytical force—displacement loops are presented in Figure 2. It may seem that the analytical model predicts well the experimental results.
Table 2 Parameter of model of isolators in roll and shear direction

<table>
<thead>
<tr>
<th>Direction</th>
<th>Isolator 1</th>
<th>Roll</th>
<th>Kf (N mm⁻¹)</th>
<th>F [mm]</th>
<th>Q (%)</th>
<th>Ff (N)</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll</td>
<td>30.2</td>
<td>0.254</td>
<td>59.0</td>
<td>65.5</td>
<td>0.1169</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roll</td>
<td>120.1</td>
<td>0.381</td>
<td>445.9</td>
<td>49.1</td>
<td>0.0331</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear</td>
<td>40.1</td>
<td>0.254</td>
<td>62.4</td>
<td>73.1</td>
<td>0.1462</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Testing of isolator no. 5 was conducted with a different arrangement which maintained constant height of the isolator during deformation in shear. The behaviour in shear of isolator no. 5 was qualitatively the same as that of the other isolators in roll. The model of equations (1) and (2) reproduced well the experimental response as illustrated in Figure 2. The parameters of the model are shown in Table 2.

Testing and modelling in vertical direction

Isolators no. 1 and 4 were tested in the vertical (compression—tension) direction. The hysteretic behaviour of the isolators in the vertical direction exhibited asymmetry due to different stiffnesses in tension and in compression. Figure 2 provides evidence for this behaviour. In compression, the isolator exhibits essentially elastoplastic behaviour while in tension it exhibits an increasingly stiffening behaviour. It should be noted that energy dissipation, as expressed by the difference between the loading and unloading branches of the loops, is different in tension from in compression.

Hysteretic models describing asymmetric behaviour of the type shown in Figure 2 could be derived by a modification of the model of equations (1) and (2). The force-displacement relation is written in the form

\[ F = F_f(U) + F_f(U)|Z| \]

(3)

\[ F = F_f(U) + F_f(U)|Z| \]

Figure 2 Comparison of experimental and analytical force-displacement loops in horizontal direction. (--), test; (---), model.

Figure 3 Comparison of experimental and analytical force-displacement loops in vertical direction. (--), test; (---), model.

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in which \( F_s \) represents the displacement-dependent skeleton curve and \( F_p \) is also the displacement-dependent half difference between the loading and unloading branches of the loop, \( Z \) is a hyperbolic dimensionless quantity taking values in the interval \([-1, 1]\) and is described by equation (2). The two parts of equation (3) describe, respectively, stiffness and hysteretic energy dissipation.

Experimental force–displacement loops, like those of Figure 3, indicate that functions \( F_s \) and \( F_p \) could be expressed in the form:

\[
F_s(U) = \mathcal{Q}_s \left( a_s \exp \left( \sum_{n=1}^{N} a_n U^n \right) \right)
\]

\[
F_p(U) = \mathcal{Q}_p \exp \left( \sum_{n=1}^{M} b_n U^n \right)
\]

in which \( \mathcal{Q}_s, a_s, a_n, N, \mathcal{Q}_p, M \) and \( b_n \) are coefficients derived from regression analysis of experimental results for each isolator. Values of these coefficients for the tested wire rope isolators are presented in Tables 3 and 4.

Based on the conclusions of analytical solutions presented by Demetriades et al.27 and comparisons of experimental and analytical compression–tension loops, the following values were selected for the remaining model parameters: \( A = 3.0, B = 0.0, \gamma = 3.0 \) (\( A = \beta + \gamma \)) and \( \beta = 1 \). Values of the displacement quantity \( Y \) were different for each isolator as shown in Table 4. Comparisons of experimental and analytical force–displacement loops of the isolators in compression–tension mode are presented in Figure 3. Each of these loops is for a specific initial compression force imposed to the isolator prior to initiation of the cyclic motion and for five cycles of motion. The loops for the stiff isolator no. 4 are for small amplitudes of displacement. Evidently the analytical model predicts the experimental behaviour with good accuracy.

Study of wire rope support systems for equipment

To determine the effectiveness of wire rope isolation systems, as well as to verify the validity of the mathematical models developed for wire rope isolators, an equipment cabinet was tested on the shake table. It was subjected to floor earthquake motions under isolated and nonisolated conditions. In all tested systems, the cabinet was supported by four wire rope isolators. Due to its slender configuration, the cabinet could undergo substantial rocking motion. Three systems of different stiffness characteristics were tested, whereas only one system was only analysed. Herein, results are presented for systems 1 and 4 which, respectively, utilized isolators no. 1 and 4 (Table 1). Further results are given elsewhere27.

Description of equipment and isolation system

The tested equipment is shown in Figure 4. The equipment is 1800 mm in height and has plan dimensions of 559 mm by 762 mm. It consists of five horizontal diaphragms (isolator level, levels 1, 2, 3 and top level) which are connected together by side walls, only in the longitudinal direction. Its weight is 1784 N and the centre of mass was determined to be at the height of level 1 and at the geometric centre of the cabinet's plan. The radius of gyration of the equipment about a horizontal axis passing through the centre of mass and parallel to the longitudinal direction of the cabinet was determined to be \( r = 580 \) mm.

The isolation system consisted of four wire rope isolators placed at a distance of 464 mm in the transverse direction as shown in Figure 4. Each isolator carried a load of 446N. Seismic excitation was applied in the vertical and transverse directions so that the isolators were subjected to combined vertical, and roll motions. The instrumentation consisted of 21 channels. The instruments monitoring the response in the vertical and transverse (testing) directions are shown in Figure 4.

| Table 3 | Coefficients in function \( F_s \) of model of wire rope isolators in vertical direction |
|---------|---------------------------------|----------------|----------------|----------------|
| Coefficient | Value | Value | Value | Value |
| \( \alpha_1 \) | \( \times 10^{-3} \) | \( \times 10^{-3} \) | \( \times 10^{-3} \) | \( \times 10^{-3} \) |
| Isolator 1 | -1283.1 | 1.00 | 3 | 70.315 | 0.544 | 12.327 |
| Isolator 4 | 4026.5 | 1.00 | 5 | 51.654 | -0.888 | 20.504 |

| Table 4 | Coefficients of function \( F_p \) of model of wire rope isolators in vertical direction |
|---------|---------------------------------|----------------|----------------|----------------|
| Coefficient | Value | Value | Value | Value |
| \( \beta_1 \) | \( \times 10^{-3} \) | \( \times 10^{-3} \) | \( \times 10^{-3} \) | \( \times 10^{-3} \) |
| Isolator 1 | 4.459 | 3 | 3.555 | 61.945 | 1.755 | 15.378 | 2.543 |
| Isolator 4 | 4.459 | 1 | 1.882 | 48.425 | - | - | 1.597 |

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The equipment was tested under isolated and nonisolated (fixed) conditions. Identification tests of the nonisolated equipment gave a fundamental frequency of 10.3 Hz, and viscous damping ratio of 0.6% in the transverse direction. The earthquake excitation consisted of the 1952 Taft (Kern County, CA, Taft Lincoln School model, component NZE and vertical), 1990 El Centro (Imperial Valley, CA, component SDOE and vertical) and 1971 Pacoima Dam (San Fernando, CA, component S/4N and vertical) records. The Taft and El Centro motions were filtered through an actual seven-story building in an attempt to generate floor motions.

The seven-story building is the reinforced concrete building tested using the full-scale pseudodynamic testing facility at Tsukuba, Japan under the US-Japan cooperative research programme. Available information and experimental data for this structure enabled the development of a detailed elastic model for the structure using program IDARC. The computed time histories of acceleration at the fifth and seventh floors of this structure were used as input to the shake table without any time scaling. The very small weight of the equipment in relation to that of a typical floor (~1/1000) let us neglect equipment-structure interaction in the analyses.

As an example of floor motion, Figure 5 shows the horizontal component of the seventh floor acceleration history (as produced by the shake table) for the El Centro input and its acceleration response spectrum. The vertical component of input accelerometer was transmitted through the structure unchanged. One may note the considerable amplification and filtering of the horizontal component of the ground motion at the seventh floor of the structure.

This amplification occurs in the period range 0.5–1.5 s, the range which contains the fundamental period of the yielding seven-story structure. In the Taft motion (see Demetriades et al.), this amplification occurs over a shorter range of periods, 0.5–0.9 s. This is because the Taft motion is weaker and causes less instant inertia action in the seven-story structure than the stronger El Centro motion.

Furthermore, the spectra of Figure 5 show amplification in a shorter range around 0.2 s period. This is due to the higher mode response of the seven-story structure. Similar amplification was evident in the floor spectra for the Taft motion. The spectrum of the seventh floor El Centro motion is consistent with published floor response spectra for the design of equipment. The motions used in this experimental program are identical to those used by Makris in testing of the same cabinet with another isolation system.

Results
The peak response of the cabinet under isolated and nonisolated (fixed) conditions is presented in Table 3. The results of this table demonstrate that system 1, which
was the most flexible with fundamental period of about 1.2 s, was effective in reducing accelerations by factors of the order of two. Furthermore, system 4, which was very stiff with a fundamental period of about 0.2 s was capable of reducing accelerations in the strongest motions (El Centro seventh floor and Pacoima) while allowing very small displacements.

In explaining the behaviour of the systems, their dynamic characteristics were determined. Puff-release tests on system 1 revealed that for displacements such as those experienced in the Taft seventh floor motion, the effective period, $T$, was equal to 1.15 s and the equivalent viscous damping ratio, $\xi$, was equal to 0.11. These dynamic characteristics were, furthermore, confirmed in a study of the moment–rotation $(M-\theta)$ relation of the isolation system. The moment, $M$, exerted to the base by the isolators was determined from the experimental histories of the vertical isolator displacements by integration of equation (2)–(5). Such $M-\theta$ loops are shown in Figure 6 for the case of the Taft seventh floor tests. It is interesting to note that these loops exhibit symmetrical hysteretic behaviour, unlike the force–displacement loops of individual isolators (see Figure 7).

In a similar manner, the dynamic characteristics of system 4 were determined to be for the Taft 7th motion: $T = 0.15$ s and $\xi = 0.23$, and for the strongest Pacoima motion to be $T = 0.20$ s and $\xi = 0.3$. Evidently, system 4 is very stiff but with significant energy dissipation capability. In contrast, system 1 has lower energy dissipation capacity because it is flexible and undergoes large displacements so that the intertwined cables lose contact and, accordingly, sliding friction is reduced. Based on these dynamic characteristics and using the spectra of Figure 5, we observe that system 4 falls within the range of highest spectral acceleration. Yet, its acceleration response is lower than that of the fixed system because of its substantial ability to dissipate energy. In contrast, system 1 falls beyond the range of high spectral acceleration and, thus, it behaves as a classical isolation system with large displacements.

It may be concluded that overall, the seismic behaviour of equipment may be substantially improved by supporting them on stiff wire rope isolators. Under such conditions, the isolators undergo small displacements, exhibit large damping capacity and prevent the occurrence of resonances.

### Analytical prediction of response

A mathematical model of equipment supported by wire rope isolators is developed for the analytical prediction of dynamic response as if the excitation consists of in-plane response as if the excitation consists of components in a vertical plane and the system exhibits no eccentricities. Furthermore, three assumptions are made:

1. The possible interaction between vertical and horizontal components of force developed at a wire rope isolator is negligible. Each isolator is modelled by two independent hysteretic elements (springs) which exhibit the characteristics described by equations (1)–(5). The two springs are placed in the vertical and horizontal directions at each location of wire rope isolator.
2. The rotational stiffness of wire rope isolators is negligible.
3. The equipment is rigid

Figure 7 shows the model of a rigid equipment supported by wire rope isolators. The springs representing wire rope isolators are symmetrically placed at distance $a$ from the centre of mass and at the same height $h$ below the centre of mass. The degrees of freedom are selected to be the horizontal, $u_x$, and vertical, $u_y$, displacements and rotation, $\theta$, of the centre of mass.
Equations of motion for large rotations

The equations of motion are first derived by considering large rotations and subsequently reduced to their geometrically linear form (small rotations). Two orthogonal coordinate systems are defined. The first, XZ, is fixed in time and defined to have its origin at the initial position of static equilibrium (at time $t = 0$) of the centre of mass. The second, $X'Z'$, is moving with the centre of mass as illustrated in Figure 7.

A point $i$ having coordinates $(X, Z)$ in the moving system can be defined in terms of coordinates in the fixed initial system by the following transformation:

$$
\begin{bmatrix}
    X \\
    Z \\
\end{bmatrix} =
\begin{bmatrix}
    u_x & u_y \\
    -\sin \theta & \cos \theta \\
\end{bmatrix}
\begin{bmatrix}
    X' \\
    Z' \\
\end{bmatrix}
$$

The displacements of point $i$ with respect to its initial position are

$$
\begin{bmatrix}
    u_x \\
    u_y \\
\end{bmatrix} =
\begin{bmatrix}
    X \\
    Z \\
\end{bmatrix} -
\begin{bmatrix}
    X' \\
    Z' \\
\end{bmatrix}
$$

At time $t = 0$ the two systems are identical and therefore $X = X'$ and $Z = Z'$. 

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The equations of motion of the equipment are derived from dynamic equilibrium in the deformed configuration

\[ m_i \ddot{u}_i + 2F_{i1} + 2F_{i2} = -m_i \ddot{u}_i \]  
(8)

\[ m_i \ddot{u}_i + 2F_{i1} + 2F_{i2} + W = -m_i \ddot{u}_i \]  
(9)

\[ I_i \ddot{\theta} - 2F_{i1}D_{i1} - 2F_{i2}D_{i2} + 2F_{i3}D_{i3} + 2F_{i4}D_{i4} = 0 \]  
(10)

where: \( I_i = \text{m} \cdot \text{r}^2 \) is the moment of inertia of the equipment about the horizontal axis passing through the centre of mass, \( r = \text{radius of gyration}, m = \text{mass}, W = \text{weight and} \ u_i, \text{and} \ \ddot{u}_i \) are the horizontal and vertical components of the input motion, respectively. Furthermore, \( F_{i1}, F_{i2}, \) and \( F_{i3} \) are the spring forces in the horizontal direction at points \( B1 \) and \( B2 \) which are given by equations (1) and (2) for displacement \( U = u_{xy} \) and \( U = u_{xy}^2 \), respectively. Moreover, \( F_{i1}, F_{i2}, \) and \( F_{i3} \) are the spring forces in the vertical direction at points \( B1 \) and \( B2 \) which are given by equations (2)–(5) for vertical displacement \( U = u_{xy} \) and \( U = u_{xy}^2 \), respectively. Distances \( D_{i1}, D_{i2}, D_{i3}, \) and \( D_{i4} \) represent the horizontal and vertical arms of isolator forces with respect to the centre of mass and are given by the following expressions:

\[ D_{i1} = a \cos \theta + b \sin \theta \]  
(11)

\[ D_{i2} = a \cos \theta - b \sin \theta \]  
(12)

\[ D_{i3} = -a \cos \theta + b \cos \theta \]  
(13)

\[ D_{i4} = a \cos \theta + b \cos \theta \]  
(14)

Equations (1)–(5) and (8)–(9) form a system of ten first-order differential equations. The initial conditions are zero except for the displacement \( u_i \) which is equal to the static vertical displacement of the isolators due to the weight of the equipment. Numerical solutions were derived by use of Gear’s implicit multistep integration scheme with adaptive time step.

**Equations of motion for small rotations**

For small rotations, \( \sin \theta \) and \( \cos \theta \) may be expanded in binomial series. Correct to \( 0(\theta^3) \), \( \sin \theta \) and \( \cos \theta \) may be replaced by \( 1 \) and \( \theta \), respectively. The error involved in such an approximation does not exceed 2% of exact values of \( \theta \) up to 0.2 rad (11 \( \text{rad} \)). In all of the tests presented herein, the angle of rotation did not exceed the limit. Accordingly, the geometrically linear equations of motion introduce errors which are insignificant for practical purposes.

The geometrically linear form of the equations of motion is:

\[ m_i \ddot{u}_i + 4K_{i1}u_i = -m_i \ddot{u}_i \]  
(15)

\[ m_i \ddot{u}_i + 4K_{i1}u_i + 4K_{i2}U = -m_i \ddot{u}_i \]  
(16)

\[ I_i \ddot{\theta} - 4F_{i1}h_i + 2F_{i3}(a + \theta h) - 2F_{i4}(a - \theta h) = 0 \]  
(17)

In these equations, \( F_{i1} \) is given by equations (1) and (2) with \( W = u_i - \theta h_i \) and \( F_{i2} \) is given by equations (2)–(5) with \( U = u_{xy} \) and \( U = u_{xy}^2 \), respectively. Integration of the geometrically linear equations of motion resulted in responses which were almost identical to those obtained from the geometrically nonlinear equations. This was the case for all analysed systems, some of which underwent rotations of up to 0.2 rad. The geometrically linear equations of motion are useful in the development of a simplified analysis procedure which can be used together with floor response spectra to obtain estimates of the peak response.

**Simplified analysis procedure**

A simplified analysis procedure is developed by assuming that each wire rope isolator may be represented by two linear springs of stiffness \( K_i \) in the horizontal direction and stiffness \( K_i \) in the vertical direction. Furthermore, the energy dissipation, is accounted for by the use of equivalent viscous damping ratios in modal analysis. Forces \( F_{i1}, F_{i2}, \) and \( F_{i3} \) are expressed as:

\[ F_{i1} = K_i(u_i - \theta h) \]  
(18)

\[ F_{i2} = K_i(u_i + \theta h) \]  
(19)

\[ F_{i3} = K_i(u_i - \theta h) \]  
(20)

Substituting equations (18)–(20) into (15)–(16), the linear equations of motion are derived after dropping higher-order terms:

\[ m_i \ddot{u}_i + 4K_{i1}u_i - 4K_{i2}U = -m_i \ddot{u}_i \]  
(21)

\[ m_i \ddot{u}_i + 4K_{i1}u_i + W = -m_i \ddot{u}_i \]  
(22)

\[ I_i \ddot{\theta} - 4K_{i1}h_i + 4K_{i2}a^2 + 4K_{i3}h_i \theta = 0 \]  
(23)

In these equations, \( u_i \) is decoupled from the other degrees of freedom because the system has no eccentricities. Accordingly, the analysis for vertical excitation may be performed independently of that for horizontal excitation.

Concentrating on the coupled horizontal–rocking response, equations (21) and (23) are expressed in the following matrix form upon division by mass \( m_i \):

\[ \begin{bmatrix} \dot{U} \end{bmatrix} = \begin{bmatrix} K \end{bmatrix} \begin{bmatrix} U \end{bmatrix} \]  
(24)

where \( \begin{bmatrix} I \end{bmatrix} \) is the identity matrix

\[ \begin{bmatrix} U \end{bmatrix} = \begin{bmatrix} u_i \cr \theta \end{bmatrix}, \quad \begin{bmatrix} K \end{bmatrix} = \begin{bmatrix} \omega_i^2 & 0 \\ 0 & \omega_i^2 \end{bmatrix} \]  
(25)

and

\[ \begin{bmatrix} \omega_i^2 h_i^2 \cr \omega_i^2 \end{bmatrix} = \begin{bmatrix} \omega_i^2 h_i^2 \\ \omega_i^2 \end{bmatrix} \]  
(26)

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In the above equation

\[ \omega_i = \left( \frac{4K_i}{m} \right)^{1/2}, \quad \omega_n = \left( \frac{4K_n}{m} \right)^{1/2} \tag{27} \]

are the roll and vertical frequencies of the isolated equipment, respectively.

Equation (24) may be solved for the modal properties of the linearized system which together with representative damping ratios may be used to obtain estimates of the peak response of the system to horizontal excitation. In free vibration, \( \phi_n = 0 \) and

\[ \{ u \} = \{ \phi_n \} e^{\gamma t} \tag{28} \]

in which \( \omega_n \) = frequency of free vibration and \( \phi_n \) and \( \phi_n \) are elements of mode shape \( \{ \phi \} \) corresponding to \( u \) and \( \phi_n \), respectively. Equation (24) takes the form

\[
\begin{bmatrix}
\omega_i^2 - \omega_n^2 & -\omega_n \left( \frac{h_i}{r} \right) \\
-\omega_n \left( \frac{h_i}{r} \right) & \omega_n \left( \frac{a_i}{r} \right) + \omega_n \left( \frac{a_n}{r} \right) - \omega_n^2 \\
\end{bmatrix}
\begin{bmatrix}
\phi_n \\
\phi_n \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
\end{bmatrix} \tag{29}
\]

The requirement for a nontrivial solution gives the characteristic equations from where the frequencies of the coupled system, \( \omega_i \) and \( \omega_n \) may be calculated:

\[
\omega_i^2 - \left( 1 + \frac{h_i}{r} \right) \omega_n^2 + \frac{a_i}{r} \omega_n^2 = 0 \quad n = 1, 2 \tag{30}
\]

With the frequencies calculated from equation (30), equation (29) is used to obtain the mode shapes. The calculation of peak response values may be performed by the modal analysis approach.\textsuperscript{60}

The simplified procedure is very useful for obtaining quick estimates of the peak response. It requires, however, the effective stiffnesses of the wire rope isolators and the effective damping ratio of the system. The effective stiffness may be obtained from experimental force-displacement loops of the isolators. This requires the employment of an iterative procedure to first calculate displacements and subsequently estimate stiffnesses and refine calculations. The effective damping ratio, however, cannot be selected a priori. A nonlinear dynamic analysis for harmonic excitation may be performed and calculated moment-rotation loops may be used to obtain values of effective damping ratio. Otherwise, damping must be selected with conservatism and based on experience.

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Comparison of experimental and analytical results

Comparisons of experimental and analytical time histories of the response of system 1 are presented in Figure 8. All analyses were based on the geometrically nonlinear formulation. Analyses with the geometrically linear equations gave almost identical results. The parameters in the analytical model were \( W = 1784 \text{ N}, \ r = 580 \text{ mm}, \ a = 231.8 \text{ mm}, \ h = 1021 \text{ mm} \) and \( h = 919.5 \text{ mm} \). The analytical time history results are in good agreement with the experimental results. Further comparisons of analytical and experimental time histories of response may be found in Demetriades et al.\textsuperscript{10}

Peak response values of system 1 as computed by time history analysis and by the simplified method are compared to experimental results in Table 6. The simplified method required estimates of isolator stiffnesses and damping ratio. They were determined by the following procedure. The system was numerically analysed for harmonic excitation and loops of moment-rotation were determined. From these loops values of effective period and equivalent viscous damping ratio were

![Figure 8](image-url)

Figure 8: Comparison of experimental and analytical time histories of response of isolated cabin (system 1) for El Centro seventh floor excitation. ---, test; ••••••, model
calculated. Furthermore, representative values of the vertical isolators stiffness, $K_v$, were determined from

$$K_v = \frac{M}{\theta} = 4K_ea^2$$  \hspace{1cm} (31)

These quantities are listed in Table 7 as a function of the amplitude of rotation ($2\theta$). Moreover, representative values of the isolator horizontal stiffness, $K_e$, were determined from experimental lateral force-displacement loops.

The analysis was performed by assuming a representative value of rotation, $\theta$, then selecting appropriate values of the vertical stiffness, $K_v$, and damping ratio, $\xi$, and performing a response spectrum analysis. Subsequently, the calculated value of rotation was used to improve the estimates of $K_v$ and $\xi$, using the data of Table 7 and the analysis was repeated.

The results shown in Table 6 demonstrate good agreement between analytical and experimental peak response values. It may be stated that the simplified method is sufficiently accurate to represent this useful design tool.

**Combined wire rope and caster support system for equipment: comparative experimental study**

Description of equipment and support system

An experimental study was performed with IBM 9370 computer equipment installed on top of a raised floor and supported by casters. The casters support the weight of the equipment and allow for easy relocation on the raised floor. When in service, the casters are locked and a 90° plate angle, called the foot, is attached in the front of the body for stability. To prevent excessive displacements and overturning in earthquake excitation, positive connection of the equipment to the floor below the raised floor is normally provided by means of bungee cords or long helical steel springs. These elements run through holes in the tiles of the raised floor.

In general, earthquakes may cause effects to computer equipment which may be catastrophic when overturning occurs, or serious when damage occurs due to excessive acceleration and impact, or minor when execution is interrupted due to large accelerations or pull-out of cables. Installation methods which can reduce accelerations and displacements to acceptable levels while allowing for easy relocation of the equipment are particularly interesting to computer manufacturers. As a part of a NCEER–IBM joint research project, various computer equipment installation methods were tested. One of them consisted of wire rope isolators.

The wire rope installation method followed the standard approach in which the equipment is supported by four locked casters with the foot installed in the front. Four helical wire rope isolators no. 5 (see Table 1) were connected to metal bars in sets of two isolators each as shown in Figure 9. The two sets of isolators were placed under the equipment and bolted to the frame above and to the tiles of the raised floor below. During testing, the isolators deformed only in their shear direction.

The IBM 9370 computer equipment has plain dimensions of 920 mm by 650 mm and height of 1578 mm. Its centre of mass is located at coordinates $X = 305.6$ mm, $Y = 637.1$ mm and $Z = 391.3$ mm according to the coordinate system of Figure 9 at point O. Its weight is 3700 N. The fundamental frequency of the equipment when fixed at its base was experimentally determined to be 4.1 Hz in the testing ($X$) direction. Attempts were

<table>
<thead>
<tr>
<th>Table 6</th>
<th>Comparison of experimental and analytical time response values of system 1 for Tah: 7th floor input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>Time History</td>
</tr>
<tr>
<td>Top horizontal acceleration (g)</td>
<td>0.625</td>
</tr>
<tr>
<td>Top horizontal displacement (mm)</td>
<td>105.74</td>
</tr>
<tr>
<td>Isolator horizontal displacement (mm)</td>
<td>7.67</td>
</tr>
<tr>
<td>Isolator S vertical displacement (mm)</td>
<td>26.19</td>
</tr>
<tr>
<td>Isolator N vertical displacement (mm)</td>
<td>20.47</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 7</th>
<th>Properties of system 1 extracted from moment - rotation loops</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{10}$ (N·m)</td>
<td>$2\theta_{m}$ (mm)</td>
</tr>
<tr>
<td>1886.2</td>
<td>14.98</td>
</tr>
<tr>
<td>2675.5</td>
<td>26.40</td>
</tr>
<tr>
<td>3255.1</td>
<td>38.43</td>
</tr>
</tbody>
</table>

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made so determine the coefficient of friction at the interface of locked casters and angle foot and supporting raised floor. The procedure described by Constantinou [11] was used, however, it was not possible to exactly determine the coefficient of friction. The coefficient was found to be in the range 0.20–0.30. For such a high value of friction, the ability of wire rope isolators to dissipate energy is not important and the isolators act only as restoring force devices. From the data of Table 2, each isolator has horizontal (shear) stiffness of 42 N mm−1 so that the frequency of the isolated equipment is 5.4 Hz. This frequency is close to that of the equipment on top of the isolators so that the combined system does not behave as a rigid body.

Experimental results and comparison to other installation methods

The wire rope system was tested on the shake table. The tests were conducted with the vertical component being equal to 1/3 of the horizontal component of excitation. The horizontal component was applied in the X-direction as shown in Figure 9. The instrumentation diagram is shown in the same figure. The displacement transducers, which are shown mounted on the equipment, measured displacements of the part just above the casters with respect to the raised floor. The table excitation was filtered through the raised floor and arrived amplified at the supported equipment.

The recorded peak responses for the Taft seventh floor and El Centro earthquake floor motions (both with vertical component equal to 1/3 of horizontal component) are presented in Table 8. The response values included in these tables are the maximum among all of the recorded peak values. It should be noted that the equipment responded with some torsional motion and motion in the transverse (Y) direction. This is due to asymmetry in the distribution of the equipment’s weight (center of mass located close to the west side casters). At the level of the support system, this asymmetry was partially counterbalanced by an additional frictional force on the east side where the foot plate angle was installed.
Table 8  Comparison of peak response of equipment with different installation methods for Talh and El Centro 7th floor input

<table>
<thead>
<tr>
<th>Installation system</th>
<th>Talh 7th acceleration (g)</th>
<th>El Centro 7th acceleration (g)</th>
<th>Displacement of castors (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Locked casters</td>
<td>0.434</td>
<td>0.710</td>
<td>159.51</td>
</tr>
<tr>
<td>Locked casters and</td>
<td>0.705</td>
<td>1.221</td>
<td>239.07</td>
</tr>
<tr>
<td>bounce cords</td>
<td></td>
<td></td>
<td>569.78</td>
</tr>
<tr>
<td>Locked casters and</td>
<td>0.783</td>
<td>3.075*</td>
<td>348.25</td>
</tr>
<tr>
<td>wire rope isolators</td>
<td>0.969</td>
<td>1.056</td>
<td>285.95</td>
</tr>
</tbody>
</table>

* "peak and impact"

In general, in the case of the wire rope system, displacements are small and accelerations are at levels which cannot cause any interruption of operation of the computer. This was verified in all tests by monitoring the execution of a computer program during shake table testing.

The wire rope support system performed considerably better than other commonly used installation methods for computer equipment. Evidently, the wire rope system reduced displacements by an order of magnitude while maintaining accelerations at the same level as the other installation methods. Interestingly, the computer equipment sustained accelerations of more than 3g (see Table 8, installation method with springs) without any interruption of its operation.

Conclusions

Wire rope systems for the seismic protection of equipment in buildings have been studied experimentally and analytically. Two installation methods were considered in which the equipment is supported by wire rope isolators and the other in which the equipment is supported by locked casters and wire rope isolators are used for providing restoring force.

It has been found that wire rope isolators exhibit hysteretic damping which decreases with increasing amplitude of motion. Typical values of equivalent damping ratio are about 0.01 of critical for large deformations and about 0.2 to 0.3 of critical for small deformations.

Based on these results, it was concluded that stiff wire rope systems may provide a degree of protection to equipment in buildings while allowing very small displacements. In contrast, the classical isolation approach of increasing the period of the system to values beyond the predominant period of the input motion is impractical because; firstly, floor seismic motions are rich in long period components; and secondly, displacements are unacceptable for equipment.

Analytical models for describing the hysteretic behaviour of wire rope isolators have been developed and experimentally calibrated and verified. Analytical predictions of response of an equipment supported by wire rope isolators were in good agreement with experimental results. Furthermore, a simplified analysis method was developed and shown to be capable of providing reliable estimates of the peak response of equipment supported by wire rope isolators. The method makes use of floor response spectra which is the usual design specification for equipment.

The second installation method for equipment, consisting of locked casters to support the weight and wire rope isolators, was tested with IBM computer equipment. The equipment was placed on top of a raised floor as it would have been in service. The response of the equipment in terms of peak acceleration and displacements of the locked casters was monitored in shake table tests and compared to the response of the equipment supported by other commonly used systems. It was found that the used stiff wire rope system reduced or maintained accelerations at the same level while reducing displacements by a factor of about 10.

Acknowledgments

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References

Wire rope systems for equipment seismic protection: G. F. Emeritides et al.

19 Papageorgiou, Y. (editor), 'Seismic isolation and response control for nuclear and non-structural structures, Special Issue for the 11th International Conference on Structural Mechanics in Reactor Technology (SMiRT 11), 28–31 August 1991, Tokyo, Japan
20 Makha, S. A. 'Seismic and experimental investigations of viscoelastic dampers in applications of seismic and vibro-isolation', PhD Diss., State University of New York at Buffalo, Buffalo, NY, 1992
25 Constantinescu, M. C. and Adam, M. A. 'Dynamics of soil-based-island structures: evaluation of two models for site-coupling systems', Report of the National Science Foundation, Department of Civil Engineering, Drexel University, Philadelphia, PA, 1993
26 'Indirect, H. Nonlinear transient dynamic analysis of yielding trusses', PhD Diss., University of California at Berkeley, Berkeley, CA, 1976