TORSIONAL COUPLING IN ANTI-EISMIC DESIGN
OF
TALL BUILDINGS

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Multistory building structures, subjected to earthquake ground motion, respond often in coupled lateral and torsional vibrations. The destructive effects of torsional vibrations are well recognized and have stimulated search for simple models which could give good confidence to the expected earthquake effects in torsionally coupled structures. Such a model was found to be the single story analogue structure.

Recent researches showed that the above simplified model can be useful for understanding the coupling effects of some classes of structures including most of the types of usual structural systems in tall buildings. When structures are subjected to either lateral or torsional earthquake components, the lateral and torsional responses are coupled if the lateral resisting structural elements have nonsymmetric distribution referred to the mass center of the various floors.

The coupling between the torsional and lateral vibrations is generally governed by the distance between the rigidity center to the mass center of the various floor levels—distance defined as static eccentricity. The effect of such coupling is the appearance of torsional accelerations in addition to the lateral ones, which are usually idealized by the building code provisions as torques additional to the induced lateral loads. As provided by the codes, this torque is obtained by multiplication of the lateral shear by a load factor (specifically a distance) dependent on the above defined static eccentricity and upon the maximum plane dimension of the structure. Many researches show a complete different dynamic response compared with that resulting from code provisions. It was shown that the induced torque in the dynamic response is dependent not only on the static eccentricity but also on the following factors: distribution of the stiffening element in plane of the various floor levels.
regularity of stiffening elements with height of the structural dynamic properties of the structure (modal shapes and associated natural frequencies in lateral and torsional vibrations) and ground motion components during earthquakes.

The coupling effect under lateral component excitation, investigated intensively, was shown to reduce the lateral induced shear forces and amplify the induced torque especially for small eccentricities and within the range of close natural frequencies. While the influence of the lateral component seems to be predominant, the torsional ground motion component induces additional larger torques which are amplified in structures with coupled vibrations. These effects are characteristic for structures which are regular with height and respond in the coupled vibration similar to the analogous single story structure. However a large number of structural systems with shear walls, multi-story frames or framed tubes can be treated by single story analogy (S.S.A.) either accurately or approximately.

The main objective of this paper is to summarize the principal effects of dynamic torsional coupling, to emphasize the main structural parameters influencing that behavior and to provide practical guidelines for evaluation of the earthquake equivalent loads, showing the limitation of the static approach.

RESPONSE OF SINGLE STORY STRUCTURES

The coupled torsional-lateral vibration of a single story structure with different rigidity and mass centers was treated in previous works and may be represented mathematically as follows:

$$K \ddot{y} + M \ddot{y} = N \ddot{u}$$

or in a more explicit form:
\[
\begin{bmatrix}
\kappa_x & 0 & 0 \\
\kappa_y & 0 & 0 \\
\text{Symm.} & \kappa_y & \text{Symm.}
\end{bmatrix}
\begin{bmatrix}
u_x \\
u_y \\
\text{Symm.} \quad \nu_y
\end{bmatrix}
= 
\begin{bmatrix}
m & -m & 0 \\
n \kappa_x + m \kappa_y & m & 0 \\
\text{Symm.} \quad n \kappa_y & m & \text{Symm.}
\end{bmatrix}
\begin{bmatrix}
u_x \\
u_y \\
\text{Symm.} \quad \nu_y
\end{bmatrix}
\]

(1a)

where the coordinate reference and geometrical definitions are presented in Fig. 1. \(k_x, k_y, k_\theta\) = lateral stiffness in \(x, y\) directions and torsional stiffnesses, respectively; \(m\) = mass; \(I_\theta\) = polar moment of inertia \((I_\theta = m l^2)\), where \(l\) = mass radius of gyration; \(\nu_x, \nu_y, \nu_\theta\) = lateral and torsional, respectively; time-dependent displacement components and \(\ddot{u}_x, \ddot{u}_y, \ddot{u}_\theta\) = lateral and torsional, respectively, acceleration time history components.

The torsional component \(\ddot{u}_\theta\) was derived from the lateral component of the time history integrating wave influence along the foundation length \(l^6,10\).

Performing adequate transformations to Eq. 1 with the aim to obtain uncoupled variables and using the definition of uncoupled frequencies as \(\omega_j^2 = k_j/m_j\) \((j = x, y, \theta)\) reads:

\[
\ddot{\mathbf{u}} + \mathbf{K} \ddot{\mathbf{u}} = -\mathbf{M} \ddot{\mathbf{u}}
\]

or in a more explicit form:

\[
\begin{bmatrix}
\omega_x^2 & 0 & 0 \\
\omega_y^2 & 0 & 0 \\
\text{Symm.} \quad \omega_y^2 \\
\end{bmatrix}
\begin{bmatrix}
u_x \\
u_y \\
\text{Symm.} \quad \nu_y
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 & 0 \\
1 & \frac{\omega_x^2}{\omega_y^2} + \frac{\omega_y^2}{\omega_x^2} & 0 \\
\text{Symm.} \quad 1
\end{bmatrix}
\begin{bmatrix}
u_x \\
u_y \\
\text{Symm.} \quad \nu_y
\end{bmatrix}
\]

(2a)
To obtain the maximal induced inertia forces in the elastic response the well known definition of spectral forces $^2$ is used, treating each component separately:

$$ P_{ij} = M \dot{u}_{ij} \omega_{ij} \xi_j $$

(3)

where $S_{ij}$ = acceleration response spectra for $j$-th ground component, $\xi_j$ = $i$-th modal damping factor, $\omega_{ij}$ = $i$-th eigenvector and frequency of the coupled vibration obtained from the equation of free motion:

$$ \begin{bmatrix} \omega_x^2 & 0 & 0 \\ \omega_y^2 & 0 & 0 \\ \omega_z^2 & 0 & 0 \end{bmatrix} - \omega_1^2 \begin{bmatrix} 1 & -\frac{2}{1+\zeta} & \frac{2}{1+\zeta} \\ \frac{2}{1+\zeta} & 1 & \frac{2}{1+\zeta} \\ \frac{2}{1+\zeta} & \frac{2}{1+\zeta} & 1 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = 0 $$

(4)

$\Gamma_{ij}$ = $i$-th modal participation factor for $j$-th component of ground motion, given by:

$$ \Gamma_{ij} = \frac{\omega_{ij}}{\omega_1} \chi_{ij} $$

(5)

where $\chi_{ij}$ = orientation vector defined for the $j$-th component of ground motion given by:

$$ \begin{bmatrix} \chi_x \\ \chi_y \\ \chi_z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} $$

(6)

$M^*$ = nondimensional mass matrix, and $\vec{P}_{ij}$ = vector of induced forces due to the $i$-th mode of vibration and $j$-th component of ground motion. The total maximal load vector is obtained by the corrected root - sum - square superposition:

$$ \vec{P}_j = \left( \sum_{i=1}^{n} \left[ \left( \frac{\omega_{ij}}{\omega_1} \right)^2 + 2 \sum_{k=1}^{n} \sqrt{\frac{\omega_{ij}}{\omega_{ik}}} \right] \right)^{1/2} $$

(7)

where

$$ e_{ik} = \left| \frac{\omega_i - \omega_k}{\omega_i + \omega_k} \right| $$

(8)
The induced inertia forces obtained by Eqs. 3 and 7 are the torsional affected loads which should be applied to the rigidity center of the structure and then distributed to the various stiffening elements.

In order to check the coupling influence let us define the uncoupled loads in y direction as:

\[ F_{oy} = M S_{ay} \omega_y, \dot{y} \]  

(9)

and \( \overline{F}_{ij} \) the relative non-dimensional forces obtained by dividing Eq. 3 and 9 as follows:

\[ \overline{F}_{1j} = \frac{M S_{ij} \omega_i}{S_{ay} \omega_y} \]  

or

\[ \overline{F}_{1j} = \frac{M S_{ij} \omega_i}{S_{ay} \omega_y} \]  

(10a)

where \( \overline{F}_{ij} = M S_{ij} F_{ij} \) leading to:

\[
\overline{F}_{ox} = \begin{pmatrix}
-\frac{g}{1} & 0 \\
1 & 1 \\
0 & 1 \\
\end{pmatrix} \quad \overline{F}_{oy} = \begin{pmatrix}
0 \\
1 \\
0 \\
\end{pmatrix} \quad \overline{F}_{by} = \begin{pmatrix}
0 \\
1 \\
\frac{g}{1} \\
\end{pmatrix}
\]  

(11)

When considering the ground motion component in x direction, \( S_{ax} \) in Eq. 10 is to be replaced by \( S_{ax} \).

Two different influences of torsional coupling may be shown. First the influence of torsional coupling on the relative loads when lateral ground component is considered and second when only the torsional component excites the structure. The first influence is plotted in Figs. 3 and 4 for three idealized smoothed spectra, mostly used in design practice and for an actual response spectrum as shown in Fig. 2. The graphs presented in Figs. 2, 3 and 4 show the coupling influence in a structure with one axis of symmetry \( (x_a = 0) \), additional graphs for the general case may be obtained by using Eqs. 7 and 10. The influence of the coupling depends
on the coupling parameter $e_{x}/1$, especially in the range of closed moncoupled natural frequencies $\omega_{0}^2/\omega_{y}^2$. The graphs plotted in Fig. 4 present the ratio between the induced torque and the product of the noncoupled lateral load and the geometrical eccentricity $F_{oy}/F_{oy}$.

These graphs show great differences from the code's prescribed ratio (10) especially in the range of closed moncoupled natural frequencies.

It may be seen that the lateral relative forces calculated for different types of spectra differ essentially in the range of small frequency ratios. The second main influence which is due to the torsional component of excitation was checked for idealized torsional spectra based on the actual spectra obtained by Hart, Di Julio, and Law 6 (see Fig. 5).

The ratio of torsional spectrum and lateral moncoupled spectrum by using Eq. 10a leads to

$$\frac{S_{ao}(\omega_{0},i_{1})/S_{ay}(\omega_{oy},i_{y})}{S_{as}(\omega_{0},i_{1})/S_{ay}(\omega_{oy},i_{y})} = \frac{a_{s}(\omega_{0},i_{1})}{a_{s}(\omega_{oy},i_{y})}$$

$$= R_{a}(\omega_{0},i_{1}) R_{a}(\omega_{oy},i_{y})$$

(12)

The ratio between the normalized forces and $R_{a}$ are plotted in Fig. 6 for two idealized rotational spectra. From this plot it may be seen that the lateral forces tend to be small for high and very small frequencies and that the spectrum type influences the torque very strongly, especially for high rotational frequencies while the lateral forces are influenced much less. It is worth to be mentioned that the coupling effect gets stronger for increasing coupling parameters (e/i) for lateral induced loads while the relative torque increases for small coupling parameters. The actual relative forces are to be associated with the relative influence of rotational to lateral component spectra which is easily derived from the expressions in Figs. 2 and 5. This influence represented by the factor $R_{a}$ in Eq. 12 (see Table 1) are dependent on the mass radius of gyration (i) and on the lateral natural period.
of the structure and shows reduced influences of coupling effects in flexible structures.

Before additional comments are made about torsional effects as a dependence of coupling parameter and natural frequencies the similarity of effects obtained in tall buildings is presented.

ANALOGIC RESPONSE OF TALL BUILDINGS

Tall buildings are provided with vertical structural systems with the aim to resist lateral loads due to wind and earthquakes. These systems comprise shear walls, multistory frames and trusses or combined elements. Under lateral loads these elements are assumed to be vertical cantilevers with different types of behavioural characteristics. The shear wall behaves as a flexural cantilever, the frames as shear cantilever while the trusses and framed tubes behave as a combined flexure-shear cantilever. In torsion the cantilevers behave with only St. Venant torsional properties or behave as thin walled elements. Under lateral static loads such systems respond usually with coupled lateral and torsional displacements. In some cases where the origins of the principal axes of each story lie on a common vertical axis defined as rigidity axis, the lateral and torsional displacements are not coupled. Although a limited class of structures seems to have uncoupled lateral behaviour it was shown in early works that a large class of structures may be assumed noncoupled, either accurate or approximate.

The dynamic equation of the structure having uncoupled lateral and torsional deformations, related to the rigidity axis as defined in Fig. 7 reads:

\[ K \ddot{\xi} + M \dddot{\xi} = M \dddot{\gamma} \]

(13)
where $k_x$, $k_y$, $k_o$ = global lateral stiffness submatrices of the structure in $x$, $y$ directions and torsional stiffness submatrix respectively.

$\vec{d}_x$, $\vec{d}_y$, $\vec{d}_o$ = global lateral displacement vectors in $x$, $y$ directions and torsion angle respectively, and $\vec{1}$ = unity vector.

Let us define the frequencies $(\omega_j)$ and modal shapes $(\vec{\psi}_j)$ of the noncoupled system ($a_x = a_y = 0$), as solution of the equation:

$$(x_j - \omega_j^2 M) \vec{\psi}_j = 0 \quad (j = x, y, o),$$

and define the transformation of the response vector as:

$$\vec{\bar{d}} = \vec{\psi} \vec{\bar{u}}$$

where:

$$\vec{\psi} = \begin{bmatrix} \psi_x & 0 & 0 \\ 0 & \psi_y & 0 \\ \text{Sym.,} & \psi_o & 0 \end{bmatrix}$$

Substituting Eq. 15 in Eq. 13a, premultiplying with $\vec{\psi}^T$ and using the orthonormality of the noncoupled dynamic properties (mass orthonormalized:

$$\vec{\psi}_j^T M \vec{\psi}_j = \vec{1}, \quad \vec{\psi}_j^T k_j \vec{\psi}_j = \text{diag}(\omega_j^2),$$

and after transformations with aim to obtain dimensionally homogeneous terms, the dynamic equilibrium equation is given by:
\[
\begin{align*}
\begin{bmatrix}
Q_{x} & 0 & 0 \\
Q_{y} & 0 & 0 \\
\xi_{ym} & Q_{y} & \xi_{ym}
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_{x} \\
\ddot{u}_{y} \\
\ddot{u}_{\phi}
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
I & -\left(\epsilon_{y}/i\right) & 0 \\
0 & I & -\left(\epsilon_{x}/i\right) \\
\left[1 + \left(\epsilon_{y}/i\right) + \left(\epsilon_{x}/i\right)\right] & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_{x} \\
\ddot{u}_{y} \\
\ddot{u}_{\phi}
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\ddot{u}_{x} \\
\ddot{u}_{y} \\
\ddot{u}_{\phi}
\end{bmatrix}
= - \begin{bmatrix}
\bar{\psi}_{x}^{T} & 0 & -\bar{e}_{y} \bar{\psi}_{x}^{T} \\
\bar{\psi}_{y}^{T} & 0 & -\bar{e}_{x} \bar{\psi}_{y}^{T} \\
0 & 0 & \left[1 + \left(\epsilon_{y}/i\right) + \left(\epsilon_{x}/i\right)\right] \bar{\psi}_{x}^{T} + \frac{\bar{e}_{x} \bar{\psi}_{y}^{T}}{i} \bar{\psi}_{x}^{T} - \bar{e}_{y} \bar{\psi}_{x}^{T} \bar{\psi}_{y}^{T} \end{bmatrix}
\begin{bmatrix}
\ddot{u}_{x} \\
\ddot{u}_{y} \\
\ddot{u}_{\phi}
\end{bmatrix}
\end{align*}
\]

where \( Q_{x} = \text{diag} \left( \omega_{x}^{1}, \omega_{x}^{2}, \omega_{x}^{3}, \ldots, \omega_{x}^{n} \right) \), \( \xi_{ym} = \text{number of reference levels} \). The mixed terms in Eq. 13 are not orthogonal, however, due to the similarity between the natural noncoupled modes shapes (characteristic for cantilever beams) the above terms yield predominantly diagonal matrices with very small off-diagonal terms. Assuming an approximative orthogonality in further developments, it has been shown in earlier work by using a perturbation technique, that appropriate solutions may be obtained for a variety of structural systems. Assuming that these mixed produces are orthogonal, i.e., equaling unity, Eq. 17 may be decomposed in \( n \) similar triplets of coupled equations completely analogous to single scory equations, as shown in Eq. 2a:

\[
\begin{align*}
\begin{bmatrix}
Q_{x} & 0 & 0 \\
Q_{y} & 0 & 0 \\
\xi_{ym} & Q_{y} & \xi_{ym}
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_{x} \\
\ddot{u}_{y} \\
\ddot{u}_{\phi}
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
I & -\left(\epsilon_{y}/i\right) & 0 \\
0 & I & -\left(\epsilon_{x}/i\right) \\
\left[1 + \left(\epsilon_{y}/i\right) + \left(\epsilon_{x}/i\right)\right] & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_{x} \\
\ddot{u}_{y} \\
\ddot{u}_{\phi}
\end{bmatrix}
\end{align*}
\]

or in a more explicit form:

\[
\begin{align*}
\begin{bmatrix}
\omega_{x}^{2} & 0 & 0 \\
\omega_{y}^{2} & 0 & 0 \\
\xi_{ym} & \omega_{y}^{2} & \xi_{ym}
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_{x} \\
\ddot{u}_{y} \\
\ddot{u}_{\phi}
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
1 & -\epsilon_{y}/i & 0 \\
0 & I + \left(\epsilon_{y}/i\right) + \left(\epsilon_{x}/i\right) & \epsilon_{x}/i \\
\xi_{ym} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_{x} \\
\ddot{u}_{y} \\
\ddot{u}_{\phi}
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\ddot{u}_{x} \\
\ddot{u}_{y} \\
\ddot{u}_{\phi}
\end{bmatrix}
= - \begin{bmatrix}
\bar{\psi}_{x}^{T} & 0 & -\bar{e}_{y} \bar{\psi}_{x}^{T} \\
\bar{\psi}_{y}^{T} & 0 & -\bar{e}_{x} \bar{\psi}_{y}^{T} \\
0 & 0 & \left[1 + \left(\epsilon_{y}/i\right) + \left(\epsilon_{x}/i\right)\right] \bar{\psi}_{x}^{T} + \frac{\bar{e}_{x} \bar{\psi}_{y}^{T}}{i} \bar{\psi}_{x}^{T} - \bar{e}_{y} \bar{\psi}_{x}^{T} \bar{\psi}_{y}^{T} \end{bmatrix}
\begin{bmatrix}
\ddot{u}_{x} \\
\ddot{u}_{y} \\
\ddot{u}_{\phi}
\end{bmatrix}
\end{align*}
\]
Defining the participation factors of the m-coupled system as

\[ \gamma_{jn} = \frac{\psi_{jn}^2 \Omega_{jn}}{Z_{jn}}, \quad (j = x, y, \theta, y) \]

and using for the space coupling parameters, \( \Omega_{in} \) and the associated natural circular frequency, \( \omega_{in} \), similar definition as given in Eq. 4 for the single story system, the induced load \( f_{mn} \) by the mode shape \( \{ \gamma_{jn} \} \) following the common definition of equivalent loads, yields:

\[ \{ f_{m1} \} = \frac{\psi_{yn}^k}{\Omega_{yn}} \Gamma_{yn} \Gamma_{yn}^T \{ \omega_{yn} \} \{ \gamma_{yn} \} \quad (i = 1, 2, 3; j = x, y, \theta) \]

(19)

and the associated moncoupled load for \( y \)-th ground component is given by:

\[ \{ f_{m1} \} = \frac{\psi_{yn}^k}{\Omega_{yn}} \Gamma_{yn} \Gamma_{yn}^T \{ \omega_{yn} \} \{ \gamma_{yn} \} \quad (20) \]

where \( \gamma_{yn} \) is the \( k \)-th term of \( n \)-th mode shape vector \( \{ \gamma_{yn} \} \). Dividing Eq. 19 by Eq. 20, the relative loads at \( k \)-th level are obtained as follows:

\[ \{ f_{m1} \} = \frac{\gamma_{yn}^k}{\gamma_{yn}^k \Omega_{yn}^k} \Gamma_{yn} \Gamma_{yn}^T \{ \omega_{yn} \} \{ \gamma_{yn} \} \]

(21)

where \( \gamma_{yn}^k \) = diagonal matrix of mode shape ratio at reference level \( k \):

\[ \gamma_{yn}^k = \text{diag} \left[ \frac{\psi_{xn}^k}{\psi_{yn}^k}, \frac{\psi_{yn}^2}{\psi_{yn}^k}, 1 \right] \]

(22)

and \( \gamma_{yn}^k \) = diagonal matrix of participation factors ratio:

\[ \gamma_{yn}^k = \text{diag} \left[ \frac{\Gamma_{yn}}{\Gamma_{yn}}, \frac{\Gamma_{yn}}{\Gamma_{yn}}, 1 \right] \]

(23)

Comparing the relative forces at a specific reference level \( k \) with those obtained in a single story structure (see Eqs. 21 and 10a) it may be seen that their expressions are similar and differ by the mode shape's

* Explicit presentation of matrices in Eq. 19 are given in Appendix I.
ratios, $\alpha_m^k$ and participation factor ratio $\beta_{m,n}^k$. In the particular case where the lateral and torsional mode shapes are identical, i.e.,

$$\overline{v}_m = i \overline{v}_m = \overline{v}_n = \overline{v}_n$$

both matrices $\alpha_{m,n}^k$ and $\beta_{m,n}^k$ become unit matrices and therefore Eqs. 21 and 26 become identical, which means that forces induced in the stories of a tall building are equal to those induced in a single-story structure. While the terms in the matrix $\beta_{m,n}^k$ are close to unity, they do not contribute to the differences between the loads in tall buildings and single-story structures, the mode shape ratio matrix $\alpha_{m,n}^k$ yields substantial differences in the distribution of the relative forces with the height of the building.

Performing further expansions of Eq. 21 an approximate relation [13] can be written for the relative forces at the $k$-th level of a tall building having one symmetry axis ($c_y$, $n$), due to $n$-th triplet (obtained by superposition of loads in Eq. 21 by Eq. 7), the shape ratio $1/\beta_{m,n}^k \beta_{m,n}^k = \alpha_{m,n}^k$ and the participation factor ratio, $\Gamma_{m,n} / \Gamma_{m,n} = \gamma_{m,n}$, as follows:

$$\begin{align*}
\begin{bmatrix}
\phi_{m,n}^{k} (v/1) \\
\phi_{n,m}^{k} (v/1)
\end{bmatrix}
&= \begin{bmatrix}
\overline{v}_m^{k} \\
\overline{v}_n^{k}
\end{bmatrix} + (\alpha_{m,n}^k - 1) \begin{bmatrix}
L_1 \\
L_2 (v/1)
\end{bmatrix} + (\gamma_{m,n} - 1) \begin{bmatrix}
L_3 \\
L_4 (v/1)
\end{bmatrix}
\end{align*}
$$

(24)

for the lateral $y$-th ground component and

$$\begin{align*}
\begin{bmatrix}
\phi_{m,n}^{k} \\
\phi_{n,m}^{k}
\end{bmatrix}
&= \gamma_{m,n} \begin{bmatrix}
\phi_{m,n}^{k} \\
\phi_{n,m}^{k}
\end{bmatrix} + \gamma_{m,n} (\alpha_{m,n}^k - 1) \begin{bmatrix}
L_4 \\
L_5 (v/1)
\end{bmatrix}
\end{align*}
$$

(25)

for the torsional $\theta$ ground component. The influence functions $L_i$ derived from Eqs. 21 and 7 are plotted in the graphs shown in Figs. 8 and 9.

The above relations show the resultant differences between the forces yielded by a dynamic response in a tall building and a single-story structure given by the first vector in Eqs. 24 and 25. The relative forces in
Eqs. 24 and 25 are to be interpreted as load multipliers for uncoupled
induced forces in order to obtain the torsionally coupled loads. The main
differences between the load multipliers in the single story structure and
the tall building depend on the parameters \(x_n\) and \(y_n\) or on the non-
coupled dynamic properties, i.e. natural mode shapes in lateral and torsion-
al vibration and the associated frequencies.

Since the coupling behaviour is mainly influenced by the composition
of the structural system and mass distribution in order to fulfill complete
ana
y between the tall building and single story behaviour these two cha-

(a) The structural system must have the property to be decoupled
statically into independent lateral and torsional subsystems, or in other
forms of uncoupled substructures [13,15].

(b) The natural shapes of the uncoupled behaviour must be completely
identical \((x_n = y_n)\).

All structures which do not satisfy the above two conditions cannot
be accurately treated by single story analogy (S.S.A.). However, struc-
tures which can be approximately decoupled statically or those which have
only similar and not identical modal shapes can be treated approximately
by this approach.

In the following different structural systems are discussed in view
of their suitability to be treated by Single-Story-Analogy (S.S.A.)
either accurately or approximately.

(1) Building structures with vertical shear walls with the same
vertical variation of structural properties, having a single vertical
stiffness axis and identical lateral and torsional mode shapes, can be
accurately treated by this approach.
(2) Plane framed systems with proportional vertical variation of structural properties (usually can be represented by vertical shear cantilever in the lateral motion and as St. Venant cantilever for torsion), having a single vertical rigidity axis, proportional lateral and torsional stiffness matrices and therefore identical lateral and torsional mode shapes, can be accurately treated by S.S.A. (Single Story Analogy).

(3) Plane systems, as shear walls or framed systems, with weak proportional variation of properties with building height can be treated approximately by perturbation techniques \(^{11,12}\), the S.S.A. solution being the zero perturbation solution.

(4) Combined coupled shear walls and framed systems with proportional vertical variation of properties having two vertical stiffness axes (one for flexural properties and the other for shear properties) can be transformed by a linear transformation into a statical moncoupled structure and then treated approximately by S.S.A., due to the similarity of the non-coupled properties \(^{12}\).

(5) At last combined shear walls and frames with slightly nonproportional vertical variation of structural properties can be transformed approximately into statical moncoupled systems and treated approximately by S.S.A. approach and if corrections are required this may be done by perturbation techniques. This last group comprises a large class of actual building systems.

Some other special structural systems fulfill mathematically the two main conditions but cannot be explicitly described as one of the above mentioned systems. Also these structures may be treated by the approximate approach. However, for those structures, which cannot fulfill these conditions, even approximately, a complete dynamic analysis is required.

It should be mentioned that the presented analogy refers to triplets
of coupled space modes of the same order (n) (see also Eq. 18) and the resultant loads are to be treated accordingly. The final loads or shears are to be calculated by suitable superposition of required modal loads as usually done for lateral induced shears in tall building structures.

DESIGN PROVISIONS

The existing seismic building codes treat generally the problem of coupling effects prescribing proportional torques to the provided lateral loads to be withstood by the structural system. The torques provided are a product of lateral shears by a so called "accidental eccentricity" added to that statically defined to cover both torsional and lateral ground component effects, non symmetric distribution of live loads and other imperfections of the idealized model considered. As earlier described, the coupling effect differs from code prescriptions and in many cases cannot be covered by these provisions. The lateral loads, affected by torsional coupling, are somewhat reduced in certain structures subjected to the lateral ground shaking and may exceed the uncoupled values when the torsional component is considered. While the building codes prescribe proportional torques to the lateral shear loads, a vertical variation of the multipliers should be considered due to the differences between the modal shapes of the uncoupled substructures in real building structures. The "accidental eccentricity" does not cover the effect of coupling when the torsional component of ground shaking is considered, especially for low coupled structures with stiff torsional systems (\(\frac{\omega_{ki}^{2}}{\omega_{i}^{2}} > 1\)). The limitation of equivalent statics methods of analysis to low building structures, as provided by building codes seems to be improper for torsional coupling problems. Lower structures which do not fulfill the conditions, earlier described herein, could give totally different responses as those provided by codes.
To allow for torsional coupling effects, the structural properties of tall buildings have to be well defined and checked in view of the conditions for suitability of simplified analogy with the single story structure behaviour. If these conditions are accurately or approximately fulfilled the S.S.A. approach may simplify substantially the dynamic analysis or the static equivalent method. Else, complete dynamic analysis should be used to predict torsional effects.

Therefore a more real estimation of torsional-coupled-affected loads could be obtained by S.S.A. (Single Story Analogy), when the principal parameters are well defined and computed in the design process:

(a) The Coupling Parameters \( a_x / f \) and \( a_y / f \) have the main influence on the coupling relative forces (the load multipliers). For large values of these parameters the load multipliers tend to unit value which shows no amplification or reduction of "statical effects" — (the lateral shear is identical with that of non-coupled structures and the torques equal the produce of lateral shear by eccentricity \( e \)). In the range of small coupling parameter values the induced torques are strongly amplified while for unit values of these parameters, lateral induced loads are substantially reduced (to 70% of the non-coupled value). If the torsional ground component is considered, the coupling parameter contributes to an increasing effect of lateral induced loads and to a reduction of the induced torques. However, while the coupling parameter vanishes, the induced torques tend to a finite value due to torsional ground component influences.

(b) The Ratio of Torsional to Lateral Non-coupled Frequencies \( \omega_T / \omega_L \) has amplifying effects on the coupling in the range of close frequencies \( \omega_T / \omega_L \pm 1 \) for lateral component (or reduces induced lateral loads), while for the torsional component the amplifications grow with increasing values of the parameters. In the range of small frequencies ratio the effect is
substantially reduced for actual earthquake response spectra (see Figs. 4d, 6b and 6e). This effect was expected because of the decreasing value of the response for high frequencies.

(c) The Ratio of Noncoupled Participation factors \( \gamma_n \) is the main measure for the similarity of the noncoupled modal shapes in a tall building structure. While previous parameters suffice for Single Story Analogy, in the modal analysis of multi-story structures the natural shapes play the most important role. If the ratio of the participation factor is close to unity or differs slightly from these values \( 2 > \gamma_n > 0.5 \), S.S.A. is approximately correct and gives practical good results. Otherwise this parameter will correct the modal multipliers for tall buildings if the S.S.A. is used (see Eqs. (24) and (25)).

(d) The Modal Shape Ratio \( \left( \frac{k}{\gamma} = 1 \right) \) \( \gamma_n \) influences mainly the vertical variation of relative loads (coupling multipliers). For unity values of this parameter the coupling multipliers become identical with those of single story structures while variation of this parameter with height indicates vertical variation of relative loads. While lateral load ratios are weakly influenced by variable \( \gamma_n \) with height, the induced torques are strongly affected.

The previously mentioned parameters allow more realistic estimation of coupling effects when using the proposed expressions of equivalent loads or the additional graphs.

The combined effect of the different components of ground shaking was obtained by statistical combination of the separated effects. The problem merits further investigation due to the statistical correlation between the ground components which do not allow simple root-sum-square superposition.
A sixteen story structure of 48.0 m (159 ft) height with five identical frames and six planar walls as stiffening elements was chosen to exemplify the S.S.A. (Single Story Analogy) for a structure without vertical rigidity axis (or without vertical rotational axis). The stiffness and geometrical properties of structural elements associated with the layout in Fig. 10, are presented in Table II. A static analysis of the above system was previously carried out by Gluck5, Rutenberg & Haudebrecht14, using the continuum approach. According to these references the dynamic equilibrium equation related to a C.t. center (see Fig. 10) can be written:

\[ \begin{bmatrix}
I & E_1 & 0 \\
0 & E_1 & 0 \\
0 & 0 & E_1
\end{bmatrix}
\begin{bmatrix}
R_y(z,t) \\
R_x(\mathcal{A}) \\
R_x(\mathcal{A})
\end{bmatrix}
+ \begin{bmatrix}
E A_1 & a E A_1 & 0 \\
E A_1 & E A_1 & a E A_1 \\
E A_1 & E A_1 & E A_1
\end{bmatrix}
\begin{bmatrix}
\mathcal{A} \mathcal{F}_A + x_1 \mathcal{G}_A \mathcal{G}_A \\
\mathcal{A} \mathcal{F}_A + x_1 \mathcal{G}_A \mathcal{G}_A \\
\mathcal{A} \mathcal{F}_A + x_1 \mathcal{G}_A \mathcal{G}_A
\end{bmatrix}
\begin{bmatrix}
R_y(z,t) \\
R_x(\mathcal{A}) \\
R_x(\mathcal{A})
\end{bmatrix}
\]

where \( E_1 \) is the total flexural stiffness of the planar walls in y direction; \( E A_1 \) is the total stiffness of frames considered as vertical cantilevers as usual in continuum approaches5,14; \( a \) = vertical distributed mass of the structure; \( m_e \) = vertical distributed polar inertia moment of the story mass; \( a \) = distance between frames centroid S.C. the walls centroid S.C. as shown in Fig. 10.

Using a dimensional transformation to obtain homogeneous dimensions and dividing Eq. 26 by \( E_1 \), yields:
\[
\left[ \begin{array}{c}
J^4 - \frac{Eg A_k b^2}{E_1 I_1} \\
\frac{Eg A_k b^2}{E_1 I_1} \\
\frac{Eg A_k b^2 \rho}{E_1 k} \end{array} \right] \left[ \begin{array}{c}
\frac{Eg A_k}{E_1 I_1} \\
\frac{Eg A_k}{E_1 I_1} \\
\frac{Eg A_k}{E_1 I_1} \end{array} \right] \left[ \begin{array}{c}
D_1 (x,t) \\
D_2 (x,t) \\
D_3 (x,t) \\
\end{array} \right] + \left[ \begin{array}{c}
\frac{m}{E_1 I_1} \\
\frac{m}{E_1 I_1} \\
\frac{m}{E_1 I_1} \\
\end{array} \right] \left[ \begin{array}{c}
\frac{Eg A_k b^2}{E_1 k} \\
\frac{Eg A_k b^2}{E_1 k} \\
\frac{Eg A_k b^2}{E_1 k} \end{array} \right] \left[ \begin{array}{c}
D_1 (x,t) \\
D_2 (x,t) \\
D_3 (x,t) \\
\end{array} \right] = \left[ \begin{array}{c}
1 - e^{-\frac{t}{k}} \\
1 - e^{-\frac{t}{k}} \\
1 - e^{-\frac{t}{k}} \\
\end{array} \right] \left[ \begin{array}{c}
U_{\gamma} \\
U_{\delta} \\
U_{\zeta} \\
\end{array} \right]
\]

where \( r_1 = \frac{E_2 A}{E_1 I_1} \), \( r_2 = \frac{E_2 A}{E_1 I_1} \), \( a_1 = \frac{E_2 b^2}{E_1 I_1} \), and \( D \) is the differentiation operator. The stiffness parameters and dynamic coupling parameters are tabulated in Table III. Using the following transformation of coordinates, \( \ddot{u} = T \cdot \ddot{u} \), Eq. (27)

\[
\left[ \begin{array}{c}
D_L \\
D_R \\
\end{array} \right] \cdot \left[ \begin{array}{c}
0.462 \\
0.887 \\
\end{array} \right] \cdot \left[ \begin{array}{c}
\ddot{u}_L (z,t) \\
\ddot{u}_R (z,t) \\
\end{array} \right] = \frac{T}{\zeta} \cdot \ddot{u}
\]

and premultiplying by the transpose of transformation matrix \( T \), Eq. (27) is statically coupled and reads:

\[
\left[ \begin{array}{c}
J^4 - \frac{p_1 b^2}{2} \\
0 \\
\end{array} \right] \cdot \left[ \begin{array}{c}
\ddot{u}_L (z,t) \\
\ddot{u}_R (z,t) \\
\end{array} \right] + \left[ \begin{array}{c}
1 - e^* \\
1 - e^* \\
\end{array} \right] \cdot \left[ \begin{array}{c}
\ddot{w}_L (z,t) \\
\ddot{w}_R (z,t) \\
\end{array} \right] = \left[ \begin{array}{c}
0.462 U_{\gamma} \\
0.462 U_{\gamma} \\
\end{array} \right]
\]

where \( p_1 = 10.8 \times 10^{-4}, \quad p_2 = 1.04 \times 10^{-4} \) and the inertia geometric properties (dimensionsless) are: \( a^* = 2.32 \times 10^{-7}, \quad e^* = 0.106, \quad f^* = 0.649 \) as are obtained by processing the matrix multiplications.

Eq. (29) is now statically uncoupled. The dynamic coupling being present is the mass matrix only. The case analyzed here completely fits the first requirement for usage of S.S.A. Defining the noncoupled dynamic
properties of the structure as the solution of the uncoupled system (a = c)

\[(\omega^4 - \omega_p^2 \omega_s^2 - \omega_y^2 m^s) \psi_y(z) = 0\]

(30)

\[(\omega^4 - \omega_p^2 \omega_s^2 - \omega_y^2 m_e^t) \psi_e(z) = 0\]

a set of eigenfunctions \( \psi_y(z) \) and \( \psi_e(z) \) are obtained associated with the eigenvalues (natural circular frequencies - \( \omega^2 \)). These dynamic properties can be easily calculated using the approximate solutions in Appendix II and the computed values of the first three lateral and rotational modes are listed in Table IV. The continuum model of the exxemplified structure can be replaced by a discrete formulation, however, the dynamic properties will be quite similar. The continuum and discrete formulation are identically treated in analysis of coupling effects as illustrated in the present work.

To obtain the coupled influenced loads the uncoupled defined loads are first computed according to Eq. (20) (see Table IV (9) - (11)) and then the coupling factors for lateral and torsional influence are taken from the graphs in Figs. 3, 4, 6 for the hyperbolic spectrum as listed in Table V(7). The combined influences of both components are obtained by superposition according to the components contribution as presented in the right hand of Eq. (29):

\[ F_n = 0.387 \bar{F}_{ny} + 0.462 \left( \frac{\bar{F}_{ny}}{B_p} \right) t^2 \]

(31)

\[ C_n = 0.387 \bar{C}_n + 0.462 \left( \frac{\bar{C}_n}{B_p} \right) t^2 \]

Using the multiplication factors the loads influenced by coupling are obtained by multiplication of the uncoupled loads and the final loads in the actual system are obtained by reverse transformation as follows:

(7) The coupling parameter \( a^p/a^s = 0.163 \) was used in these graphs and the values of coupling coefficients were obtained by...
\[
\begin{bmatrix}
Q^m_1 \\
Q^m_2
\end{bmatrix}
= 
\begin{bmatrix}
0.462 & 0.867 \\
0.867 & 0.462
\end{bmatrix}
\begin{bmatrix}
Q^m_1 \\
Q^m_2
\end{bmatrix}
\]

The coupled affected base shears in the actual system presented in Table V are compared with the base shears computed in a complete dynamic analysis listed in parentheses in the same table. The values obtained in the complete analysis differ from those obtained by the approximate method by about 5% for the first pair of modes, about 2% for the second and less than 1% for the third. The results are quite satisfying for practical purposes. It is worth mentioning that the calculations were available on a simple desk computer with reduced memory.

CONCLUDING REMARKS

The analogy between the behavior of nodal response of tall buildings and of single story structures to earthquake excitation is explained and used to account for the effects of torsional coupling in high rise buildings. While the existing code provisions cannot cover many effects of such coupling, the S.S.A. should give more efficient tools for approximate static analysis. At this stage, the S.S.A. approach seems to be the simple rational method to provide earthquake equivalent loads, torsionally affected in tall buildings. Therefore the conditions established for the use of this approach should be considered as the limit conditions for static load methods used in design of torsionally coupled structures.

(*) \( Q \) are the shear forces at the base of the structure and are obtained by integration of induced loads toward the base.
Eq. (15) in the text reads:
\[ \{ \phi_{i,j} \}^k = m_k x \gamma^k x \omega_{i,n} x \Gamma_{i,n} x S_{i,j}(\omega_{i,n}, \omega_{i,m}) \]  
(Al-1)

or explicitly:
\[
\begin{pmatrix}
\phi_{i,j}^k \\
\Gamma_{i,n}^k \\
\omega_{i,n}^k
\end{pmatrix}
= m_k \begin{pmatrix}
- m_k x \gamma^k \\
1 + \frac{a_s x \gamma^k}{\gamma} + \frac{b_s x \gamma^k}{\gamma} \\
1
\end{pmatrix}
\begin{pmatrix}
\omega_{i,n} x \gamma^k \\
\omega_{i,m} x \gamma^k \\
\omega_{i,m}
\end{pmatrix}
\Gamma_{i,n} S_{i,j}(\omega_{i,n}, \omega_{i,m})
\]  
(Al-2)

and
\[
\begin{pmatrix}
\omega_{i,n}^k \\
\omega_{i,m}^k
\end{pmatrix}
= \begin{pmatrix}
\omega_{i,n}^T \\
\omega_{i,m}^T
\end{pmatrix}
\begin{pmatrix}
\bar{a}_{i,n} x \gamma^k \\
\bar{a}_{i,m} x \gamma^k
\end{pmatrix}
\begin{pmatrix}
\bar{a}_{i,n} x \gamma^k \\
\bar{a}_{i,m} x \gamma^k
\end{pmatrix}
\begin{pmatrix}
\omega_{i,n}^T \\
\omega_{i,m}^T
\end{pmatrix}
\]  
(Al-3)

where \( \bar{a}_{i,j} \) is defined in Eq. 11 in the text.

**APPENDIX II**

The approximate dynamic properties of structures with combined flexural and shear properties can be obtained by application of common used formulae cited in Ref. 13, as illustrated in the following:

(a) The uncoupled circular frequencies:
\[ \bar{\omega}_{j,n} = \frac{\gamma}{\eta_n} (1 + \frac{p_j^2}{\lambda_n}) \quad (n = 1, \ldots N) \quad (j = x, y, \theta) \]  
(AlI-1)

where:
\[ \bar{\omega}_{j,n} = \text{nondimensional natural frequency} \]  
\[ \lambda_n = \text{nondimensional coefficient of flexural beam frequencies which for the first modes takes the values} \]  
\( \lambda_n = 1.075, 4.694, 7.855, 10.960 \).
(b) The natural mode shape function:

\[
\phi_n(\xi_k) = (C_1 \cos \gamma_k \xi_k + C_2 \sin \gamma_k \xi_k + C_3 \cosh \gamma_k \xi_k + C_4 \sinh \gamma_k \xi_k) \xi_k
\]

where \( \xi_k \) is the nondimensional vertical coordinate \( \xi_k = x_k/H \) for level \( k \) and the other coefficients computed as follows:

\[
\begin{align*}
\gamma_{2j'n}^2 &= \gamma_{1j'n}^2 + p_j^2, & \gamma_{1j'n} &= \gamma_{2j'n} = \xi_{j'n}^2 \\
C_{1j'n} &= 1 \\
C_{2j'n} &= \left\{ \cos \gamma_k + (\gamma_{2j'n}/\gamma_k) \cosh \gamma_k \right\} \xi_{j'n} \\
C_{3j'n} &= 1 \\
C_{4j'n} &= -\left( \gamma_{2j'n}/\gamma_{1j'n} \right) - C_{2j'n}
\end{align*}
\]
REFERENCES


**Notation**

- $\mathbf{Z}_n^n$ = diagonal matrix of mode shape ratios
- $a^k$ = geometric distance (defined in text)
- $\mathbf{C}_n^n$ = diagonal matrix of participation factors
- $c_{nj}$ = amplification factor (defined in text)
- $c_1, c_2, c_3, c_4$ = coefficients (defined in text)
- $\mathbf{D}$ = displacement vector
- $\mathbf{D}_x, \mathbf{D}_y, \mathbf{D}_\theta$ = lateral displacement and torsion vector
- $x^e, y^e$ = distance from elastic to mass center
- $\mathbf{F}_n^n$ = relative nondimensional forces
- $\mathbf{F}_{1j}, \mathbf{F}_{1e}, \mathbf{F}_{1e}$ = vector of induced forces
- $\mathbf{F}_{0j}$ = uncoupled loads
- $\mathbf{F}_0$ = vector of unities
- $\mathbf{I}$ = diagonal unit matrix
- $i$ = mass radius of gyration
- $K$ = overall stiffness matrix
- $k_x, k_y, k_\theta$ = lateral and torsional stiffness respectively
- $k_{x1}, k_{y1}, k_{\theta1}$ = lateral and torsional overall stiffness matrices respectively
- $L_i$ = influence functions
- $M$ = overall mass matrix
- $M^*$ = nondimensional mass matrix (coupling matrix)
- $\mathbf{M}$ = diagonal mass matrix
- $m$ = mass
- $I$ = polar moment of inertia
- $\beta_1$ = coefficient - ratio of shear-flexural properties
- $q^0$ = base shear
- $q^S$ = base shear in the actual system
- $\mathbf{R}_{0j}$ = orientation vector (defined in text)
\( R_1, R_2 \) = coefficients of spectrum ratios (defined in text)
\( r_j \) = orientation vector (defined in text)
\( R_{\text{m}}, R_{\text{k}} \) = shear and resp. flexural stiffness radius (defined in text)
\( s_{\text{a}} \) = acceleration response spectra for \( j \)-th component
\( \bar{u}_{\text{g}} \) = ground displacement vector
\( \bar{u}_{\text{gs}}, \bar{u}_{\text{gs}}, \bar{u}_{\text{g\theta}} \) = lateral and torsional resp. acceleration time history
\( u_{x}, u_{y}, u_{\theta} \) = lateral and torsional resp. time dependent displacements
\( \bar{u}_{\text{gs}}, \bar{u}_{\text{gs}}, \bar{u}_{\text{g\theta}} \) = vector of coupling coefficients
\( s_{\text{m}}, s_{\text{k}} \) = ratio of noncoupled \( k \)-th component of modal shapes
\( \gamma_{ij}, \gamma_{\text{ini}} \) = participation factor for \( j \)-th component of ground motion
\( \gamma_{n} \) = participation factor ratio
\( \gamma_{i}, \gamma_{j} \) = coefficients (defined in text)
\( e_{\text{jk}} \) = coefficient (defined in text)
\( \xi_{i} \) = modal damping factor
\( k_{i} \) = vertical dimensionless coordinate
\( \lambda_{\text{m}} \) = nondimensional coefficient of circular frequencies
\( \bar{\Omega}_{\text{i}} \) = noncoupled modal shapes
\( \bar{\Omega} \) = noncoupled modal shape matrix
\( \Omega \) = diagonal matrix of circular frequencies
\( \omega_{r}, \omega_{r}, \omega_{\theta} \) = noncoupled circular frequencies
\( \bar{\Theta} \) = lateral flexural properties
\( \bar{\Theta} \) = lateral shear properties.
### TABLE 1 - RATIO OF ROTATIONAL TO LATERAL COMPONENTS RESPONSE

<table>
<thead>
<tr>
<th>Rotational spectrum</th>
<th>Flat</th>
<th>Hyper</th>
<th>Hyper</th>
<th>Hyper</th>
<th>Hyper</th>
</tr>
</thead>
<tbody>
<tr>
<td>lateral spectrum</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(\omega)$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E \times T_L$</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E \times T_L^3/2$</td>
<td>0.11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E \times T_L^2$</td>
<td>0.201</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 2 - GEOMETRIC AND STIFFNESS OF STRUCTURAL ELEMENTS IN FIG. 10.

<table>
<thead>
<tr>
<th>Wall</th>
<th>Maximal moment of inertia (m^4)</th>
<th>Coordinates</th>
<th>Frame</th>
<th>Moment of inertia, in (\text{mm}^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(5.7166)</td>
<td>(-18.671)</td>
<td>(0)</td>
<td>(26.579) (53.158) (18.985)</td>
</tr>
<tr>
<td>(2)</td>
<td>(2.6834)</td>
<td>(5.329)</td>
<td>(0)</td>
<td>(26.579) (53.158) (18.985)</td>
</tr>
<tr>
<td>(3)</td>
<td>(2.6834)</td>
<td>(9.329)</td>
<td>(0)</td>
<td>(26.579) (53.158) (18.985)</td>
</tr>
<tr>
<td>(4)</td>
<td>(5.7166)</td>
<td>(13.329)</td>
<td>(0)</td>
<td>(26.579) (53.158) (18.985)</td>
</tr>
<tr>
<td>(5)</td>
<td>(1.2348)</td>
<td>(7.329)</td>
<td>(-4.5)</td>
<td>(26.579) (53.158) (18.985)</td>
</tr>
<tr>
<td>(6)</td>
<td>(1.2349)</td>
<td>(7.329)</td>
<td>(+4.5)</td>
<td>(26.579) (53.158) (18.985)</td>
</tr>
</tbody>
</table>

\[ E = 2 \times 10^{10} \text{ N/m}^2; \quad m = 70 \text{ kN sec}^2/\text{m}^2; \quad H = 48.0 \text{ mm} \]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_k$</td>
<td>$31.20 \times 10^4$ N</td>
</tr>
<tr>
<td>$E_{G1}/E_{A1}$</td>
<td>$8.74 \times 10^{-4}$ m$^2$</td>
</tr>
<tr>
<td>$a/r_k$</td>
<td>0.462</td>
</tr>
<tr>
<td>$r_a/r_k$</td>
<td>0.392</td>
</tr>
<tr>
<td>$a/r_k$</td>
<td>0.182</td>
</tr>
<tr>
<td>$1/r_k$</td>
<td>0.672</td>
</tr>
<tr>
<td>$a$</td>
<td>$70.16 \text{ KN m}^2/\text{sec}^2$</td>
</tr>
<tr>
<td>Ref. Level</td>
<td>Noncoupled Dynamic Properties</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td>$k$</td>
<td>Mode nr. I</td>
</tr>
<tr>
<td>10</td>
<td>0.562</td>
</tr>
<tr>
<td>9</td>
<td>0.491</td>
</tr>
<tr>
<td>8</td>
<td>0.419</td>
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<tr>
<td>7</td>
<td>0.347</td>
</tr>
<tr>
<td>6</td>
<td>0.275</td>
</tr>
<tr>
<td>5</td>
<td>0.207</td>
</tr>
<tr>
<td>4</td>
<td>0.143</td>
</tr>
<tr>
<td>3</td>
<td>0.087</td>
</tr>
<tr>
<td>2</td>
<td>0.041</td>
</tr>
<tr>
<td>1</td>
<td>0.011</td>
</tr>
</tbody>
</table>

**Base Shear Forces**

- $\xi_{yn}$
- $W_{yn}$

<table>
<thead>
<tr>
<th>Natural Freq. $w$ (rad/sec)</th>
<th>3.96</th>
<th>19.49</th>
<th>52.17</th>
<th>4.64</th>
<th>28.22</th>
<th>78.81</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particip. Factor ($f_i$)</td>
<td>+2.580</td>
<td>+1.330</td>
<td>+0.813</td>
<td>+2.553</td>
<td>+1.383</td>
<td>+0.789</td>
</tr>
</tbody>
</table>
### Table 5 - Computation of Equivalent Loads in the Coupied System

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>( \frac{\omega_1^2}{\omega_y^2} )</th>
<th>( q_n ) (t)</th>
<th>Coupling coefficients due to lateral component</th>
<th>Coupling coefficients due to rotational component</th>
<th>Coupled loads</th>
<th>Coupled load in actual system</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>( F_{x_n} ) ( F_{y_n} ) ( F_{x_n} / B_2 )</td>
<td>( C_{x_n} )</td>
<td>( C_{x_n} / B_2 )</td>
<td>( C_n )</td>
<td>( q_{y_n} ) (t)</td>
<td>( q_{y_n} ) (t)</td>
</tr>
<tr>
<td>(1)</td>
<td>1.37</td>
<td>94.28</td>
<td>0.89, 0.90, 0.93 (0.995)</td>
<td>3.45, 7.20, 5.22 (4.95)</td>
<td>87.7</td>
<td>52.2</td>
</tr>
<tr>
<td>1</td>
<td>2.09</td>
<td>-41.22</td>
<td>0.98, 0.93 (1.02)</td>
<td>2.15, 10.00, 4.90 (4.70)</td>
<td>-39.6</td>
<td>-21.4</td>
</tr>
<tr>
<td>2</td>
<td>2.28</td>
<td>22.49</td>
<td>0.99, 0.96 (1.02)</td>
<td>2.00, 11.00, 5.07 (4.65)</td>
<td>21.6</td>
<td>12.1</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Accord. to Fig. 3b</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Accord. to Fig. 6c</td>
<td></td>
<td></td>
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Accord. to Fig. 4b

Accord. to Fig. 6d

\[ q^B = 113.9 \]

\[ M_B^B = 24.9 \]
Fig. (2) Lateral Acceleration Response Spectra.
Fig. (5) Relative lateral loads due to lateral ground component.
Fig. (5) Pseudo Angular Velocity According to Ref. 5 and Idealized Spectra for Torsional Ground Component.
Fig. (9) Influence Function for Tall Building due to Rotational Ground Component.
Fig. (6) Influence Functions for Tall Buildings due to Lateral Ground Movement.