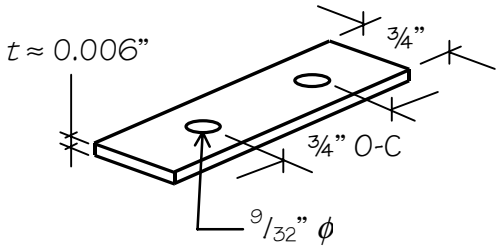
APPENDIX B
EFFECTS OF TRANSVERSE BRACING ON BEHAVIOR OF TEST STRUCTURE

B.1 Introduction

The effect of adding members for the lateral bracing of the test structure was investigated. The following three pages of hand calculations show the negligible contribution of the braces to the stiffness of the test structure in the direction of shaking. Following that is a description of simple tests performed to approximate the mechanical properties of the polyurethane material used at the connection points of the bracing to the specimen mass and the base plate.

Finally, two separate free vibration tests were performed (one without any cross bracing, and one with the metal bracing attached) to investigate the impact of the bracing on behavior. This was done using an extra specimen described in this section.

Check of Steel Strips for use as Lateral Support of Frame



Neglect holes for conservatism

Assume $t := 0.02 \cdot in$ for conservatism

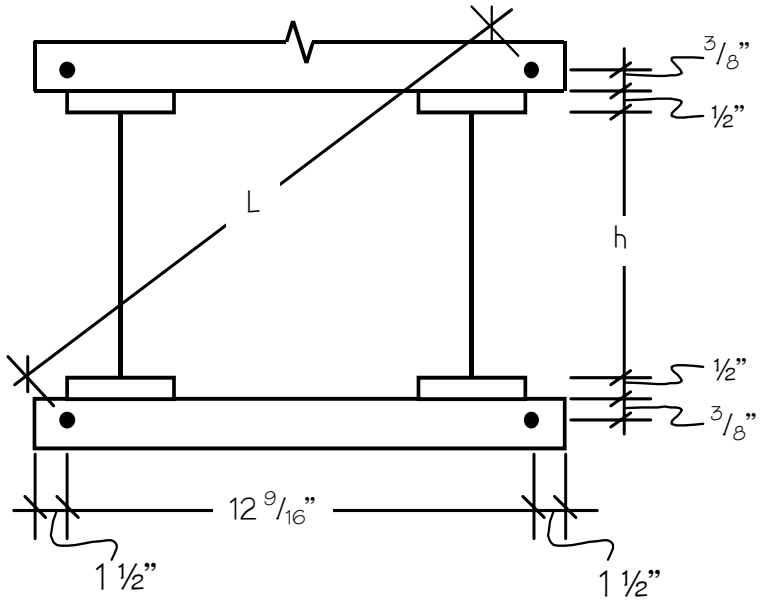
$width := \frac{3}{4} \cdot in$

$A := width \cdot t$ $\quad A = 0.015 \cdot in^2$

$I_{xx} := \frac{1}{12} \cdot width \cdot t^3$ $\quad I_{xx} = 5.00 \cdot 10^{-7} \cdot in^4$

$I_{yy} := \frac{1}{12} \cdot width^3 \cdot t$ $\quad I_{yy} = 7.03 \cdot 10^{-4} \cdot in^4$

Length Across Frame



Specimen Indices	h (in)	L (in)
3, 5	3.61	13.658
1, 2, 4, 10	5.41	14.460
15	7.22	15.436
9	8.12	15.976
8	10.81	17.764
14	10.83	17.778
7	13.51	19.766
13	14.43	20.484
6	16.24	21.942
12	18.02	23.424
11	21.65	26.559

Shortest, & Therefore Stiffest Braces

∴ Check if additional buckling strength is added to structure

Gauge Length, L, varies with specimen Height, h, according to:

$$L(h) := \sqrt{(h + 1.75 \cdot in)^2 + (12.5625 \cdot in)^2}$$

$A = 0.015 \cdot in^2$ $I_{xx} = 5.00 \cdot 10^{-7} \cdot in^4$ $I_{yy} = 7.03 \cdot 10^{-4} \cdot in^4$

$h := 3.61 \cdot in$ $L(h) = 13.658 \cdot in$

$r_x := \sqrt{\frac{I_{xx}}{A}}$ $r_x = 5.7735 \cdot 10^{-3} \cdot in$ $r_y := \sqrt{\frac{I_{yy}}{A}}$ $r_y = 0.2165 \cdot in$

For Fixed Ends: $k := 0.5$

$$\frac{k \cdot L(h)}{r_x} = 1182.8$$

Assume: $E := 29000 \cdot ksi$ & $F_y := 36 \cdot ksi$

AISC-LRFD Compressive Strength:

$$\lambda_c := \frac{k \cdot L(h)}{r_x \cdot \pi} \cdot \sqrt{\frac{F_y}{E}} \qquad \lambda_c = 13.266 \qquad \text{(E2-4)}$$

$\lambda_c > 1.5 = 1$

$$F_{cr} := \left(\frac{0.877}{\lambda_c^2}\right) \cdot F_y \qquad F_{cr} = 0.179 \cdot ksi \qquad \text{(E2-3)}$$

$$P_{n.band} := A \cdot F_{cr} \qquad P_{n.band} = 2.691 \cdot lbf \qquad \text{(E2-1)}$$

Nominal Compressive Strength of Specimen 3 & 5: $P_{n.spec} := 0.3761 \cdot kip$ (Table A-5 Strength & Stability Calcs)

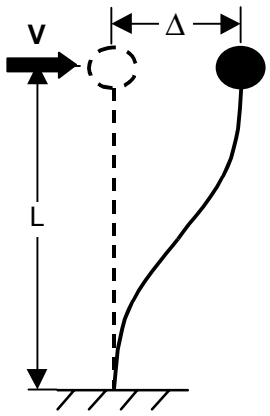
$P_{n.spec} = 1673 \cdot N$

$$\frac{P_{n.band}}{P_{n.spec}} = 0.716 \cdot \% \qquad \text{(E2-4)}$$

(Note: Both sets of compressive strength calcs (braces & specimen) are for single member, --> same ratio for entire structure (4 of each))

Lateral Stiffness of Brace

Assume fixed top & bottom, double curvature configuration:



$V = K \cdot \Delta$ $\qquad K_{brace} := \dfrac{12 \cdot E \cdot I_{xx}}{L(h)^3}$

$$K_{brace} = 6.829 \cdot 10^{-5} \cdot \frac{kip}{in}$$

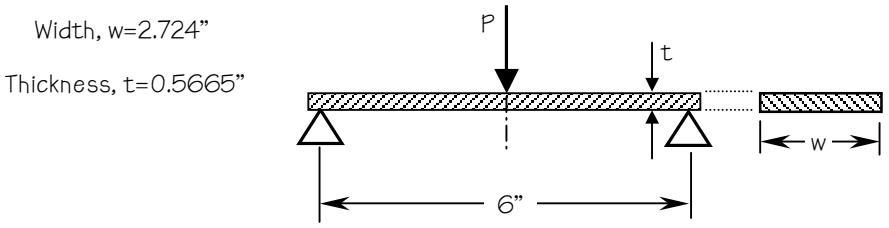
$$K_{col} := 0.1505 \cdot \frac{kip}{in}$$

$$\frac{K_{brace}}{K_{col}} = 0.045 \cdot \%$$

With extremely low axial compressive strength and lateral stiffness, steel strip appears to be good option for use as cross-bracing element.

B.2 Polyurethane Mechanical Properties

Prior to machining into the ends for the cross bracing the polyurethane material was tested to calculate a value for an approximate modulus of elasticity. Mechanical properties of the polyurethane material were investigated using a simply supported single point beam load as illustrated below:



The beam was loaded incrementally with weights that were nominally 2 and 5 Newton and listed in the table below:

2N Weights		5N Weights	
mass (g)	Weight (N)	mass (g)	Weight (N)
203.8	1.999	507.0	4.972
203.1	1.992	502.6	4.929
204.3	2.004	508.7	4.989
203.3	1.994	504.9	4.952
205.1	2.011		
204.2	2.003		
205.1	2.011		
205.5	2.015		
w_{ave} =	2.004	w_{ave} =	4.960

Deflections at the centerline of the beam were measured with a dial gauge. For a simply supported beam loaded by a concentrated point load at mid-span, the deflection at the centerline is given by: $\Delta = \frac{P \cdot l^3}{48 \cdot E \cdot I}$, which can be solved for the elastic modulus, *E*, since all other quantities are either known, or measured over the course of the test.

Five trials were performed with various increments of weights. Results for all trials are presented below, where the average value is the average of E calculated for each loading step for each trial, and the "trendline" is the *E* calculated by taking the linear best-fit generated by Excel, and dividing it by $\frac{48 \cdot I}{l^3}$. Individual trial results are presented on the subsequent pages.

Modulus of Elasticity Calculation Summary			
Trial	Average (ksi)	Trendline (ksi)	
1	5.703	5.416	
2	5.933	5.575	
3	5.646	5.550	
4	5.840	5.652	
5	5.520	5.415	
Average of all trials:	5.728	5.522	ksi
	39.494	38.071	MPa

L =	6	in
t =	0.5665	in
w =	2.724	in
I =	0.04127	in^4

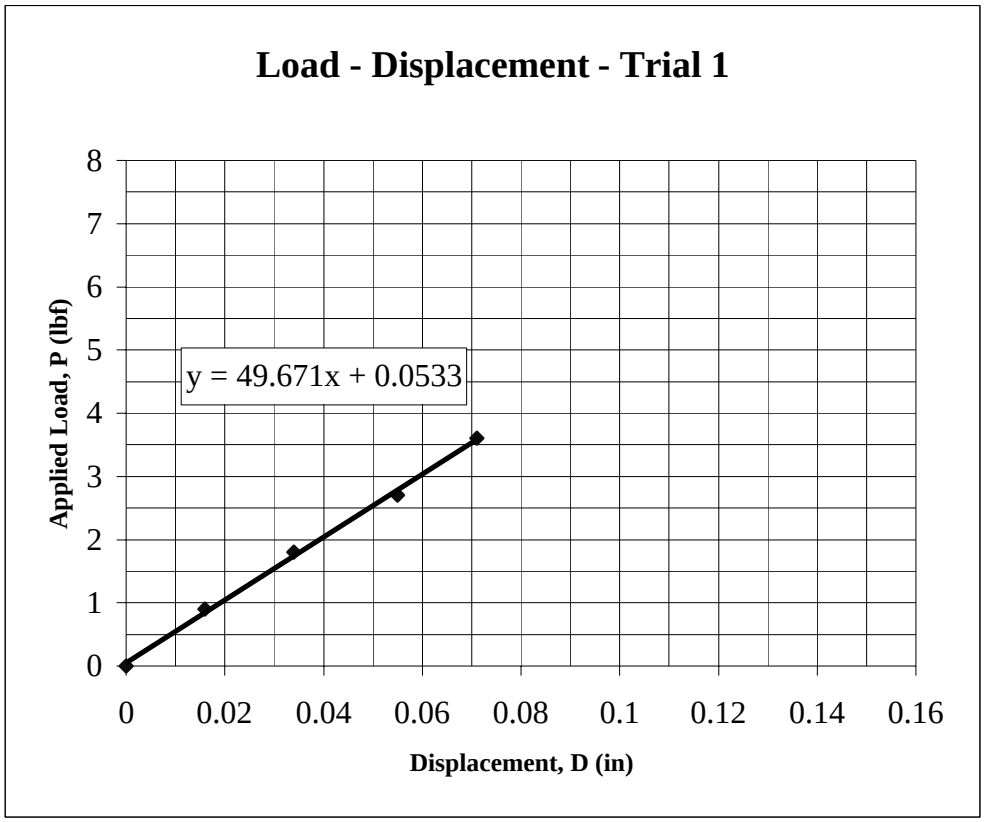
Average Value of Weights:	2N	5N
	2.00357	4.96038

Δ (in)	P (N)	P (lbf)	E (ksi)
0	0	0	0
0.016	4.0071	0.9009	6.1396
0.034	8.0143	1.8018	5.7784
0.055	12.0214	2.7027	5.3582
0.071	16.0286	3.6035	5.5342

E_{ave} = 5.7026 ksi

Slope = 49.671 lbf/in

E = 5.4161 ksi



L =	6	in
t =	0.5665	in
w =	2.724	in
I =	0.04127	in^4

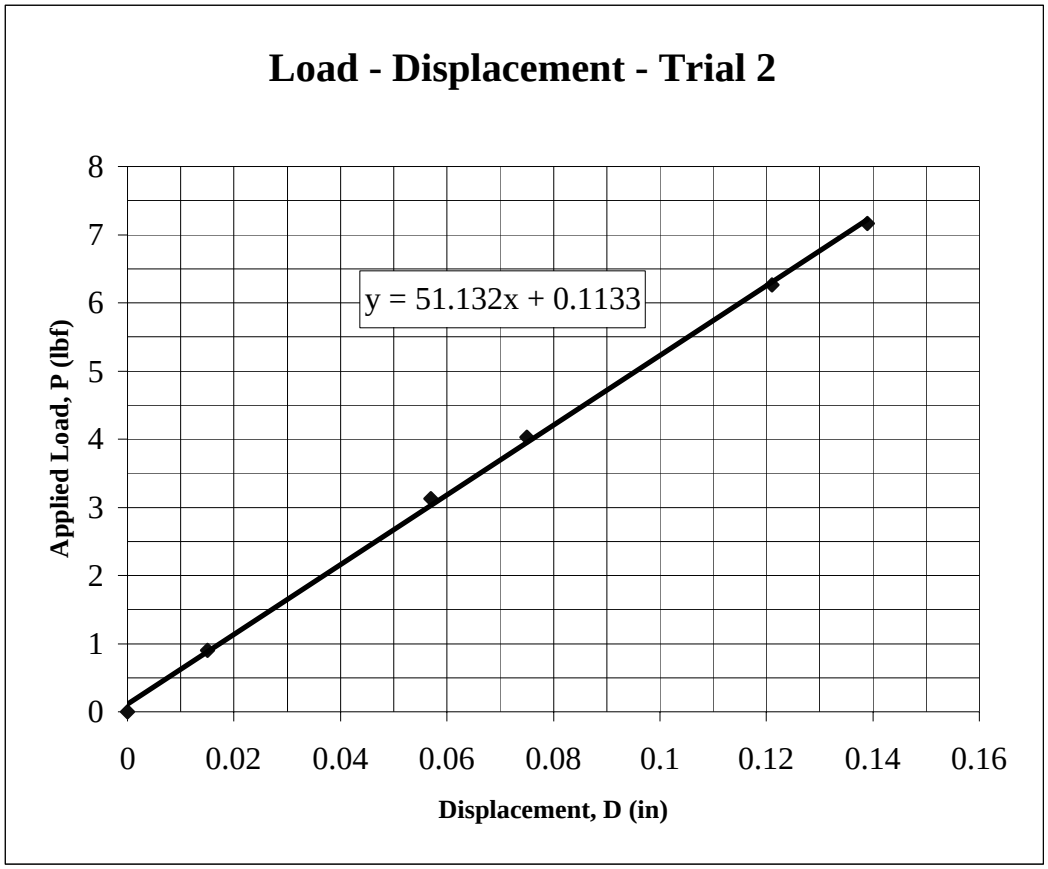
Average Value of Weights:	2N	5N
	2.0036	4.9604

Δ (in)	P (N)	P (lbf)	E (ksi)
0	0	0	0
0.015	4.0071	0.9009	6.5489
0.057	13.9279	3.1313	5.9901
0.075	17.9350	4.0322	5.8622
0.121	27.8558	6.2625	5.6436
0.139	31.8629	7.1634	5.6194

E_{ave} = 5.9328 ksi

Slope = 51.1320 lbf/in

E= 5.5754 ksi



L =	6	in
t =	0.5665	in
w =	2.724	in
I =	0.04127	in^4

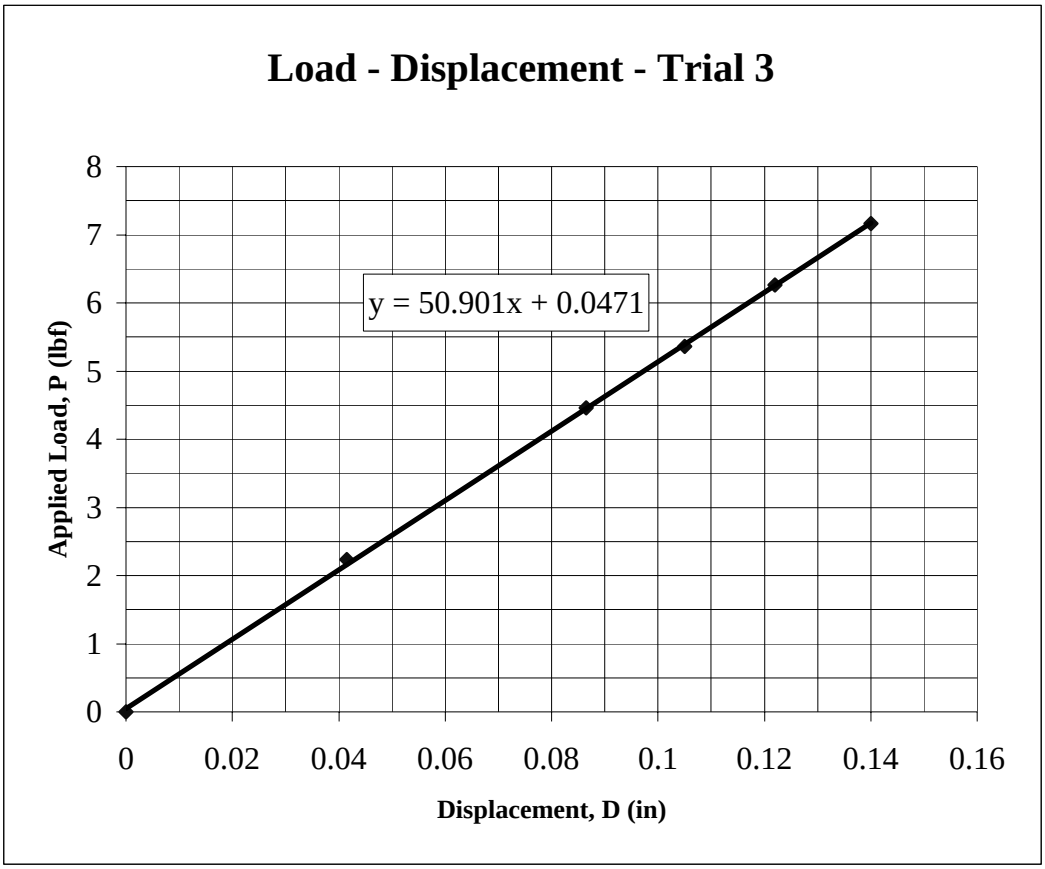
Average Value of Weights:	2N	5N
	2.0036	4.9604

Δ (in)	P (N)	P (lbf)	E (ksi)
0	0	0	0
0.0415	9.9208	2.2304	5.8603
0.0865	19.8415	4.4608	5.6232
0.105	23.8487	5.3617	5.5680
0.122	27.8558	6.2625	5.5973
0.14	31.8629	7.1634	5.5793

E_{ave} = 5.6456 ksi

Slope = 50.9010 lbf/in

E= 5.5503 ksi



L =	6	in
t =	0.5665	in
w =	2.724	in
I =	0.04127	in^4

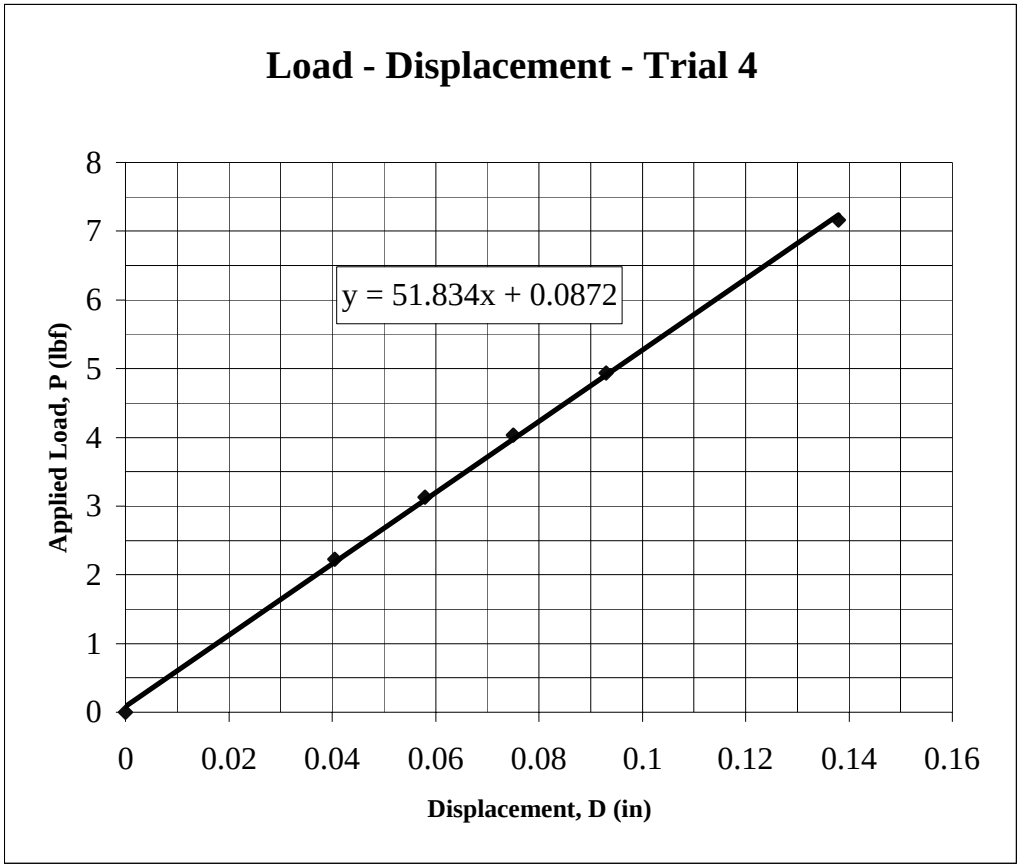
Average Value of Weights:	2N	5N
	2.0036	4.9604

Δ (in)	P (N)	P (lbf)	E (ksi)
0	0	0	0
0.0405	9.9208	2.2304	6.0050
0.058	13.9279	3.1313	5.8868
0.075	17.9350	4.0322	5.8622
0.093	21.9422	4.9330	5.7839
0.138	31.8629	7.1634	5.6602

E_{ave} = 5.8396 ksi

Slope = 51.8340 lbf/in

E= 5.6520 ksi



L =	6	in
t =	0.5665	in
w =	2.724	in
I =	0.04127	in^4

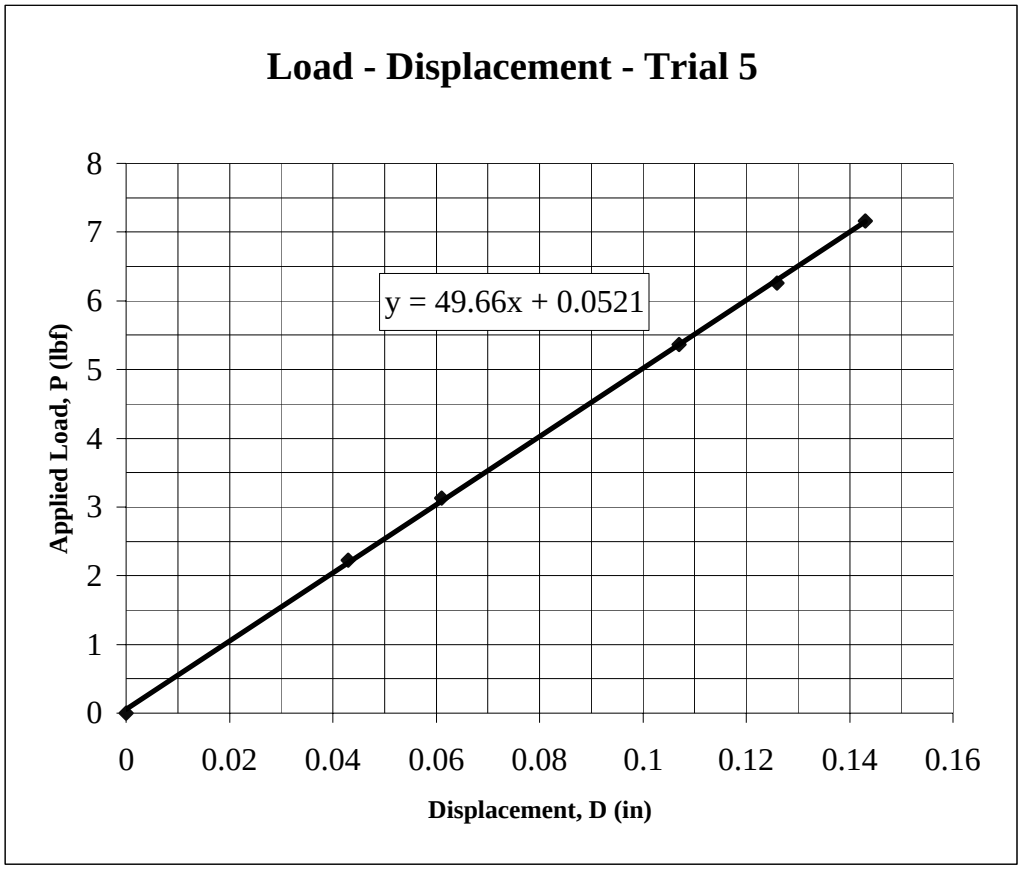
Average Value of Weights:	2N	5N
	2.0036	4.9604

Δ (in)	P (N)	P (lbf)	E (ksi)
0	0	0	0
0.043	9.9208	2.2304	5.6559
0.061	13.9279	3.1313	5.5973
0.107	23.8487	5.3617	5.4639
0.126	27.8558	6.2625	5.4196
0.143	31.8629	7.1634	5.4623

E_{ave} = 5.5198 ksi

Slope = 49.6600 lbf/in

E= 5.4149 ksi



B.3 Specimen 9* Free Vibration Tests

An extra set of columns nominally identical to Specimen 9 was used to assess the effect of transverse bracing on the specimen. Fewer mass plates were used (compared to Specimen 9) in order to avoid collapse at this stage. The column dimensions, measured and calculated, as well as column orientations, are given in Tables B-1, B-2, and B-3, respectively. These tables follow the same format as those presented in Chap 3.

Table B-4, as well as Figures B-1 and B-2 summarize the results of the two free vibration tests performed, in the same manner as Chap 4. Inherent damping of the specimen appears to be little affected by the addition of bracing in the transverse direction.

TABLE B-1 Measured Specimen Dimensions (mm)

Orientation	w_1	w_2	w_3	$Base_1$	$Base_2$	L_1	L_2	v_1	v_2
Specimen 9*									
U-D_1	4.98	4.95	5.04	50.8	50.8	207.2	205.6	-2.13	1.66
L-R_1	4.86	4.86	4.89	50.8	50.8	206.5	206.4	-0.93	0.00
U-D_2	4.9	4.9	4.9	50.8	50.8	206.4	205.8	1.6	5.0
L-R_2	4.8	4.9	4.8	50.8	50.8	205.5	206.9	-0.3	-3.2
U-D_3	5.0	4.9	5.0	50.8	50.8	207.0	206.1	-2.1	-2.1
L-R_3	4.9	4.8	4.9	50.8	50.8	206.9	206.3	-2.0	-1.8
U-D_4	4.9	4.9	4.9	50.8	50.8	206.0	205.4	0.0	2.0
L-R_4	5.0	5.0	4.9	50.8	50.8	205.5	205.9	2.0	0.0

TABLE B-2 Calculated Specimen Dimensions

Orientation	w_{avg} (mm)	L_{avg} (mm)	θ (deg)	V_{unif} (mm)	ϕ (deg)
Specimen 9*					
U-D_1	4.99	206.4	1.730	-0.23	-4.287
L-R_1	4.87	206.4	0.143	-0.46	-1.046
U-D_2	4.9	206.1	0.645	3.29	-3.870
L-R_2	4.8	206.2	-1.576	-1.73	3.253
U-D_3	4.9	206.6	1.117	-2.11	0.057
L-R_3	4.9	206.6	0.731	-1.92	-0.258
U-D_4	4.9	205.7	0.573	0.98	-2.221
L-R_4	5.0	205.7	-0.530	0.98	2.221

TABLE B-3 Column Locations and Orientations

Column Index	Location	Orientation
Specimen 9*		
1	NW	1
2	NE	4
3	SW	2
4	SE	3

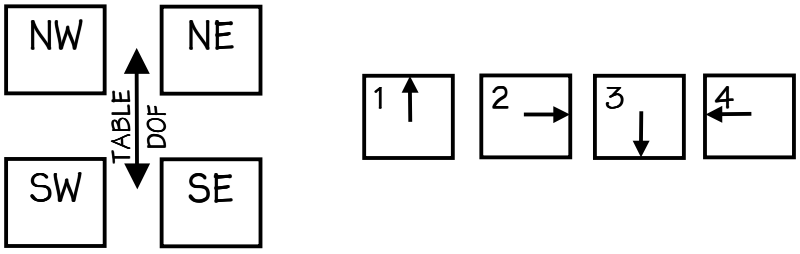


TABLE B-4 Free Vibration Damping Estimates

i	k_i	u_{ki}	k_i+j	u_{ki+j}	ξ_i (%)	Remarks
mass = 18.315 kg/col, no cross bracing						
HorWest Channel, u_o = 3.023 mm						
1	1	2.997	14	1.702	0.69	--
2	14	1.702	28	0.686	1.03	--
3	28	0.686	42	0.076	2.50	--
AccEast Channel, u_o = 0.053 g						
1	1	0.191	14	0.116	0.61	--
2	14	0.116	28	0.061	0.73	--
3	28	0.061	42	0.006	2.64	--
AccWest Channel, u_o = 0.050 g						
1	1	0.125	14	0.118	0.07	--
2	14	0.118	28	0.045	1.10	--
3	28	0.045	42	0.005	2.50	--
mass = 18.315 kg/col, with metal strip cross bracing						
HorEast Channel, u_o = 0.508 mm						
1	2	2.464	13	1.55	0.67	--
2	13	1.549	27	0.58	1.11	--
3	27	0.584	35	0.20	2.10	--
HorWest Channel, u_o = 0.483 mm						
1	2	2.337	13	1.58	0.57	--
2	13	1.575	27	0.53	1.23	--
3	27	0.533	35	0.08	3.87	--
AccEast Channel, u_o = 0.015 g						
1	2	0.169	13	0.106	0.68	--
2	13	0.106	27	0.041	1.08	--
3	27	0.041	35	0.013	2.29	--
AccWest Channel, u_o = 0.014 g						
1	2	0.162	13	0.105	0.63	--
2	13	0.105	27	0.042	1.04	--
3	27	0.042	35	0.011	2.67	--

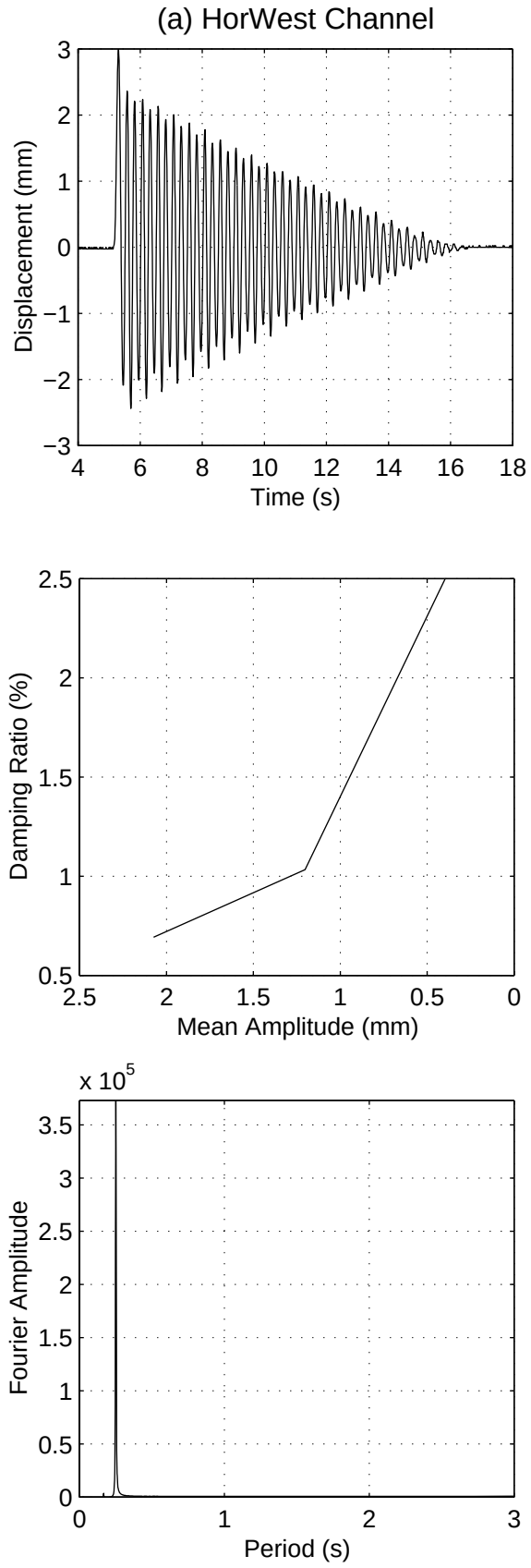


FIGURE B−1 Free Vibration Test of Specimen 9* − no Bracing

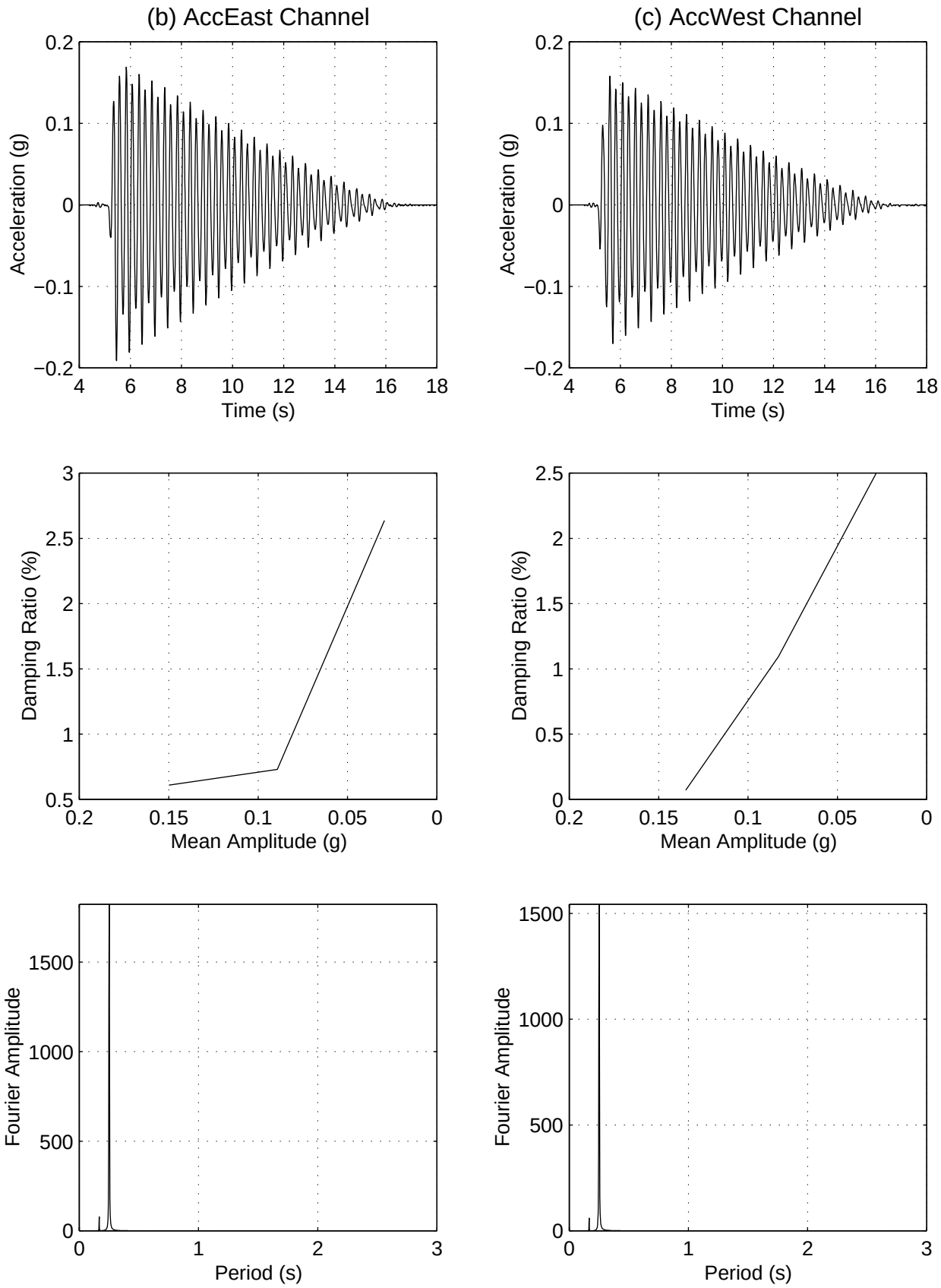


FIGURE B−1 (cont'd) Free Vibration Test of Specimen 9* − no Bracing

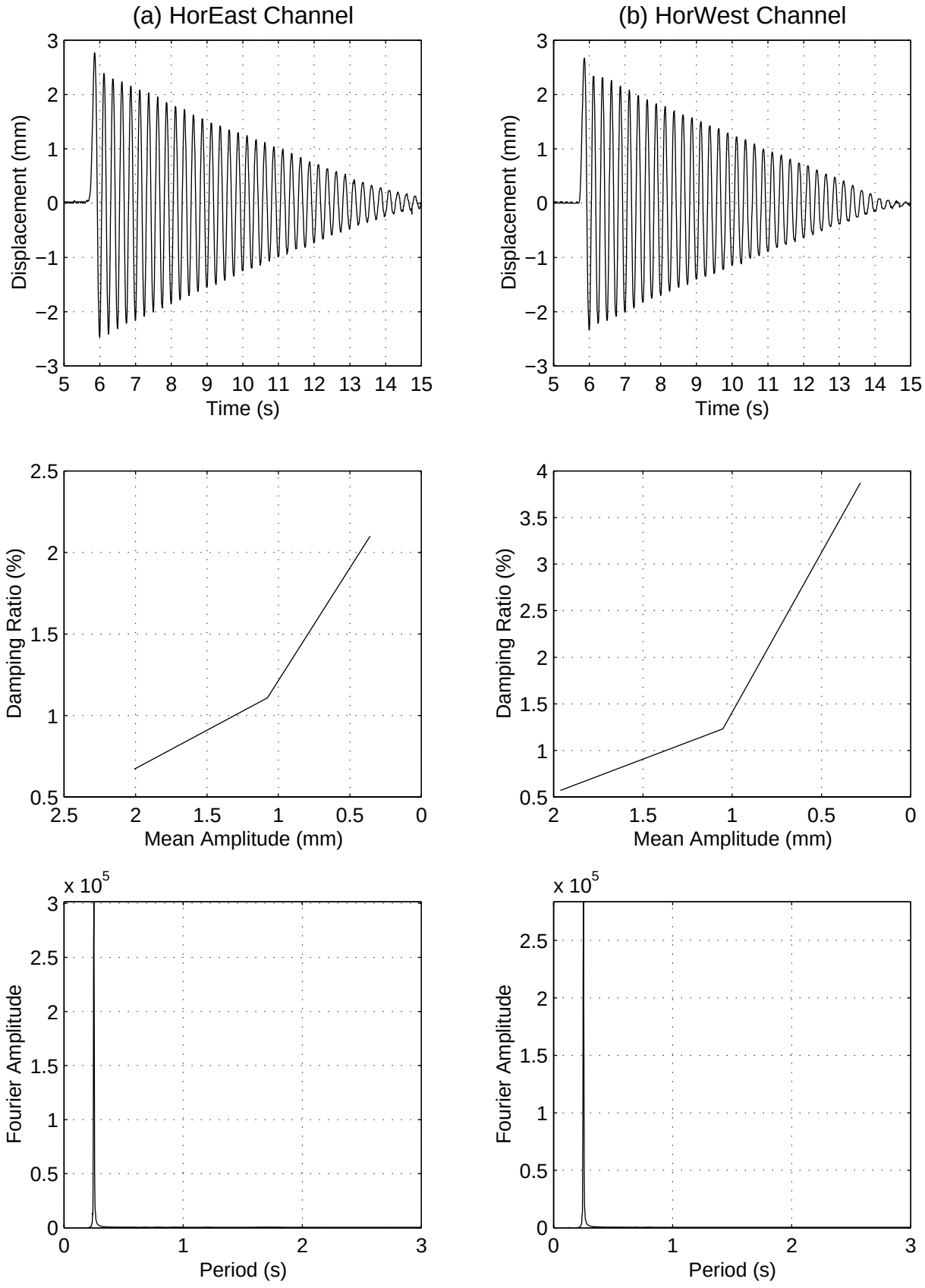


FIGURE B−2 Free Vibration Test of Specimen 9* − with Bracing

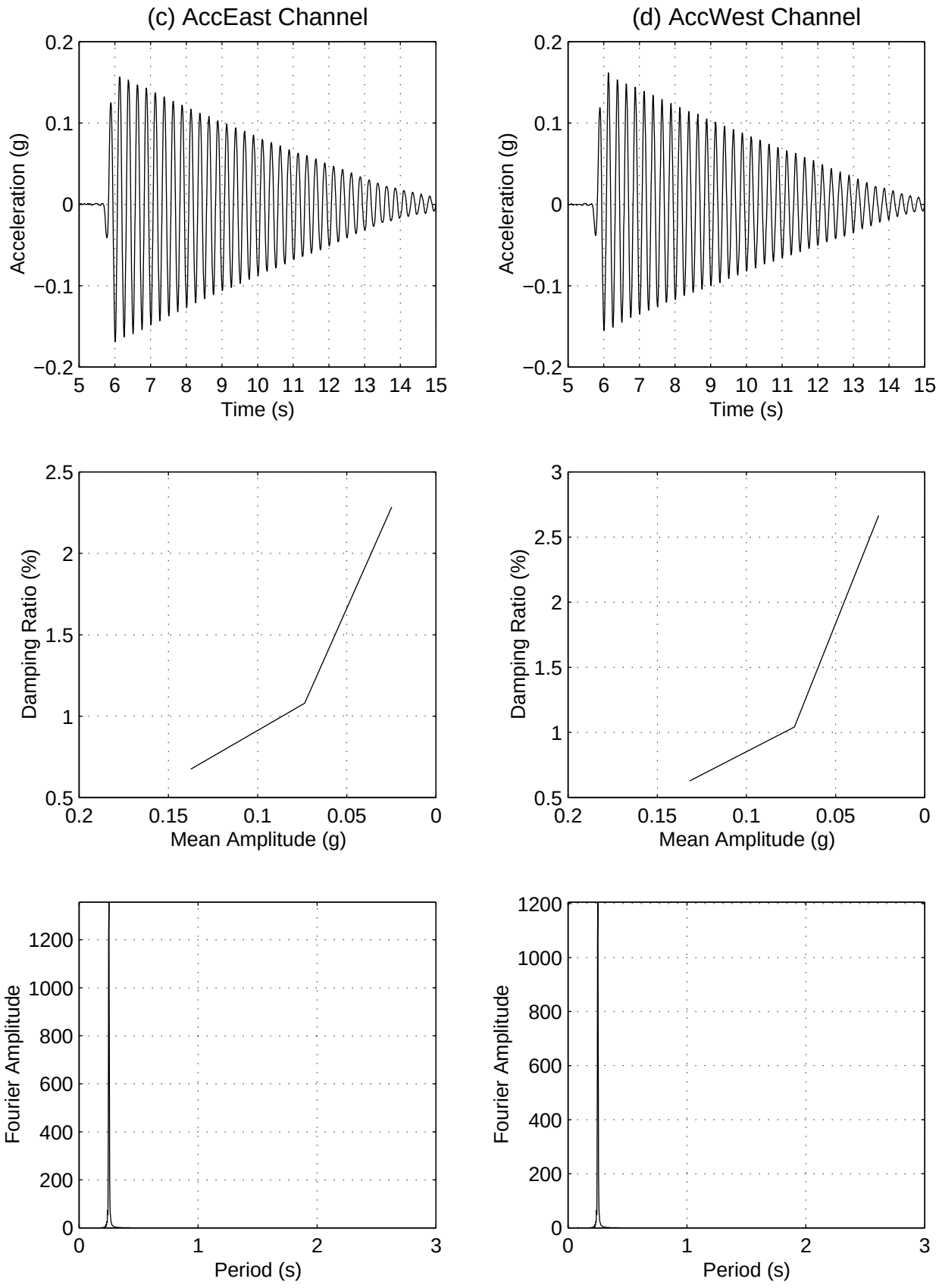


FIGURE B−2 (cont'd) Free Vibration Test of Specimen 9* − with Bracing