SEISMIC FRAGILITY ANALYSIS

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Abstract

Seismic fragility is the probability that a geotechnical, structural, and/or nonstructural system violates at least a limit state when subjected to a seismic event of specified intensity. Current methods for fragility analysis use peak ground acceleration (PGA), pseudo spectral acceleration (PSa), velocity (PSv), or spectral displacement (Sd) to characterize seismic intensity. While these descriptions of seismic intensity are attractive for applications, they cannot capture the essential properties of the ground motion, since the probability law of a stochastic process cannot be specified by, for example its maximum over a time interval. The paper presents a method for calculating system fragility as a function of moment magnitude \(m\) and source-to-site distance \(r\), referred to as fragility surface. The seismic ground motion intensity is characterized by seismic activity matrix, i.e., the relative frequency of the earthquakes with various \(m\) and \(r\). According to the specific barrier model the probability law of the ground motion process is completely characterized by \(m, r\), underlying soil at the site and other parameters. A structural/nonstructural system located in New York City is used to demonstrate the methodology. Fragility surfaces for different limit states are obtained for the system and its components.

Introduction

Fragility curves show the probability of a system reaching a limit state as a function of some measure of seismic intensity such as peak ground acceleration \(PGA\) (Hwang and Huo, 1994; Hwang and Jaw, 1990), pseudo spectral acceleration \(PSa\) (Singhal and Kiremidjian, 1996), or moment magnitude \(m\) and source-to-site distance \(r\) of the seismic event (Seidel et al., 1989).

A single parameter such as \(PGA\) or \(PSa\) cannot represent completely an entire function of time. \(PGA\) is an inadequate parameter for characterizing ground motion (Swell, 1989) and correlates weakly with both observed and theoretically computed structural damages. This paper presents an alternative method for indexing fragility. The method is based on seismic activity matrix and a ground acceleration model called specific barrier model (Papageorgiou and Aki, 1983a; Papageorgiou and Aki, 1983b).
Fragility Analysis

Common ground motion intensity characterization

Suppose that the ground acceleration process $W(t)$ can be modeled as a Gaussian white noise (GWN) excitation and consider a single degree of freedom (SDOF) system representing a structure. The initial frequency and the damping ratio of the system are $\omega_0 = 7$ rad/sec and $\zeta = 5\%$, respectively and the parameters of the Bouc-Wen model (Wen, 1976) representing a hysteretic damper are $A = 1$, $\alpha = 0.5$, $\beta = 0.5$, $n = 2$ and the rigidity ratio $\rho$. The intensity of the GWN is $g_0 = 1.0$ and the Nyquist frequency is 250 rad/sec. Figure 1 reflects the lack of correlation between $PGA$ and

![Figure 1. Lack of correlation](image)

the spectral displacement ($S_d$) for $\rho = 1$ (linear system) and for $\rho = 0.5$ (nonlinear system), and between $PS_a$ calculated using the initial linear stiffness and $S_d$ for the nonlinear system using 50 samples of $W(t)$.

Proposed ground motion intensity characterization

Example-1: Unlike $PGA$, the intensity $g_0$ completely defines the GWN excitation. Consider the same parameters for the GWN input and the same SDOF system used previously with $\rho = 1$, so that the system is linear. The correlation between $g_0$ and the maximum response of the system is estimated using (i) Monte Carlo simulation and (ii) crossing theory of stochastic processes. For the simulation approach 10 samples of $W(t)$ are generated for 20 different values of $g_0$ covering the range from 0.1 to 2.0 and corresponding $S_d$'s are calculated. Figure 2 shows spectral density of the GWN excitation and the correlation between $g_0$ and $S_d$. Probability of system failure given a limit state, referred as the fragility, is also estimated using Monte Carlo simulation and crossing theory. For the simulation approach the fragility at a given intensity level $g_0$ is approximated by the ratio of the number of samples of $S_d$ exceeding a given limit state $d$ to the total number of samples. The fragility of the system can also be approximated using the crossing theory. Accordingly, the probability that the (stationary) system response process $R(t)$ leaves the safe set $D = (-d, d)$ in a time interval of length $t_w$ is

$$P[\text{System failure}] \approx 1 - P[R(0) \in D] P[N_D(t_w) = 0],$$

in which $N_D(t_w)$ is the average number of $D$-outcrossings of $R(t)$ in $t_w$ and $P[N_D(t_w) = 0] \approx \exp(-\nu_D t_w)$ is the reliability of the system, the probability that $R(t)$ stays in the safe set $D$ during $t_w$ (Veneziano et al., 1977). It is assumed that $P[R(0) \in D] \approx 1$ for the selected safe
set. The mean crossing rate of the system $\nu_D$ is the mean rate at which the response process $R(t)$ leaves the safe set $D$ and is given by $\nu_D = \frac{1}{\pi} \frac{\dot{\sigma}}{\sigma} \exp \left( -\frac{d^2}{2\sigma^2} \right)$ assuming $R(t) \sim N(0, \sigma^2)$ and $\dot{R}(t) \sim N(0, \dot{\sigma}^2)$ (Veneziano et al., 1977). Hence, the fragility of the system can be approximated by

$$P[\text{System failure}] \simeq 1 - \exp \left( -\nu_D t_w \right).$$

(1)

Figure 2 shows the fragility of the system for $d = 0.4$ against $g_0$ estimated using Monte Carlo simulation and crossing theory.

Example-2: Consider the same linear SDOF system used previously. The seismic ground acceleration at a site is modeled by a zero-mean, stationary Gaussian process $F(t)$ with spectral density $s_{FF}$ given by the specific barrier model (Papageorgiou and Aki, 1983a; Papageorgiou and Aki, 1983b). Accordingly,

$$s_{FF}(\omega, r) = \frac{1}{2\pi t_w} |a(\omega, r)|^2$$

(2)

where $t_w$ is the duration of the strong ground motion (Halldorsson et al., 2002), $r$ denotes the distance from the seismic source to site, $|a(\omega, r)|$ is the Fourier amplitude spectrum of the strong ground acceleration at the site and $f = \omega/2\pi$ is the frequency in Hertz. The spectral density function in Eq. 2 completely defines the zero-mean, stationary Gaussian seismic ground acceleration process $F(t)$ and is a function of the moment magnitude of the earthquake $m$, source to site distance $r$, and the soil type at the site. Figure 3 shows the spectral density of ground acceleration for $m = 5.3$ and $r = 150$ km, the correlation between $(m,r)$ and $S_d$, and the fragility of the system for $d = 0.5$ against $(m,r)$.

**Numerical Example**

A simplified mathematical model is developed for a hospital building constructed in 1970’s and located in Southern California, referred to as the MCEER Demonstration Hospital Project, WC70. It is assumed that the structure (i) is linear elastic and does not fail, (ii) has a proportional damping, and that (iii) translation in the weak, $x$, and the strong, $y$, directions are decoupled, and (iv) cascade analysis applies, that is, the nonstructural system does not affect the dynamics of the supporting
structure. The modal properties corresponding to the first 12 modes are calculated using a three dimensional model of the structure. The direction of the seismic ground motion is assumed to coincide with the weak direction of the structure. The non-zero modal participation factors in the $x$ direction correspond to modes 1, 4, 7 and 10. An illustration of the WC70 model with a nonstructural system ($NS$) consisting of two components $C_1$ and $C_2$ attached to it, and the required modal properties of the structure are shown in Table 1. It is assumed that damping ratio is 3% for all modes.

![Figure 3. Specific barrier model](image)

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\omega_i$</th>
<th>$\Gamma_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.22</td>
<td>15.83</td>
</tr>
<tr>
<td>4</td>
<td>20.98</td>
<td>-6.00</td>
</tr>
<tr>
<td>7</td>
<td>37.41</td>
<td>3.47</td>
</tr>
<tr>
<td>10</td>
<td>56.81</td>
<td>2.09</td>
</tr>
</tbody>
</table>

Table 1. Illustration and modal properties of WC70

**Structural system**

Equation of motion of a multi degree of freedom system subjected to a seismic ground acceleration $F(t)$ is given by

$$
\mathbf{m}\ddot{\mathbf{Z}}(t) + \mathbf{c}\dot{\mathbf{Z}}(t) + \mathbf{k}\mathbf{Z}(t) = -\mathbf{m}\mathbf{1}F(t)
$$

(3)

where $\mathbf{Z}(t)$ is the relative displacement response in $x$ direction, $\mathbf{1} = [1, \ldots, 1]^T$, and $\mathbf{m}$, $\mathbf{c}$ and $\mathbf{k}$ are the mass, stiffness and the damping matrices of the structural system, respectively. The response $\mathbf{Z}(t)$ is a stationary Gaussian process since it is assumed that the structural system is linear and $F(t)$ is a stationary Gaussian process. The spectral density between the absolute acceleration responses $G_k(t)$ and $G_l(s)$ at joints
$k$ and $l$ can be written as

$$S_{G_k G_l}(\omega) = \alpha_k \alpha_l S_{FF}(\omega) - S_{FF}(\omega) \sum_{i=1}^{n} \Gamma_i (\alpha_k \phi_i(l) \tilde{H}_i^*(\omega) + \alpha_l \phi_i(k) \tilde{H}_i(\omega))$$  \tag{4}$$

and

$$S_{FF}(\omega) \sum_{i,j=1}^{n} \phi_i(k) \phi_j(l) \Gamma_i \Gamma_j \tilde{H}_i(\omega) \tilde{H}_j^*(\omega),$$  \tag{5}$$

where $\alpha_k = 1 - \sum_{j=1}^{n} \Gamma_j \phi_j(k)$, $\phi_j(k)$ is the $k^{th}$ coordinate of the modal shape $j$,

$$\tilde{H}_i(\omega) = \left( \frac{\omega_i^2}{\omega_{d,i}^2} - \frac{2 \zeta_i \omega_i \omega_{d,i}}{\omega_i + h \omega} \right) H_i(\omega) - \frac{2 \zeta_i \omega_i}{\omega_i + h \omega}$$  \tag{6}$$

with $\omega_{d,i} = \omega_i \sqrt{1 - \zeta_i^2}$, $h = \sqrt{-1}$, $H_i(\omega) = 1/ (\omega_i^2 - \omega^2 + 2 h \zeta_i \omega \omega)$ and $\tilde{H}_i^*(\omega)$ is the conjugate of $\tilde{H}_i(\omega)$. Mean and correlations, namely the second moment properties, define the response process completely.

**Nonstructural system**

The nonstructural system consists of a water tank and a power generator located at the roof (joint-24) and at the first floor (joint-5), respectively (Table 1). It is assumed that (i) the components are not interacting, (ii) $C_1$ is drift sensitive and $C_2$ is velocity sensitive, and (iii) both components are linear SDOF oscillators with parameters $\omega_{C_1} = 8.0$ rad/sec, $\zeta_{C_1} = 0.02$, $\omega_{C_2} = 20.0$ rad/sec, $\zeta_{C_2} = 0.03$. Equation of motions for $C_i$ is

$$\ddot{R}_i(t) + 2 \zeta_{C_i} \omega_{C_i} \dot{R}_i(t) + \omega_{C_i}^2 R_i(t) = -G_k(t)$$  \tag{7}$$

where $R_i(t)$ is the relative displacement response of $C_i$ for $i = 1, 2$, $k = 24$ for $i = 1$ and $k = 5$ for $i = 2$.

**Fragility surfaces by crossing theory**

Mean crossing rates are used to obtain an upper bound for the probability of failure of $C_1$, $C_2$ and $NS$. It can be shown that the mean crossing rate $\nu_{NS}$ of the nonstructural system can be bounded by $\nu_{NS} \leq \nu_{C_1} + \nu_{C_2}$, where $\nu_{C_i}$ is the mean crossing rate of the component $C_i$, $i = 1, 2$ (Veneziano et al., 1977). The limit states for the relative displacement response of $C_1$ and the relative velocity response of $C_2$ are $d = 15$ cm and $v = 40$ cm/sec, respectively. Figure 4 shows the fragilities of the components and an upper bound for the system fragility for the linear nonstructural system defined previously.

**Conclusions**

Fragility is the probability of a system reaching a limit state as a function of some measures of seismic intensity and it needs to be plotted against parameters of the probability law of ground acceleration rather than properties of its samples such as

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Figure 4. Fragility surfaces of the nonstructural components and system

$PGA$ or $PS_a$. This paper presents an alternative method for indexing fragility. The method is based on a ground acceleration model called specific barrier model.

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References


