LOADING SYSTEMS: Dynamic Structural Testing

Loading Devices- Dynamic

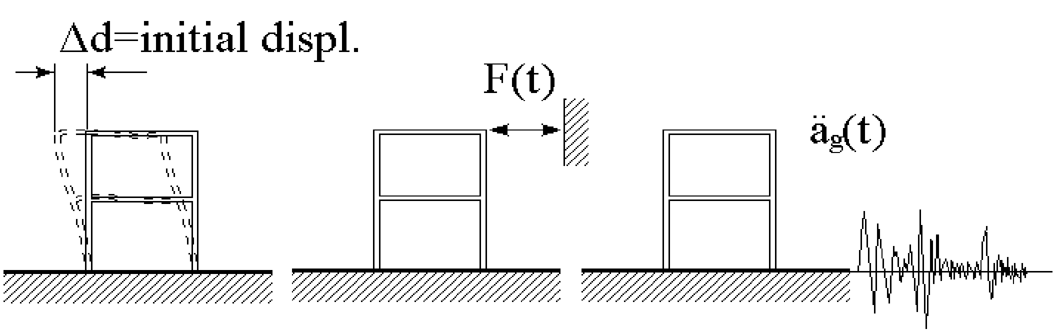
1. Objective of loading device

- Produce inertial load effects in structure
- Produce response in ‘real time’ or ‘deformed time’
- Determine high strain rate effects on member properties.

$$\left(\frac{d\varepsilon}{dt} = \dot{\varepsilon} = \text{strain rate}\right)$$

2. Testing for Inertial load effects

- Free vibrations – produced by changing initial conditions
- Forced vibrations – produced by time varying loads, or
- Base vibrations – produced by base movement

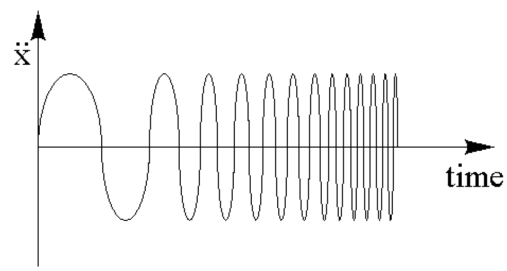


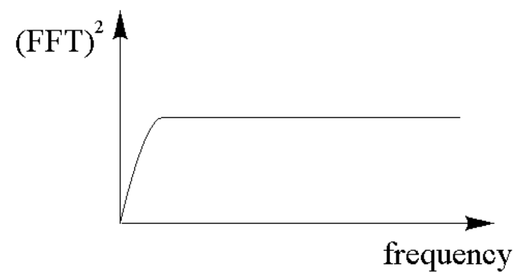
3. Loading Systems

- ***Impact devices*** – for free vibration – changes initial velocity

- Cable snap-back (quick release) – free vibration – changes initial displacement

- Rotating motors – for forced vibration – sinusoidal sweep





- Vibrating tables (forced vibration by base movement) – harmonic and random motion
  - Harmonic
  - Sine Sweep (like the rotating motors)
  - White noise
  - Random motions
    - Natural
    - Simulated
      - Spectrum compatible
      - Empirical

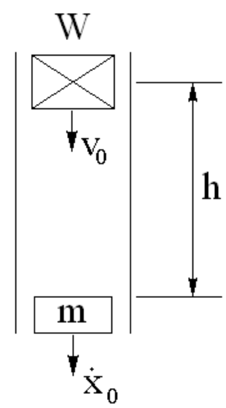
- Hydraulically driven actuators (forced vibration) – sinusoidal or random loading, and pseudo dynamic effects

- Hybrid systems

Impact

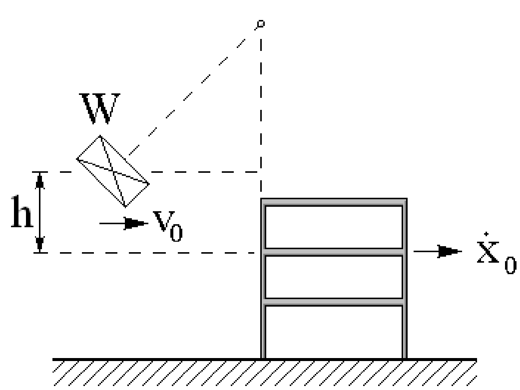
Falling weights

$$V_o = \sqrt{2gh} \qquad \dot{X} = \frac{W}{g} \cdot \frac{V_0}{m}$$



(controlled impulse transfer)

Swinging weights



(controlled impulse transfer)

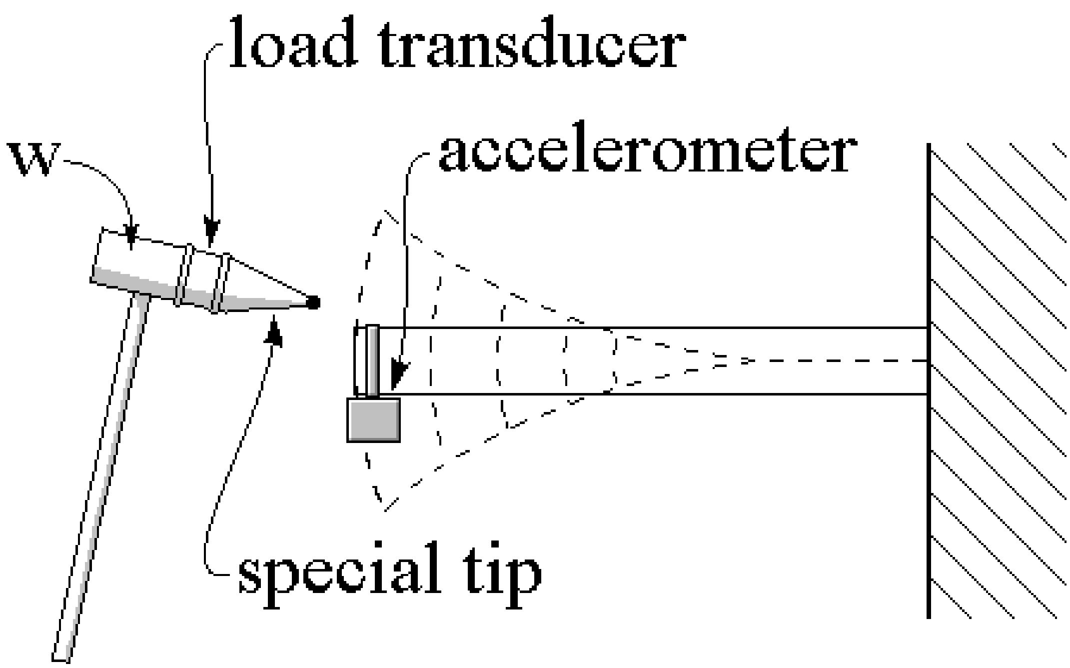
$$V_o = \sqrt{2gh} \qquad \dot{X} = \frac{W \cdot V_0}{g \quad m}$$

This can produce local damage from plastic impulse transfer (on the same principle as a wrecking ball).

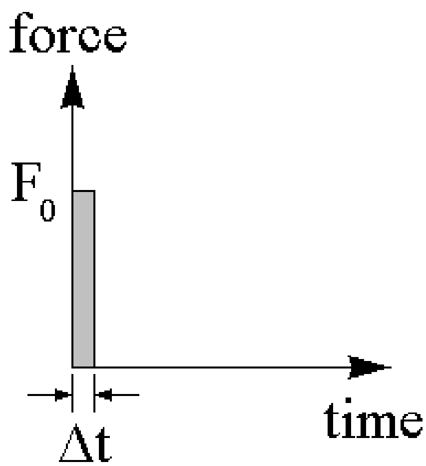
These devices can be used for strength qualifications.

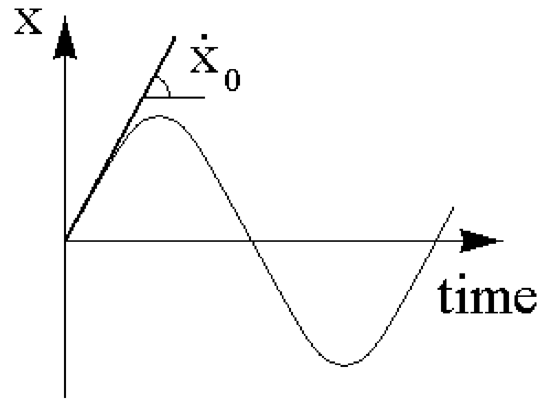
'Impact hammers'

- Produce vibrational effects without affecting strength.
- The vibrations are usually measured using accelerometers or velocity meters
- Built to measure the load (impact) history.



e.g. $m\ddot{x}+kx=0$ (an undamped free vibration system)





measured with a displacement transducer…

$$\text{Impulse} = \int F dt = m\Delta v$$

$$\Rightarrow F_0 \Delta t = m\dot{x}_0$$

$$\dot{x}_0 = \frac{F_0 \Delta t}{m}$$

$$x(t) \to x(\omega) \to \ddot{x}(\omega) \to \dot{x}(\omega)$$

$x(\omega)$ = frequency domain response function

In the frequency domain:

$$X(\omega) = H^{-1}(\omega)\ F(\omega) \qquad H(\omega)F(\omega) = X(\omega)$$

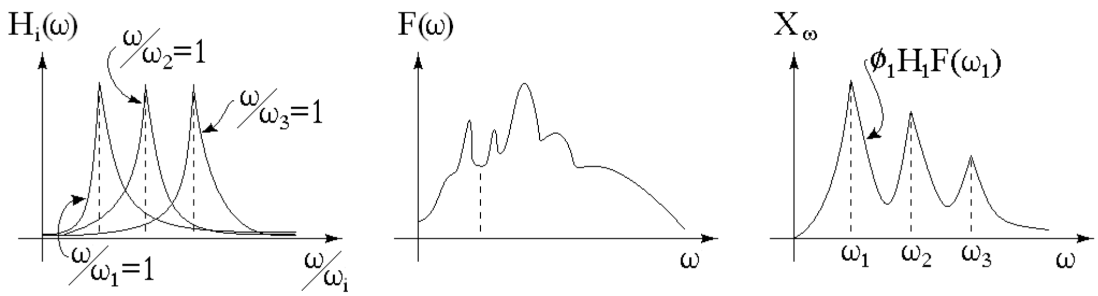
where $H^{-1}(\omega)$ is the frequency complex transverse function:

$$H(\omega) = \frac{\dfrac{\omega_0^2}{m}}{\left\{\left[1-\left(\dfrac{\omega}{\omega_0}\right)^2\right]+\left[2\xi_i \dfrac{\omega}{\omega_0}\right]\right\}}$$

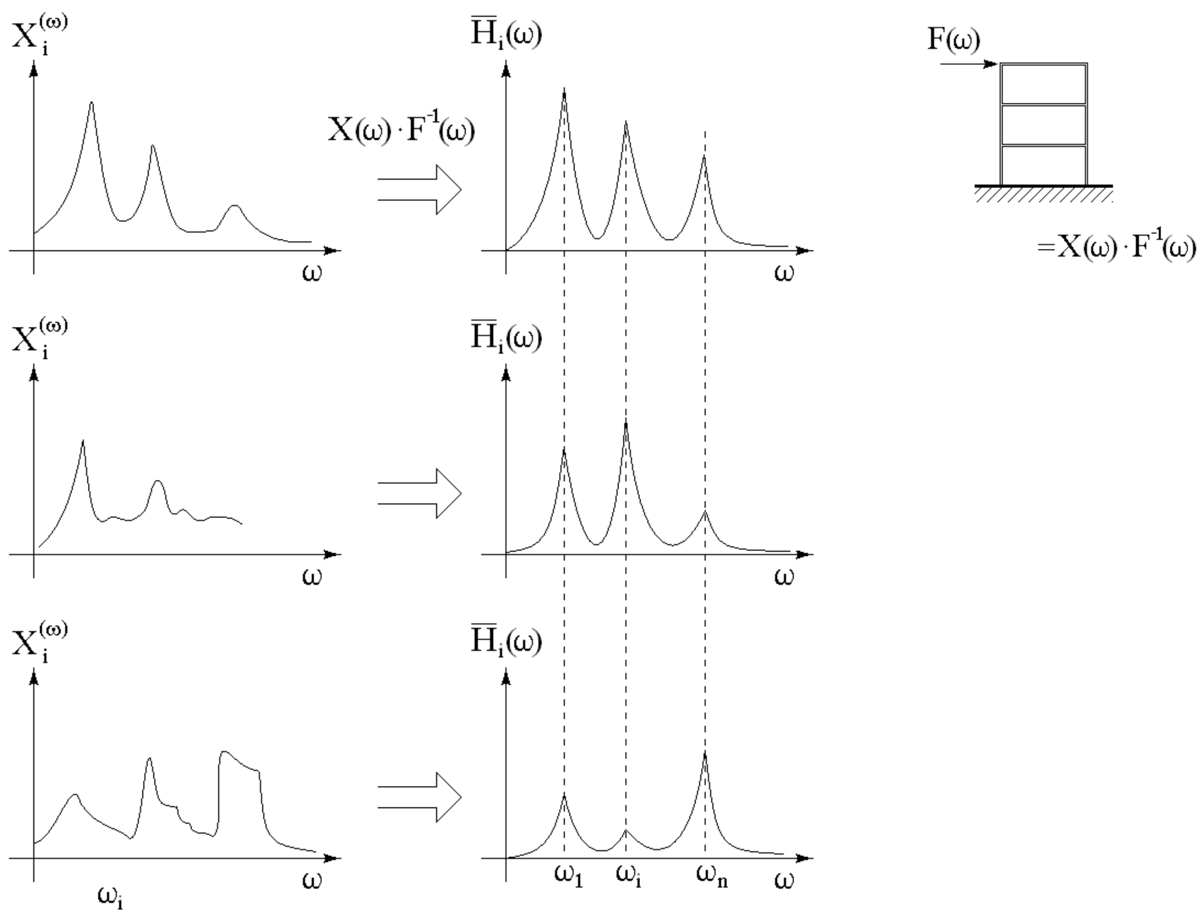
This function has information about frequencies.

If the structures have multiple degrees of freedom (N):

$$X(\omega)=\sum_{1}^{N} H_i(\omega)\cdot F(\omega)\cdot \varphi_i(\omega_i) \Rightarrow$$



Modal identification with single impact source:



$$\bar{H}_n(\omega)=\sum_j H_j(\omega)\cdot\varphi_{nj}(\omega_j)$$

for well separated peaks:

$$\bar{H}_n(\omega_j)=H_j(\omega)\cdot\varphi_{nj}(\omega_j)$$

by studying $H_j(\omega_j)$; determine damping ratio using peak value, or half power band width.

Divide $$\bar{H}_n(\omega_j)\Big/\bar{H}_i(\omega_j)\Rightarrow \varphi_{nj}(\omega_j)\Big/\varphi_{1j}(\omega_j)=\bar{\varphi}_{nj}\ \text{(normalized)}$$

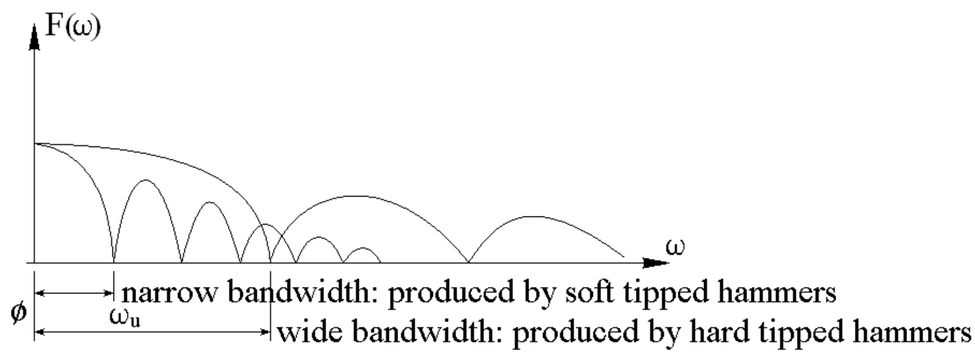
The impact hammer produces $F(\omega)$ and the accelerometer produces $X(\omega)$.

Structure identification: determine $H(\omega)$ and all dynamic characteristics…

$$H(\omega)=X(\omega)\cdot F^{-1}(\omega)$$

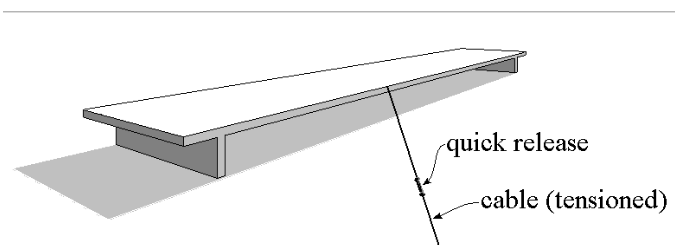
Usually

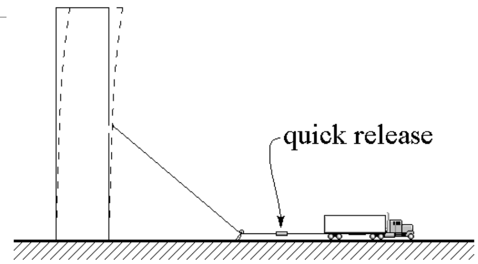
$F(\omega)\Rightarrow$



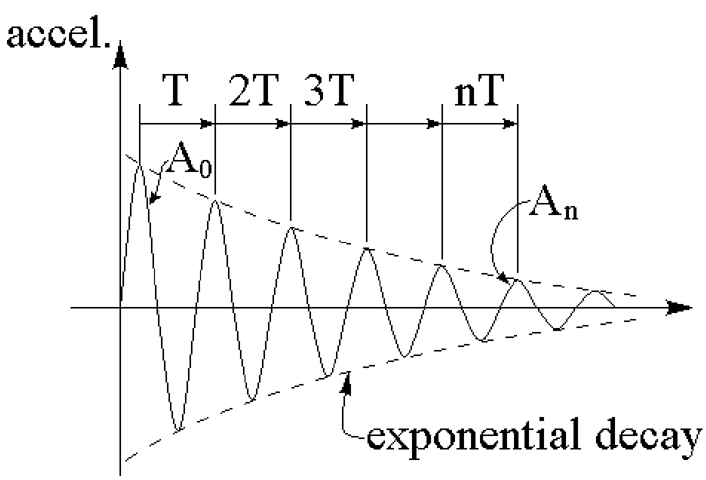
(Use hard tips for multimode identification)
(Use soft tips for high-resolution identification of lower modes)

a. Snap-back (cable) tests





- The loading produces an initial static deformation at the loading level.
- Once ‘snapped’, <u>free</u> vibrations are produced in the structure, primarily in the lower modes.
- Loading can be applied symmetrically or asymmetrically (for torsional modes), based on the attachment location.
- Record time domain response with accelerometers
- Determine low (first) mode damping using the logarithmic displacement method:



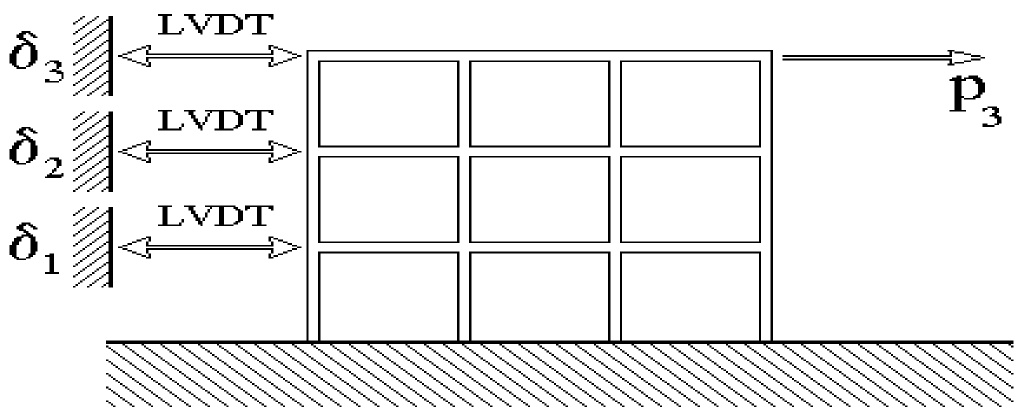
$$\xi = \frac{1}{2\pi_n} \ln \frac{A_n}{A_0}$$

- Record the transfer functions.

- The damping value is equivalent viscous damping created in response to a number of phenomena, such as cracking, slip, friction and the like…

- The damping value will therefore vary depending on the displacement amplitude.

- For snap back tests, displacement amplitudes should vary and an average equivalent viscous damping value should be determined.



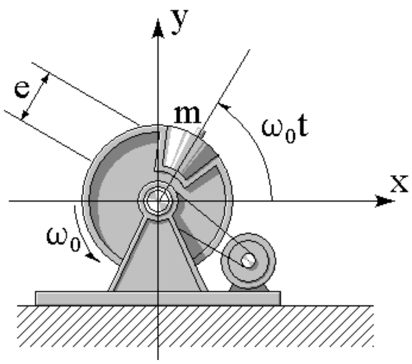
- During the static application of loading, deflections can be measured to find the structural stiffness matrix and inverted to find the structural stiffness matrix if the structure behaves as a shear building (i.e. with rigid floors). TEST ALSO NAMED "PULL BACK" TEST

$$f_{ij} = \begin{bmatrix} \delta_3/p_3 & \delta_2/p_3 & \delta_1/p_3 \\ \delta_3/p_2 & \delta_2/p_2 & \delta_1/p_2 \\ \delta_3/p_1 & \delta_2/p_1 & \delta_1/p_1 \end{bmatrix} \text{in}/\text{kip}$$

$$k_{ij} = f_{ij}^{-1}$$

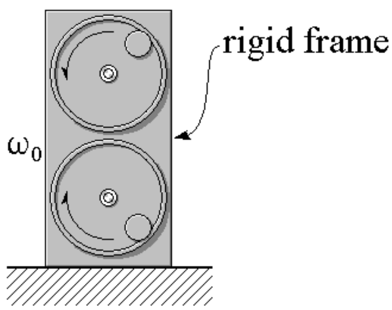
Rotating motors

Generated forces:



→ $F_x = me\omega_0^2 \cos\omega_0 t$

↓ $F_y = me\omega_0^2 \sin\omega_0 t$

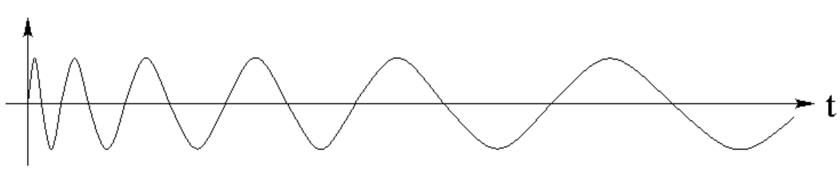


→ $F_x = 2me\omega_0^2 \cos\omega_0 t$

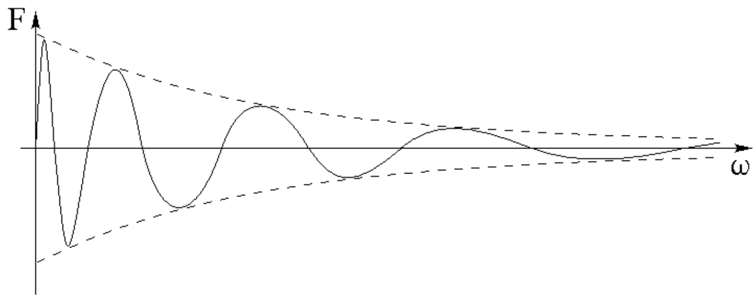
↓ $F_y = 0$

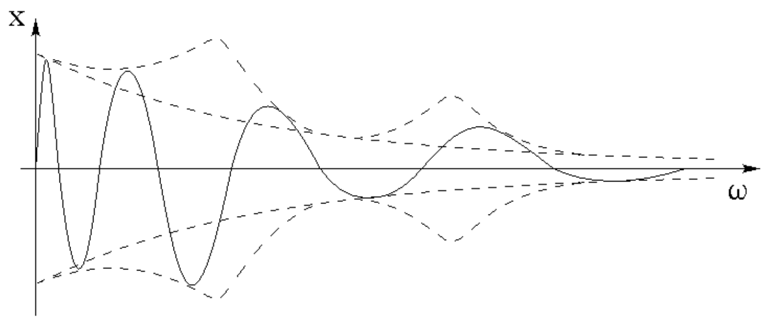
- Used in tall buildings, large bridges, dams, etc…
- Force depends on mass (m), and frequency of rotation (ω_{0max}) which is dependant on motor capacity.
- Produces a 'sine sweep history'.

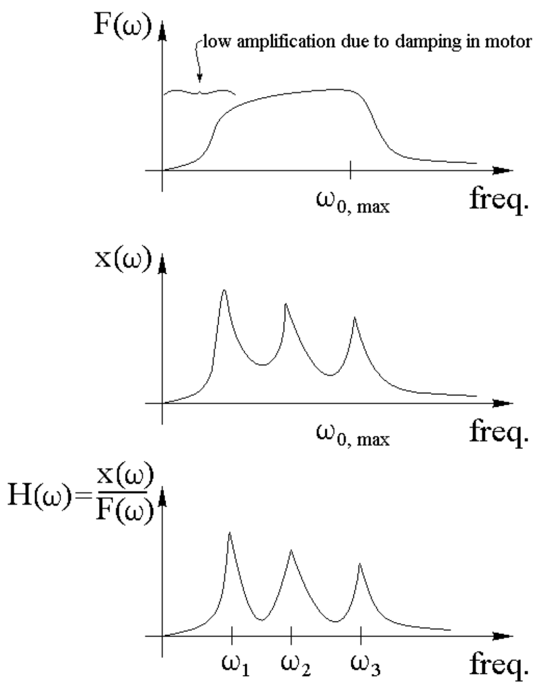
  - bring motor to highest speed (ω_{0max})
  - shut down motor excitation
  - motor slows down from ω_{0max} to 0
  - motion produced is:



- the structural response will be:



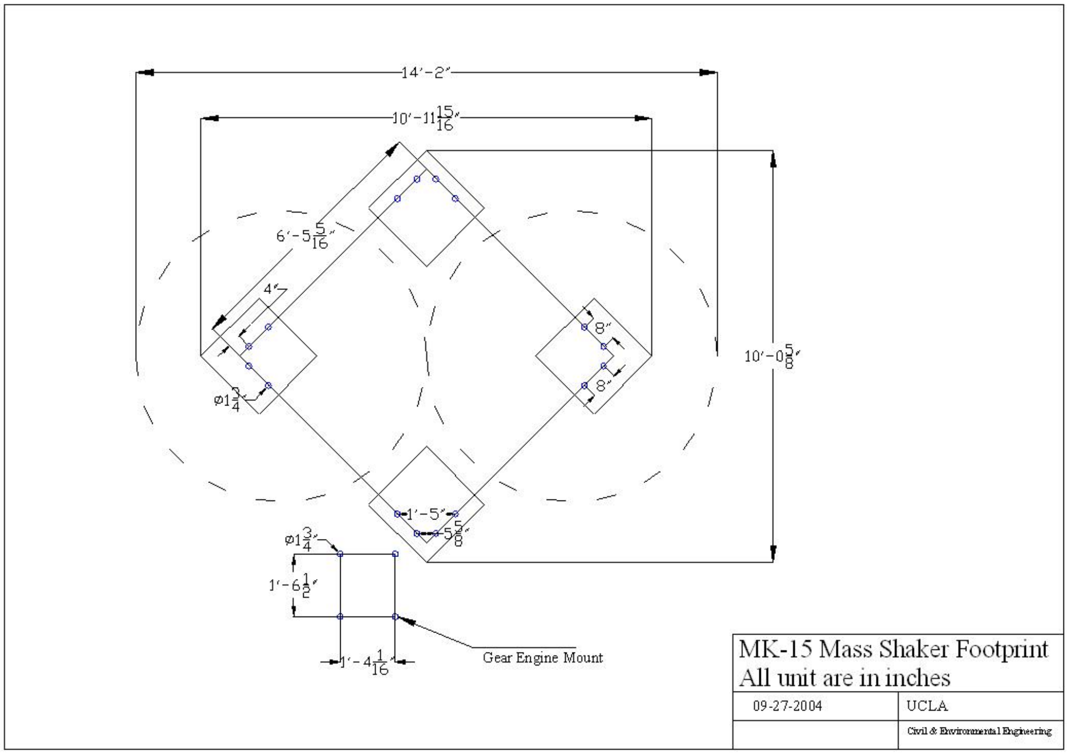




- The structural response is amplified at resonant frequencies between ω_{0max} and 0.
- The resolution depends on the damping of the motor. For a low damping motor the resolution is better. The equipment needs to be good, well lubricated, and perfectly balanced and aligned.
- The measurements require time domain recording.
- It is suitable for large structures.

Shaker of NEES@UCLA

Rotational Shaker



Linear Shaker:

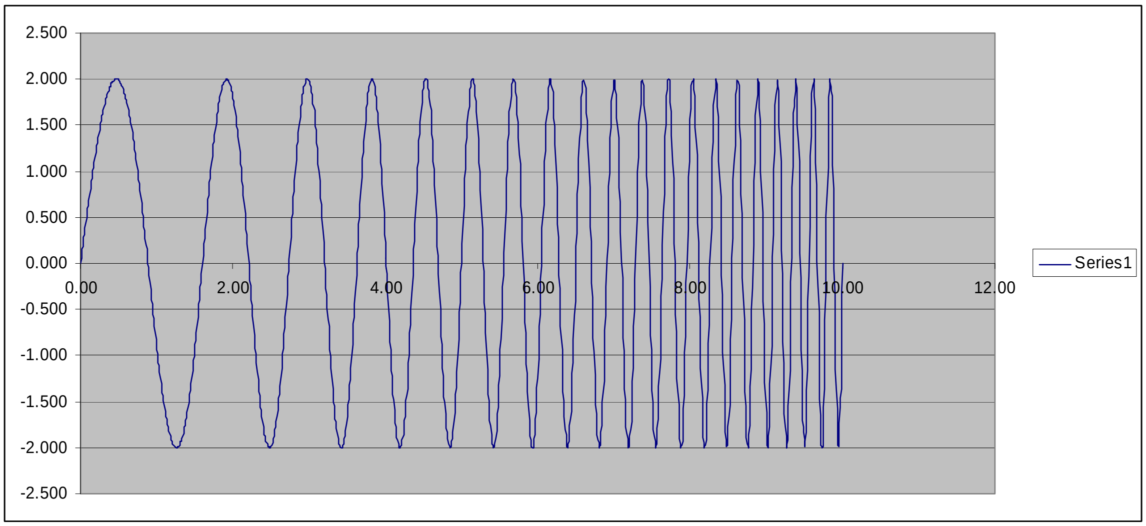
Sine Swep (may use linear exciters otr shake tables):

(a) First Formulation:

Based on octave: here double the frequency every t_o seconds

$$f(t_i) = f_o \cdot 2^{t_i/t_o}$$
$$x(t_i) = x_o \sin[2\pi t_i \cdot f(t_i)]$$

For t_o = 5 sec, f_o=0.5 Hz, x_o=2 cm



Second formulation:

$$f(t_i) = f_o + (f_n - f_o) * t_i / T_D$$
$$t_i = \Delta t * (i-1)$$
$$x(t) = x_o \sin[2\pi t_i \cdot f(t_i)]$$

t(i)=dt * (i-1) , i=0 to Duration / dt
Frq(i) = IniFrq + [(FinFrq-IniFrq)*(t(i) /Duration)]
Ampl(i) = MaxAmpl *sine [2*PI*Frq(i) * t(i))]

```
function u = isinesweep (dt,tend,delta,tdly,frq0,frq1,ampl)
        // u = isinesweep (dt,tend,delta,tdly,frq0,frq1,ampl)
        // Outputs:
        //      u =  sine sweep input signal
        //
        // Inputs:
        //      dt  = time step
```

```
//      tend = duration of signal in seconds
//      delta = duration of sine sweep in seconds
//      tdly = the start time of the sine sweep
//      frq0 = the initial frequency (Hz)
//      frq1 = the final frequency (Hz)
//      ampl = amplitude of sine sweep
//

   twopi = 2*4*atan(1);
   frq0 = frq0 * twopi;
   frq1 = frq1 * twopi;
   nsw  = ceil(delta/dt);
   nst  = ceil(tdly/dt);
   dfrq = (frq1 - frq0)/delta;
   tsw = dt*(0:nsw-1)';
   frq = dfrq*tsw + frq0;
   frs =  ampl*sin(frq*tsw);

   npts = ceil(tend/dt) - nsw - nst;
   if (npts < 0)
      nsw=nsw +npts
   end
   u = [zeros(nst,1);frs;zeros(npts,1)];

end
```

Examples of Identification of properties using free and forced vibrations:
See reference: Bracci et al. (1992) NCEER-Report-92-0027 – Section 4.0

Special Topics
Identification of Damping
- Half Power Bandwith
- Logarithmic Decrement
- Peak Transfer Function

Identification of Dynamic Characteristics
Identification of Structural Properties

Types of tests:
Impact Hammer Test
Snap Back Test
Pull Back Test
Shake Table – White noise Test.

Table 4-2 Dynamic Characteristics of the Unloaded Model

Test	f_i^u (Hz.)	Φ_{ij}^u	K_{ij}^u (kip/in)	k_i^u (kip/in)	ξ_i^u (%)	C_{ij}^u
HAMMER	$\begin{pmatrix}3.40\\11.00\\17.60\end{pmatrix}$	$\begin{pmatrix}1.00 & -0.67 & -0.62\\0.82 & 0.18 & 1.00\\0.47 & 1.00 & -0.63\end{pmatrix}$	$\begin{pmatrix}70.1 & -72.1 & 10.3\\-72.1 & 115.5 & -59.2\\10.3 & -59.2 & 97.3\end{pmatrix}$	$\begin{pmatrix}72.1\\59.2\\38.1\end{pmatrix}$	$\begin{pmatrix}2.7\\1.5\\1.0\end{pmatrix}$	$\begin{pmatrix}0.028 & -0.007 & -0.003\\-0.007 & 0.028 & -0.005\\-0.003 & -0.005 & 0.033\end{pmatrix}$
STAAD (0.565 EI_g)	$\begin{pmatrix}3.70\\10.81\\16.50\end{pmatrix}$	$\begin{pmatrix}1.00 & -0.81 & -0.43\\0.78 & 0.53 & 1.00\\0.40 & 1.00 & -0.88\end{pmatrix}$	$\begin{pmatrix}46.7 & -50.9 & 4.4\\-50.9 & 102.7 & -56.2\\4.4 & -56.2 & 108.2\end{pmatrix}$	$\begin{pmatrix}50.9\\56.2\\52.0\end{pmatrix}$	-	-

Table 4-3 Dynamic Characteristics of the Loaded Model

Test	f_i (Hz.)	Φ_{ij}	K_{ij} (kip/in)	k_i (kip/in)	ξ_i (%)	C_{ij}
PULL	$\begin{pmatrix}1.76\\5.34\\8.15\end{pmatrix}$	$\begin{pmatrix}1.00 & -0.82 & -0.41\\0.76 & 0.55 & 1.00\\0.40 & 1.00 & -0.88\end{pmatrix}$	$\begin{pmatrix}47.4 & -52.7 & 3.2\\-52.7 & 109.3 & -59.8\\3.2 & -59.8 & 113.9\end{pmatrix}$	$\begin{pmatrix}52.7\\59.8\\54.1\end{pmatrix}$	-	-
SNAP	$\begin{pmatrix}1.86\\5.66\\8.40\end{pmatrix}$	-	-	-	$\begin{pmatrix}2.5\\4.8\\4.0\end{pmatrix}$	-
WHN_B (Eq.(4.20))	$\begin{pmatrix}1.78\\5.32\\7.89\end{pmatrix}$	$\begin{pmatrix}1.00 & -0.82 & -0.46\\0.80 & 0.46 & 1.00\\0.42 & 1.00 & -0.83\end{pmatrix}$	$\begin{pmatrix}51.9 & -53.4 & 2.5\\-53.4 & 102.4 & -54.4\\2.5 & -54.4 & 104.7\end{pmatrix}$	$\begin{pmatrix}53.4\\54.4\\50.3\end{pmatrix}$	$\begin{pmatrix}2.0\\2.4\\2.0\end{pmatrix}$	$\begin{pmatrix}0.072 & -0.042 & -0.012\\-0.042 & 0.097 & -0.029\\-0.012 & -0.029 & 0.112\end{pmatrix}$
WHN_B (Eq.(4.19))	-	-	-	-	$\begin{pmatrix}1.7\\1.6\\1.4\end{pmatrix}$	-
STAAD (0.565 EI_g)	$\begin{pmatrix}1.78\\5.20\\7.94\end{pmatrix}$	$\begin{pmatrix}1.00 & -0.81 & -0.43\\0.78 & 0.53 & 1.00\\0.40 & 1.00 & -0.88\end{pmatrix}$	$\begin{pmatrix}46.7 & -50.9 & 4.4\\-50.9 & 102.8 & -56.3\\4.4 & -56.3 & 108.2\end{pmatrix}$	$\begin{pmatrix}50.9\\56.3\\51.9\end{pmatrix}$	-	-

Suggested Test Protocols

Static

Dynamic

See List:

Suggested lab exercise:

Part 1: Impact Hammer

a. Use a small impact hammer with a larger model (from lab #1).
b. Use a soft tip and determine transfer function (identify frequencies and mode shapes).
c. Use a hard tip and repeat b.
d. Record motion at first floor
e. Identify the frequencies of structure (as many as you can).

Part 2: Perform a sine sweep test using 10 seconds per octave form 0.5 Hz up to 16 Hz in acceleration (.05g) control using the shake table:

b. Record response of structure at top and at first floor
c. Identify frequencies from time domain trace
d. Calculate the FFT of the input an output
e. Identify as many frequencies as you can

Part 3: Perform a white noise test using 0.05 rms for 30 seconds in displacement control

f. Record response at first and top floor
g. Calculate transfer functions
h. Determine as many frequencies as you can.