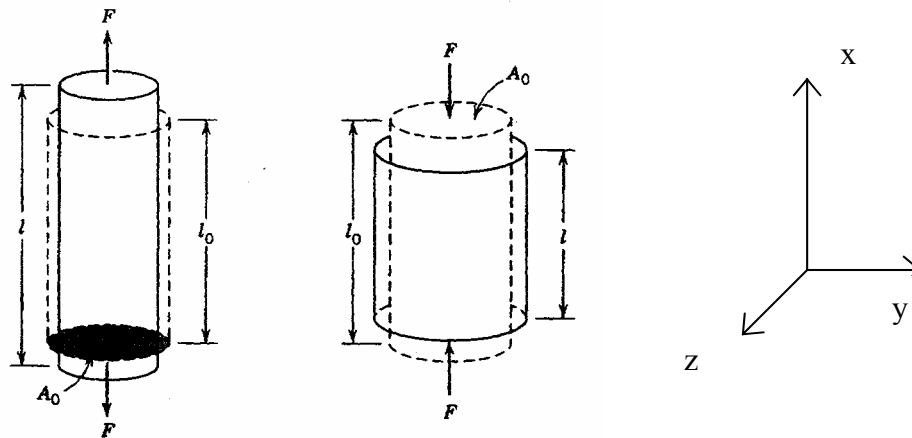


## Review from Mechanical Properties of Materials

### Concept of Stress and Strain

Figure 1 shows the behavior of a cylindrical body subjected to tension and compression.

- Tension specimen extends along its axis and contracts on the orthogonal (circumferential) axes
- Compression specimen compresses along its axis and expands on the orthogonal (circumferential) axes



**Figure 1:** Specimen under tension and compression (Callister(2000))

*Axial stress* (units of psi or MPa) is the applied axial load  $P$  divided by the original cross sectional area,  $A_0$ :

$$\sigma_x = \frac{P}{A_0} \quad (1)$$

*Axial strain* (dimensionless, in/in, m/m) relates the change in length of the specimen to the original length. It is calculated as follows:

$$\epsilon_x = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} \quad (2)$$

where  $l_0$  is the original length  
 $\epsilon_x$  is the axial strain

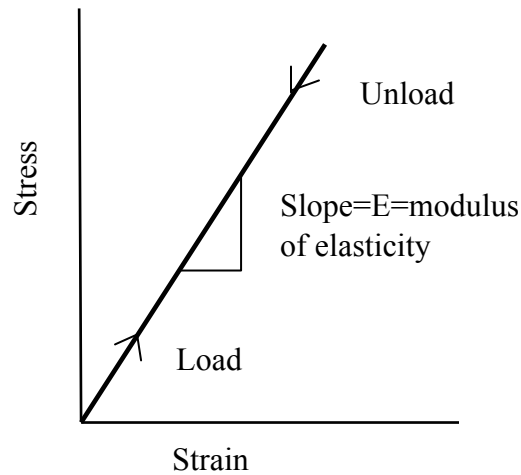
During a tension test the sample will elongate, therefore  $l > l_0$  and the strain will be positive.

## Mechanical Properties of Material

Certain mechanical properties of the material can be determined from a plot of stress against strain.

### *Modulus of Elasticity*

In the linear elastic range of response, the stress is proportional to the strain as shown in Figure 2 below.



**Figure 2:** Initial portion of stress-strain curve

The relationship is written as:

$$E = \frac{\sigma}{\epsilon} \quad \text{Hooke's Law}$$

where E is known as the *modulus of elasticity* or *Young's modulus*.

Typical values of E for steel, aluminum and titanium are given in Table 1.

**Table 1:** Typical values of modulus of elasticity

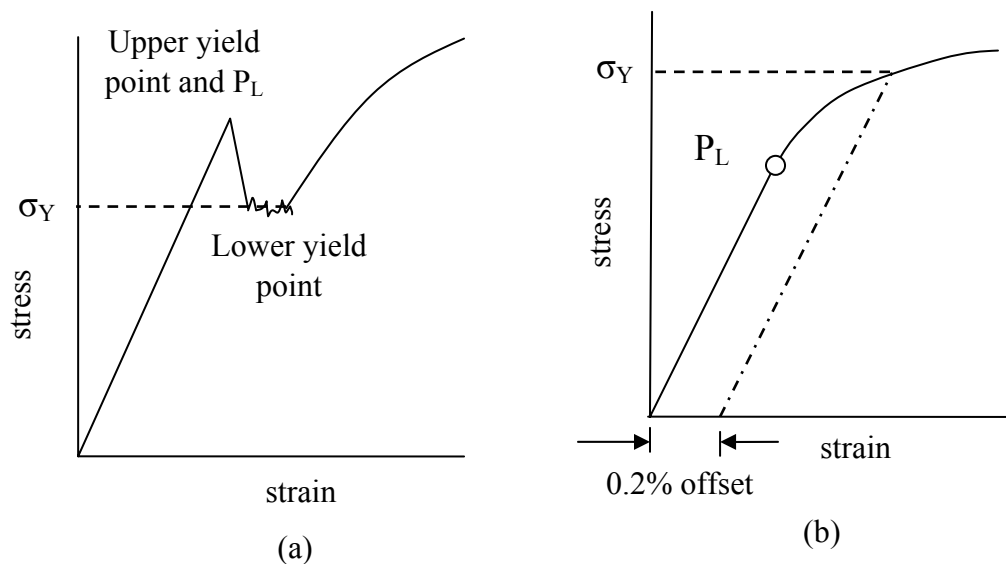
Metal	Modulus of elasticity, E	
	GPa	ksi
Steel	207	30000
Aluminum	69	10000
Titanium	115	16500

The largest value of the stress for which Hooke's law can be used for a given material is known as the proportional limit,  $P_L$ , of that material. It marks the change from linear elastic behavior to non-linear elastic behavior (see Figure 3).

### *Yielding and yield strength*

The yield point marks the transition from essentially elastic behavior (strain is recoverable upon unloading) to plastic behavior (some permanent deformation upon unloading). Some metals (including mild steel) exhibit the stress-strain relationship shown in Figure 3, part (a) where the yield point is clearly defined. In this figure, the yield point and the proportional limit coincide. The elastic to plastic transition is well defined by an *upper yield* point. However, for such materials the yield strength is taken as the average strength associated with the *lower yield* point.

For metals exhibiting a stress-strain relationship similar to that shown in part (b), with no well defined yield point, the 0.2% offset strain is used to determine the yield strength. The yield strength is typically denoted as  $\sigma_y$ .



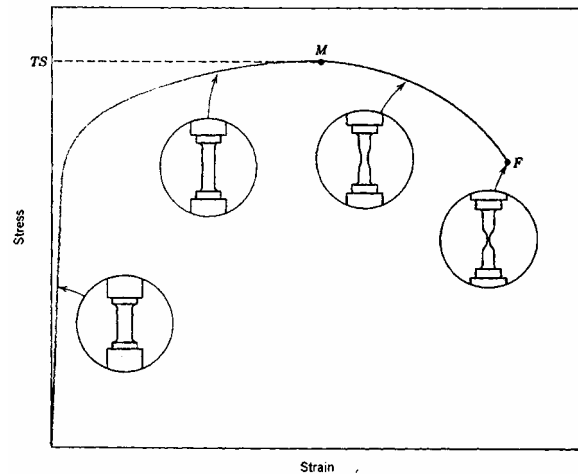
**Figure 3: Yield Point**

### ***Plastic Behavior***

As a metal deforms past its yield point, plastic deformations will result. Plastic deformation is associated with slip in crystalline materials such as metals (you will talk about this more in CIE 327). The process is not reversible.

### ***Tensile strength***

After yielding, the stress necessary to increase deformation increases to a maximum, shown as point *M* in Figure 4. Also shown in this figure are the states of deformation of a test specimen at various points along the stress-strain curve. The maximum stress is termed the tensile strength, typically denoted as  $\sigma_u$ .



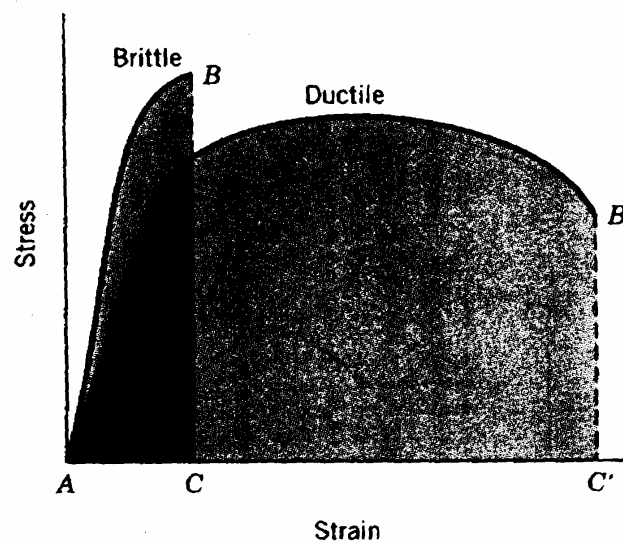
**Figure 4:** Stress-Strain curve to failure (Callister (2000))

### *Breaking Strength*

Point F in Figure 4 corresponds to the breaking (or fracture) strength.

### *Ductility*

Ductility is a measure of the plastic deformation capacity of a metal. Consider Figure 5 below:



**Figure 5:** Brittle and ductile behavior (from Callister)

A brittle (opposite of ductile) material is one with limited (or no) plastic deformation capacity. One metric for brittleness is a fracture strain of 5% or less.

A measure of ductility can be obtained by calculating either % elongation or % reduction in area. The percent elongation is the percentage of plastic strain at fracture and is given by

$$\%EL = \frac{l_f - l_0}{l_0} 100 \quad (3)$$

where  $l_f$  is the fracture length. When reporting %EL, the original gage length (often 2 in or 50 mm) must also be reported. For commonly used steels, the specified minimum elongation is 21%.

% reduction in area is calculated from the following equation:

$$\%AR = \frac{A_0 - A_f}{A_0} 100 \quad (4)$$

High values of %AR indicate necking of the specimen and ductile behavior. Typical values for structural steel are 60 to 70%.

Table 2 below gives values of yield strength, tensile strength and ductility for mild steel, aluminum and titanium.

**Table 2:** Typical values of yield strength, tensile strength and ductility

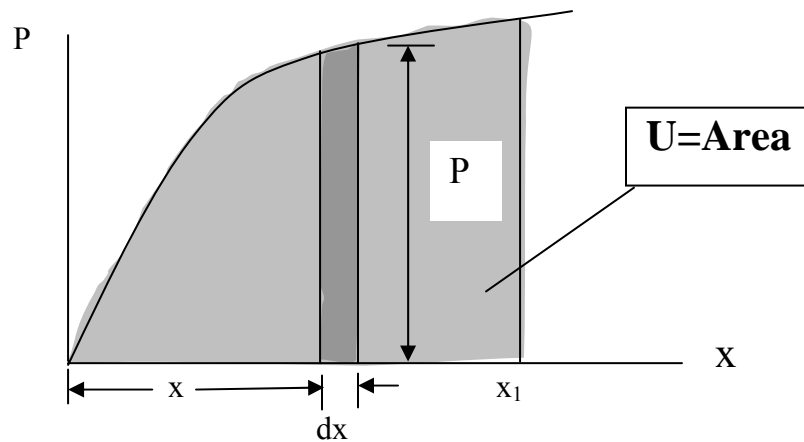
Metal	Yield strength		Tensile strength		Ductility %EL (in 50 mm)
	MPa	ksi	MPa	ksi	
Structural Steel	248	36	400	58	21%
Aluminum (6061)	240	35	260	38	17%
Titanium	830	120	900	130	10%

### *Strain Energy*

The work done by the load P as it is slowly applied to the specimen results in an increase in stored energy associated with deformation of the rod. This energy is referred to as the strain energy of the rod.

$$\text{Strain Energy} = U = \int_0^{x_1} P dx \quad (4)$$

which is equal to the area under the load deformation diagram between  $x=0$  and  $x=x_1$  as shown in Figure 6



**Figure 6:** Strain Energy

For a linear elastic material,

$$U = \frac{1}{2} P_1 x_1 \quad (5)$$

where  $P_1$  is the value of the load corresponding to the deformation  $x_1$ .

The units of energy are Joule (N.m) in SI units or in.lb in U.S. Customary units.

The strain energy is dependent on the length and cross-sectional area of the test specimen. Strain energy density or strain energy per unit volume is independent of the dimensions of the specimen. *Strain energy density* is actually the area under the stress-strain curve.

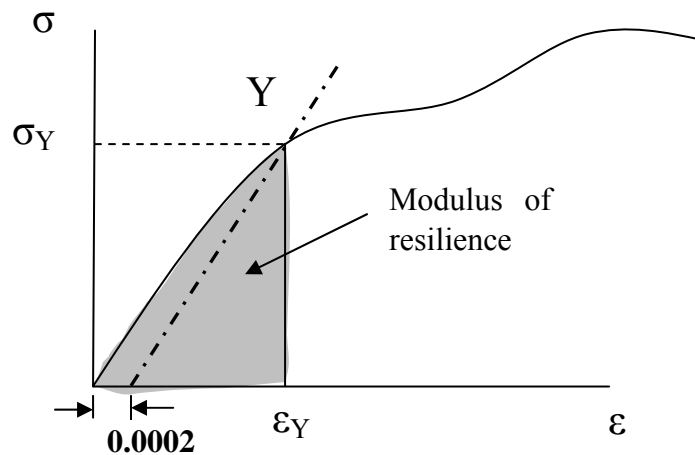
### *Resilience*

The strain energy density at *yield* is known as the resilience modulus. Resilience is the capacity of a material to absorb energy elastically upon loading. Mathematically, this can be expressed as

$$U_r = \int_0^{\varepsilon_y} \sigma d\varepsilon \quad (6)$$

The *modulus of resilience* represents the energy per unit volume that the material can absorb without yielding. Figure 7 shows that for a linear elastic material

$$U_r = \int_0^{\varepsilon_y} \sigma d\varepsilon = 0.5 \sigma_y \varepsilon_y = \frac{\sigma_y^2}{2E} \quad (7)$$



**Figure 7:** modulus of resilience

Units of resilience are joules per cubic meter or inch-pounds force per cubic inch. Resilient materials have a high yield strength and a low modulus of elasticity, properties useful for components such as springs. The capacity of a structure to withstand an impact load without being permanently deformed also depends on its resilience,

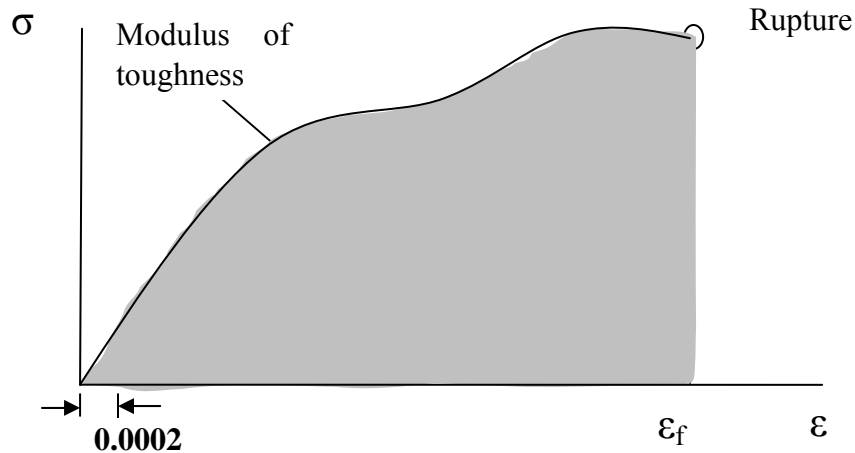
### *Toughness*

Qualitatively, the toughness is the strain energy density (area under the stress-strain curve) up to the point of *fracture* (see Figure 8 ) and is given by:

$$\int_0^{\epsilon_f} \sigma d\epsilon \quad (8)$$

The units for toughness are the same as those for resilience (energy per unit volume). Similar to the calculation of resilience, the toughness of a material can be determined from numerical integration of the stress-strain data.

For a material to be considered tough, it must possess significant ductility and strength.  
Why?

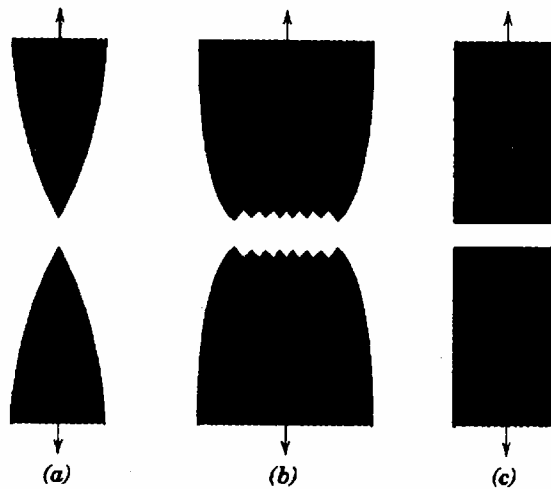


**Figure 8:** Toughness

### *Fracture*

Fracture is the separation of a body into two or more parts. Two modes of fracture are possible, ductile or brittle. The classification is based on whether the material can sustain substantial plastic deformation.

Consider the three fractured specimens shown in Figure 9.



**Figure 9:** Types of fracture (from Callister)

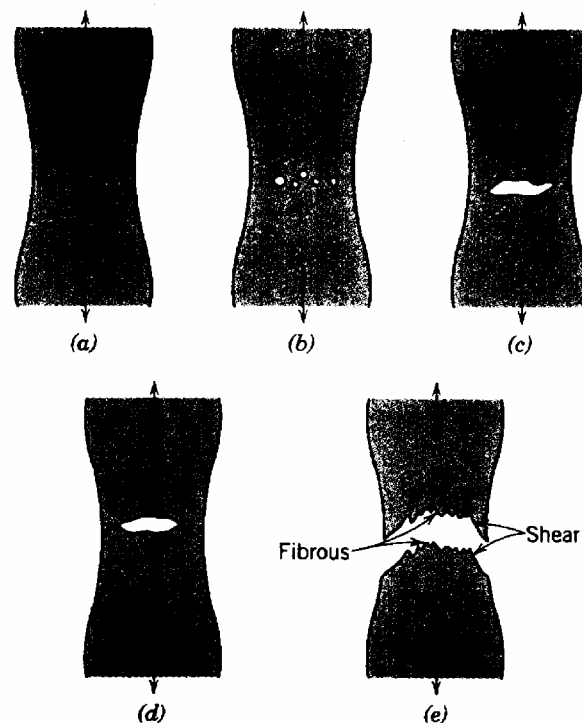
Part (a) of the figure shows a highly ductile fracture in which the specimen necks down to a point.

Part (b) of the figure shows a moderately ductile fracture after some necking (expected from steel specimens)

Part (c) of the figure shows brittle fracture without any plastic deformation.



The Figure 10 (Callister(2000) shows the progression of fracture for a material which shows moderately ductile failure. Five stages in the cup-and cone fracture are shown.



**Figure 10:** Fracture of moderately ductile material

- (a) Initial necking of the specimen
- (b) Small cavities or micro voids form in the interior of the cross section
- (c) Micro voids enlarge, come together, and coalesce to form an elliptical crack, which has its long axis perpendicular to the stress direction.
- (d) Crack continues to grow
- (e) Fracture is preceded by the rapid propagation of a crack around the outer perimeter of the neck and by shear deformation at an angle of 45 degrees to the tension axis (the angle at which the shear stresses are maximum).

In contrast to the cup and cone fracture of a ductile material, a brittle material tends to fail along a horizontal plane where the normal stresses are a maximum.

## References

Callister, W.D. **Materials Science and Engineering**, 5th ed., John Wiley and Sons, New York, 2000