Instrumentation (2)

Motion Measurement (Time)

1. Acceleration, velocity, and displacement instruments
2. Referenced transducers
3. Non-referenced transducers
4. Noise and noise elimination
5. Initial data processing

Motion Characteristics

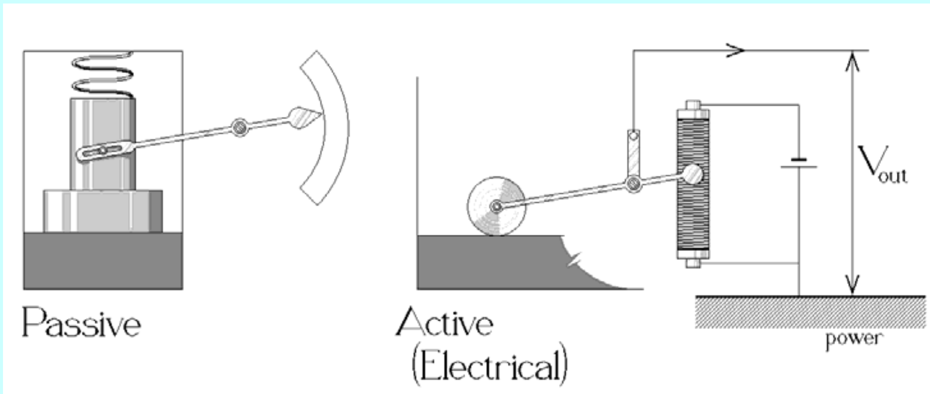
Displacement $\mu(t) \rightarrow \mu(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mu(t)e^{i\omega t}dt$

Velocity $\dot{\mu}(t) \rightarrow \dot{\mu}(\omega)=+i\omega\mu(\omega)$

Acceleration $\ddot{\mu}(t) \rightarrow \ddot{\mu}(\omega)=-\omega^2\mu(\omega)$

Instrument types

Active vs. Passive

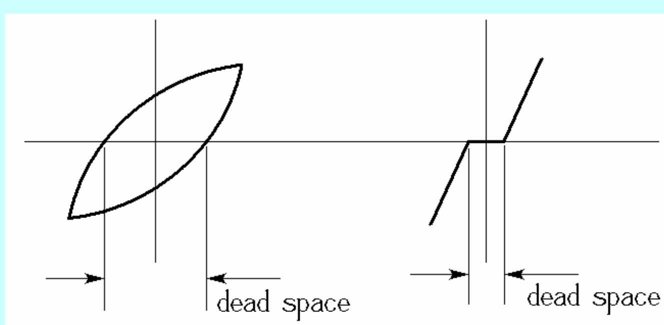


Analogue/ Digital Instruments

- Analogue output is in the form of a continuous current or voltage
- Digital output is in multiples of quantified voltage

Static Characteristics of Instruments

- Accuracy (±error, this is random)
- Tolerance (maximum acceptable error)
- Range/span (maximum-minimum values)
- Bias (constant error over full range)
- Linearity (output ~ measured quantity over all range)
- Sensitivity (≡ calibration factor)
- Sensitivity to environment (zero or sensitivity drift)
- Hysteresis
- Dead space
- Resolution (lower limit of measured quantity)



Dynamic Characteristics of Instruments

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + ...+ a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i$$

q_i = measured quantity

q_o = output reading

a_i = calibration constants

Zero Order Instrument

$$a_0 q_o = b_0 q_i \qquad q_o = \frac{b_0}{a_0} q_i = K q_i$$

K = calibration factor or instrument sensitivity

First Order Instrument

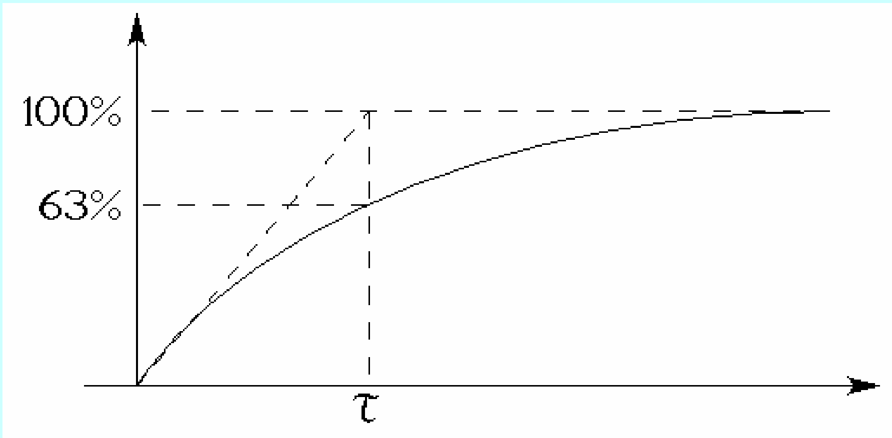
$$a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i$$

$$a_1 D q_o + a_0 q_o = b_0 q_i$$

$$q_o = \frac{\frac{b_0}{a_0} q_i}{1 + \left(\frac{a_1}{a_0}\right) D} = \frac{K q_i}{1 + \tau D} = K(t) q_i$$

$\tau = a_0/a_1 \equiv$ time constant at 63% final reading

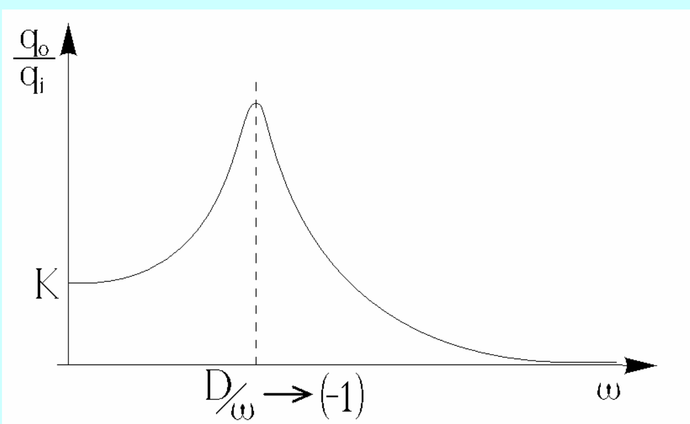
Example I: Thermocouple



$$T=Ce^{-t/15}$$

Example II: Piezoelectric Displacement Transducer

Second Order Instruments (non-reference motion transducer)



$$a_2\frac{d^2q_o}{dt^2}+a_1\frac{dq_o}{dt}+a_0q_o=b_0q_i$$

$$K=b_0a_0\ ;$$

$$\omega=\frac{a_0}{a_2}\ ;$$

$$\varepsilon=\frac{a_1}{2a_0a_2}$$

$$\frac{q_o}{q_i}=\frac{K}{1+\frac{D^2}{\omega^2}+2\varepsilon\frac{D}{\omega}}$$

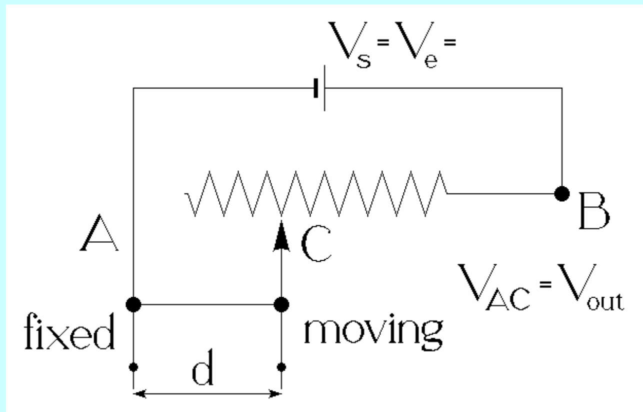
Displacement Transducers – Referenced

Resistive Potentiometer

$$\frac{V_{out}}{V_e}=\frac{AC}{AB}=\frac{R_{AC}}{R_{AB}}=\frac{R_i}{R_t}$$

$$V_{out}=\frac{AC}{AB}\cdot V_e$$

$$V_{out}=V_e\cdot\frac{1}{AB}\cdot d$$



Note that the voltage reading V_{out} is dependant on the resistance of the voltmeter (R_m).

Assume V_M – measured by voltmeter

V_{out} – result of displacement

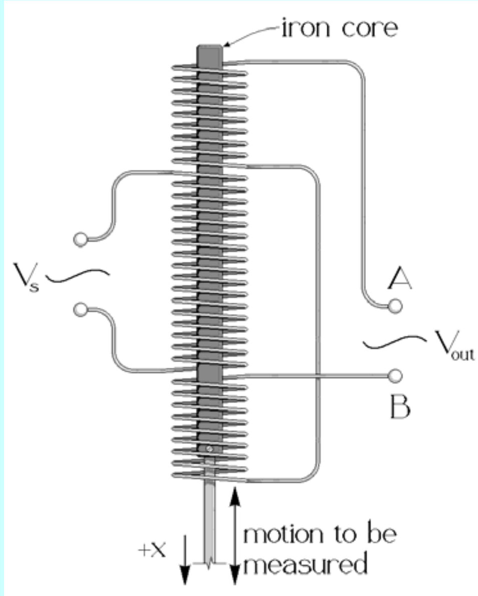
$$V_0\text{-}V_M=(\varepsilon V)=\frac{R_i^2\left(R_t\text{-}R_i\right)}{R_t\left(R_iR_t+R_mR_t\text{-}\right)R_i^2}\;;\qquad R_t=R_{AB}\,,\quad R_i=R_{AC}$$

$$V_0\text{-}V_M=(\varepsilon V)=\frac{R_i^2\left(R_t\text{-}R_i\right)}{R_t\left(R_iR_t+R_mR_t\text{-}\right)R_i^2}$$

to reduce (εV) to zero or very small requires (R_m/ R_t) → large

(See potentiometers pages.)

Linear Variable Differential Transducer



$$V_s = V_s \sin\omega t$$

$$V_A = K_a \sin(\omega t - \phi)$$

$$V_B = K_b \sin(\omega t - \phi)$$

K_a and K_b are proportional to the number of windings around the iron core.

$$V_{out} = V_A - V_B = (K_a - K_b)\sin(\omega t - \varphi)$$

$K_a = V_s\left(x_{start} - \Delta x_{core}\right)$ $K_b = V_s\left(x_{start} + \Delta x_{core}\right)$ $V_{out} \approx V_s \cdot \text{position}$

Note that this system requires an alternating current/voltage supply, as well as noise isolation.

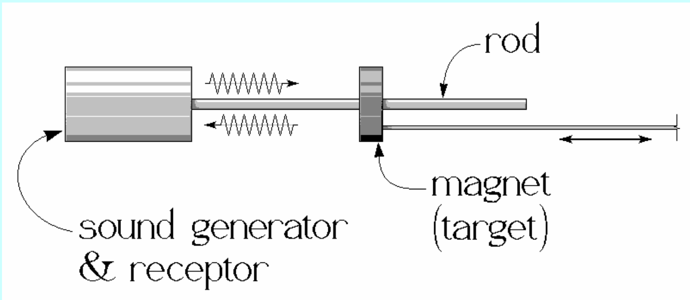
It works on the principle of mutual inductance; an AC voltage across the terminals of the primary coil induces a voltage of the same frequency in each of the two secondary coils. When centered the secondary voltages are equal, but a positive displacement of the core leads to one voltage being greater than that of the secondary coil.

i.e.: The change in voltage is proportional to the change in displacement.

(See LVDT cutaway page)

http://nees.buffalo.edu/docs/labmanual/SEESLLabManual.pdf

Ultrasonic Transducers



Pulses of acoustic energy are transmitted at high frequency, the reflections off a target object are detected and the time of flight is used to determine the distance.

The target must not touch the rod such that a sufficient acoustic reflection is produced, hence the target being magnetic so that it can slide on the rod without touching it.

Ultrasonic transducers can work over a wide frequency range, from below 8kHz to 200kHz.

http://nees.buffalo.edu/docs/labmanual/SEESLLabManual.pdf

Other range sensors include:

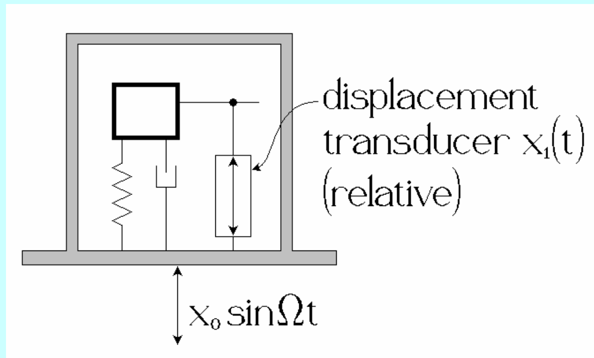
- Piezoelectric Transducers
- Capacitive Transducers
- Laser Sensors

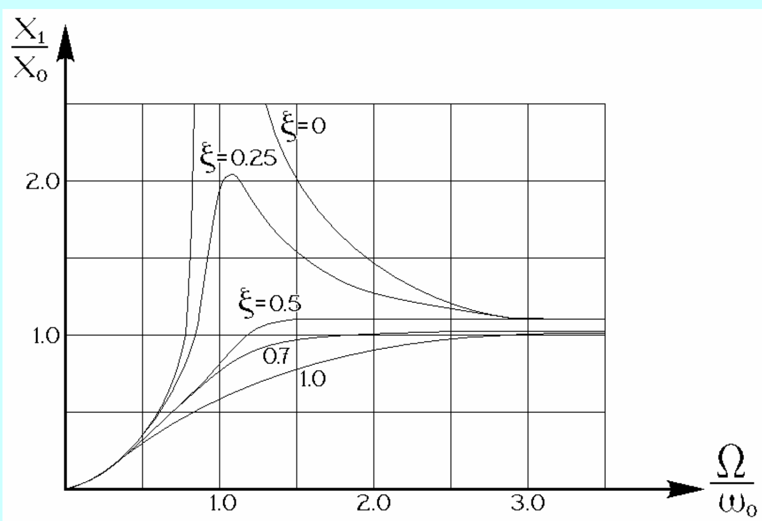
Non-reference Sensors:

Accelerometers: Seismometers:

$$x_1 = x_0 \frac{\left(\frac{\Omega_1}{\omega_0}\right)^2}{\left\{\left[1-\left(\frac{\Omega_1}{\omega_0}\right)^2\right]^2 + \left[2\xi\frac{\Omega_1}{\omega_0}\right]^2\right\}^{\frac{1}{2}}}$$

$$\varphi = \tan^{-1}\frac{2\xi\frac{\Omega_1}{\omega_0}}{1-\left(\frac{\Omega_1}{\omega_0}\right)^2}$$

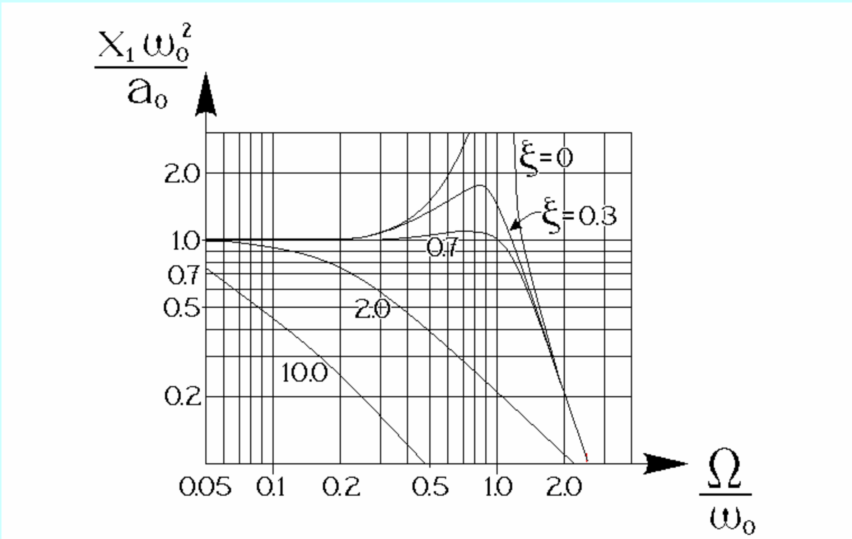




Displacement Response of Instrument

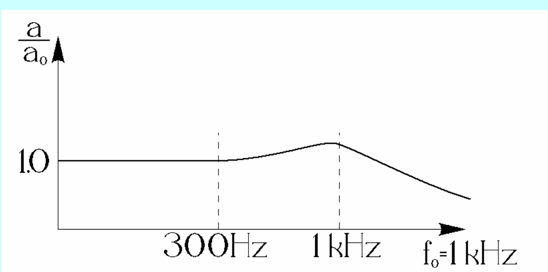
For $\xi \approx 0.7$ $\Omega/\omega_0 \geq 2$

Accelerometers-Seismic and else



Acceleration Response of Seismic Instrument
$\xi = 0.7$ $\Omega/\omega_0 < 0.5$

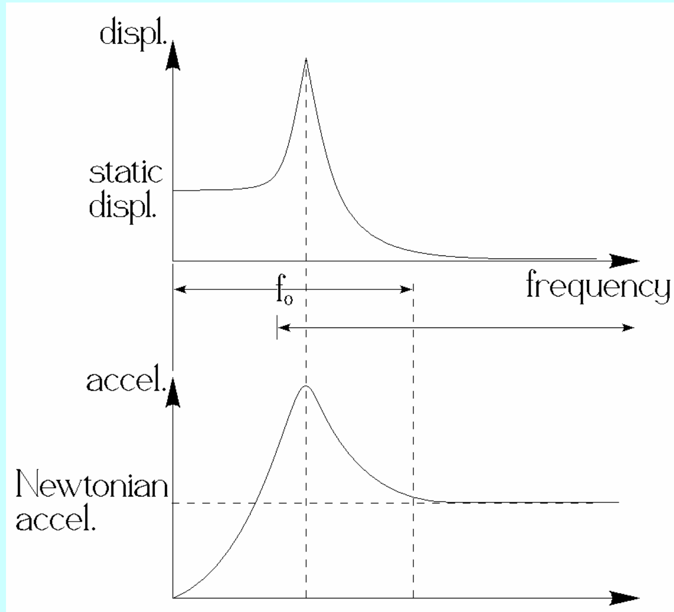
In Linear Scale



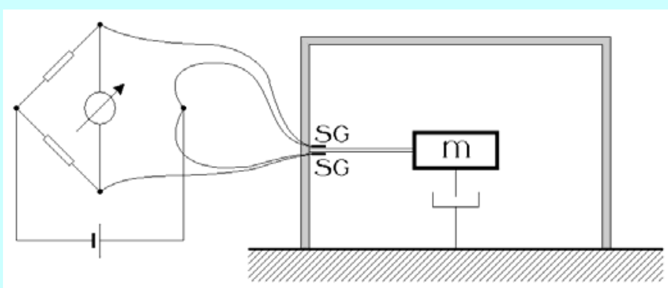
Ranges of Operation

Displacement Measurements

Acceleration Measurements

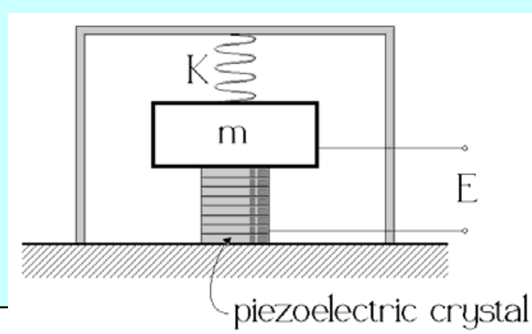


Resistive Accelerometers



Calibration:
Upside down ± 2g

Piezoelectric Accelerometers



Calibration:
Need reference and vibrator

Piezoresistive Accelerometers
(See manufacturer's information-PCB, Kiesler Instruments)

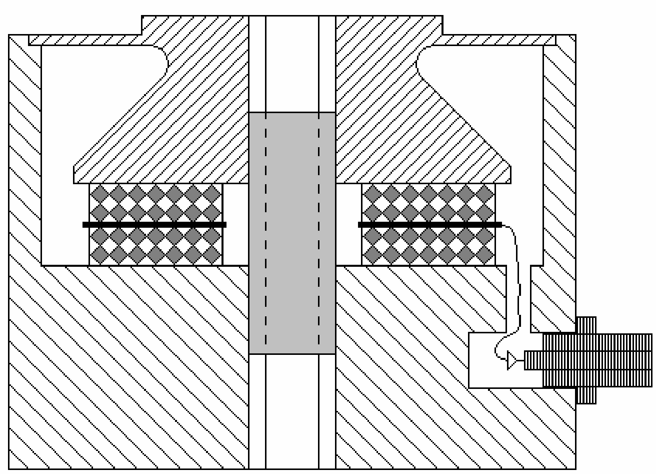
They have the advantage that they can be conditioned by a strain gauge conditioner.

(See accelerometers pages)

http://nees.buffalo.edu/docs/labmanual/SEESLLabManual.pdf

Piezoelectric Force Transducer (dynamic instruments)

Definition: Self-generating piezoelectric force transducers are stiff, elastic structures that convert deflection caused by a force into electrical signals more convenient for recording.



The quartz sensing elements have a low thermal response and a low frequency response.

Piezoelectric force transducers are modeled as a classical spring-mass seismic system.

Coordinate Tracking System – Image technology

http://nees.buffalo.edu/docs/labmanual/SEESLLabManual.pdf

Field-of-view: 17 m³, distributed into three accuracy zones as follows:

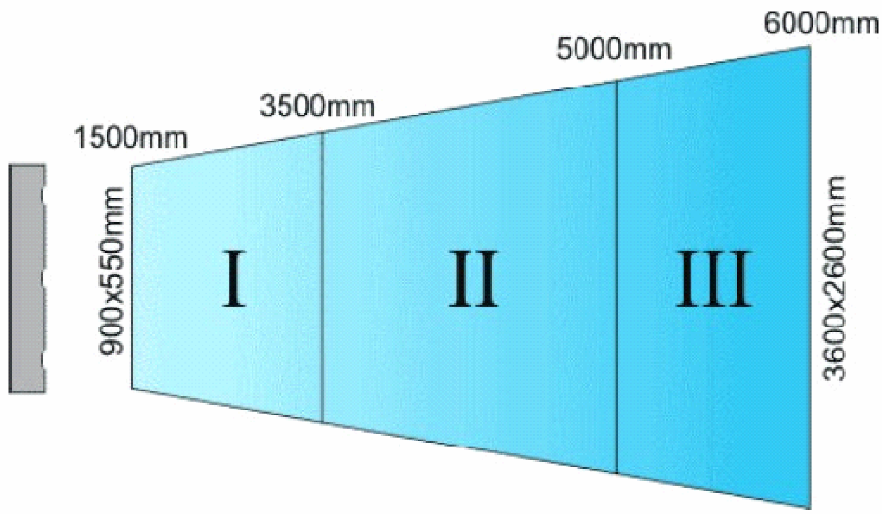


Figure 10: camera field-of-view

SEESL / NEES Coordinate tracking system: <u>Krypton K600</u>
Capabilities (abbreviated):

Measurement system / probes capabilities:

1 LED	3 degrees of freedom
3 (or more) LED	6 degrees of freedom

Sampling rate:

Rate = <u>3000 /</u> # of LED (in samples per second)
i.e. for 20 active LED's the Rate = 150 samples per second
for 50 active LED's the Rate = 60 samples per second

Field of view for K600:

Minimum distance (D) from camera 1.5 m; Maximum distance (D) from camera xx m.

The field of view is defined as noted below (H = height of image, W- width of image, D = the distance from which the max view can be captured). H and W can be interchanged. Here are the manufacturer specified field views:

0	H=0.9m, W=0.5m	D_{min} = 1.5 m
I	H=1.7m, W=1.8m	D_{max} = 3.5 m
II	H=2.4m, W=3.3m	D_{max} = 5.0 m
III	H=2.6m, W=3.6m	D_{max} = 6.0 m

Additional performance limitations see below:

Accuracy :

Single Point : 0,060 mm
Volumetric : 0,090 mm + 0,010 mm/m

The indicated measurement uncertainty is expressed for a confidence level of 95%, according to the ISO 10360 II, VDI 2617 and ANSI / ASME B89.1.12M standards for acceptance of CMMs.

Acquisition frequency :

Important notice: The K400 camera system can ***not*** *be used for dynamic measurements.*

The measurement frequency for static measurements is set to 10Hz.

K600 CAMERA UNIT

Field-of-view: 17 m³, distributed into three accuracy zones as follows:

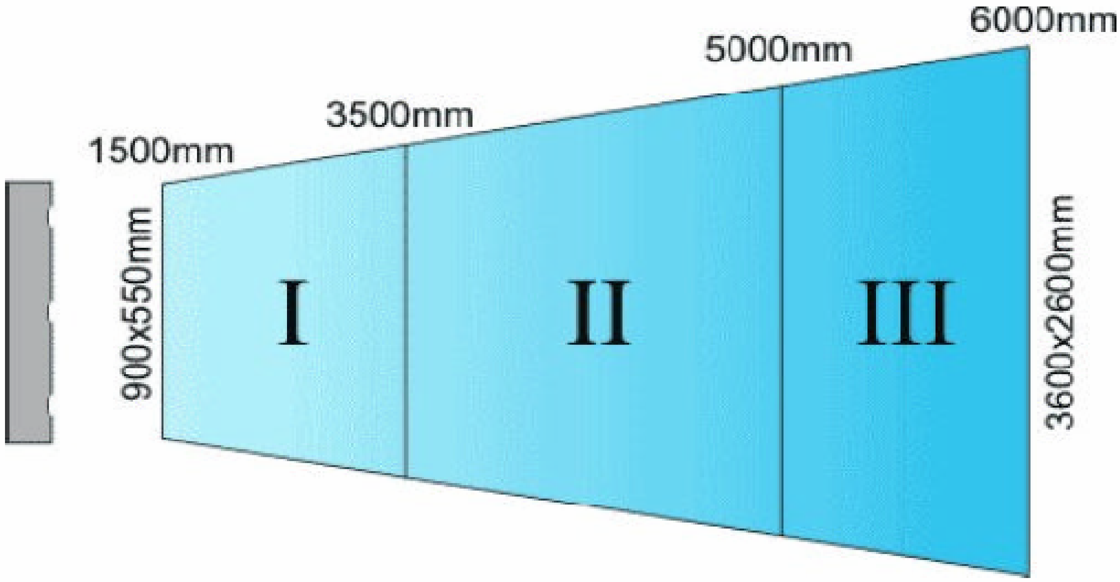


Figure 10: camera field-of-view

Resolution : 0,002 mm at 2,5 mm

Noise (1σ) : 0,010 mm

Accuracy :

Zone	Volumetric Accuracy (±2σ)	Single Point Accuracy (±2σ)
I	90μm + 10μm/m	60μm + 7μm/m
II	90μm + 25μm/m	60μm + 17μm/m
III	190μm + 25μm/m	130μm + 17μm/m

System Specifications

		K400	K600
Static Definition:			
Measurement Volume	m3	4.5	17
	cu.yd	6	22
Resolution	micron	2	2
	inches	0.000078	0.000078
Single Point Accuracy	micron	Up to 60	Up to 60
	inches	Up to -0.0024	Up to 0.0024
Volumetric Accuracy	micron	Up to 90	Up to 90
	inches	Up to 0.0035	Up to 0.0035
Temperature Range	C	15-35	15-35
	F	60-95	60-95
Dynamic Definition:			
Freq. 1 marker	(Hz)	NA	650 Hz
Freq. 3 markers	(Hz)	NA	325 Hz

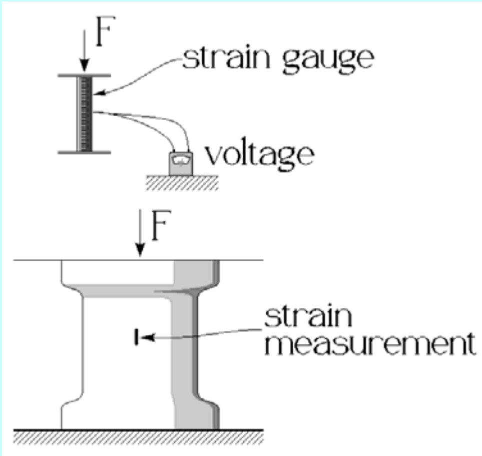
Characteristics are subject to change without notice

For more details, contact us at info@krypton.be

For a quotation, contact us at sales@krypton.be

Force Transducers (Load Cells:) See Part 1 of Instrumentation

Strain Gauge Load Cell



$F \sim \varepsilon_{gauge}$

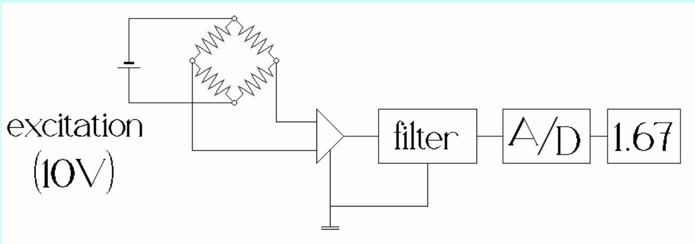
$\sim \Delta R_{gauge}$

Definition: When subjected to a force, a strain gauge network produces an electrical signal proportional to the deflected shape (ε).

Measurement types:

- Axial loads
- Bending moments
- Shear forces

Wheatstone bridge: $\varepsilon \sim \Delta R$ across bridge

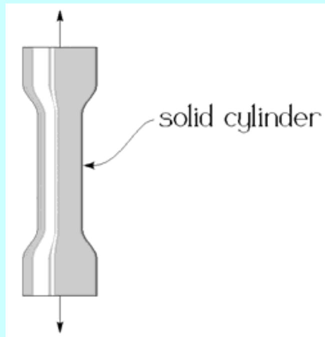


Types of Load cell Shapes

Depends on desired force measurement:

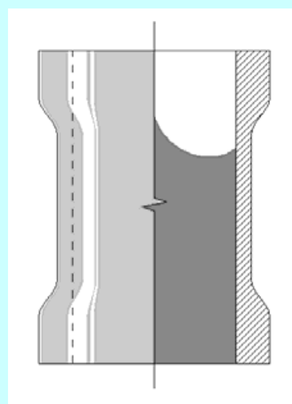
Axial Tension / Compression (uniform normal strain);

- Solid Cylinder
- Tendons
- Instrumented bar

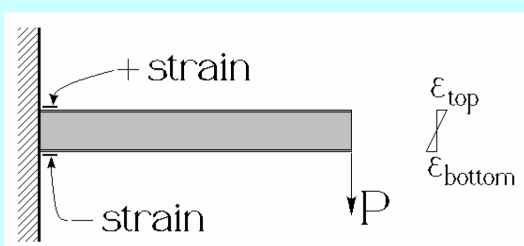


Shear Forces

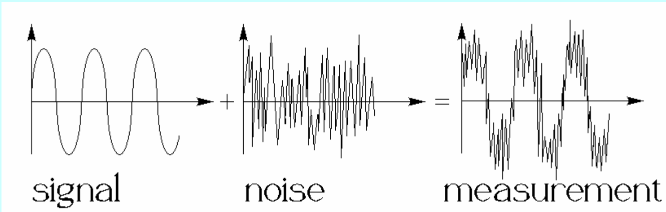
- Thin-walled cylindrical tube without stress concentrations (shear stress will be constant across the cross-section)
- A thick ring must be avoided, as shear stress will not be evenly distributed.



Moments (linear normal strain)



Measurement Noise



$$\frac{V_{noise}}{V_{measured}} \text{ small } \rightarrow 60\text{dB. or less}$$

Elimination of High Frequency Noise

- Use of low pass (or notch) filters.
- Averaging signal in high frequency (filter or digital).

(integrate (for averaging) procedures)

The best filtering and noise elimination is done digitally.

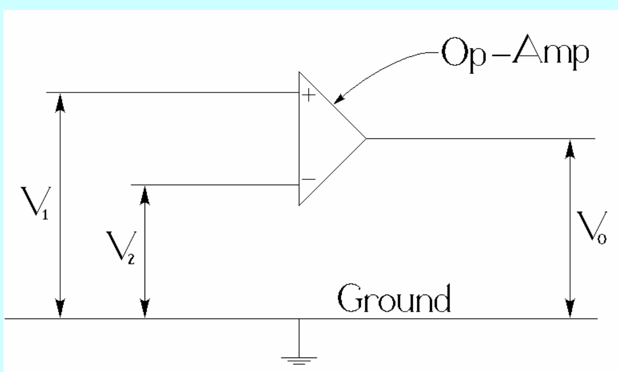
Signal Conditioning

- Provide electric source
- Amplify signal
- Provide balancing resistor
- Provide shunt calibration (a calibrated resistor which produces a simulated deviation as a given strain)
- Provide filtering capabilities

http://nees.buffalo.edu/docs/labmanual/SEESLLabManual.pdf

Basic Electrical Circuits

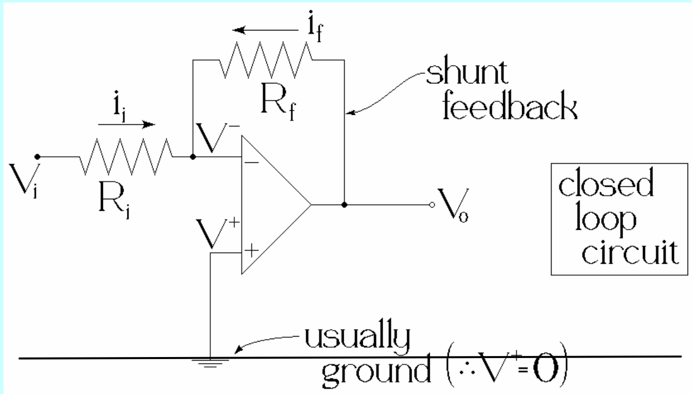
Operational Amplifier (Op-Amp)



$$V_0 = A \cdot (V_1 - V_2)$$

where A = Amplifier Gain (usually very large)
Current Input and Output is assumed negligible (0)

Inverting Operational Amplifier

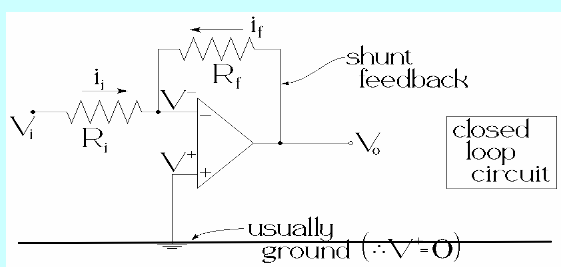


Assume input inverting voltage = V^-

Assume negligible current flowing into and out of the op-amp

$$\therefore \qquad i_i + i_f = 0$$

$$\frac{V_i - V^-}{R_i} + \frac{V_0 - V^-}{R_f} = 0$$



The Op-Amp ≡ $V_0 = AV_d = A\left(V^+ - V^-\right) = -AV^-$

where A = Operational Gain (Amplification)

Substituting for $V^- = -V_o/A$

$$\frac{V_0/A + V_i}{R_i} + \frac{V_0/A + V_0}{R_f} = 0 \qquad V_0\left[\frac{1}{R_f} + \frac{1}{A}\left(\frac{1}{R_i} + \frac{1}{R_f}\right)\right] = -\frac{V_i}{R_i}$$

Amplification of circuit:

$$\boxed{A_{CL} = \frac{V_0}{V_i} = \frac{-R_f/R_i}{1 + 1/A\beta}}$$

where β = feedback fraction

$$\boxed{= \frac{R_i}{R_i + R_f}}$$

Assume $A\beta >> 1$

$$\boxed{\text{Approximate gain} \quad A_{CL} \cong -\frac{R_f}{R_i}}$$

$$\boxed{\text{Voltage ratio} \quad V_0 = -\left(\frac{R_f}{R_i}\right)V_i}$$

Example:

i.e. $V_i = 2$ mV, $R_i = 200$ ohms, $R_f = 2{,}000$ ohms, $A = 100{,}000$;
Find β, loop gain, closed loop gain, and closed loop gain with A tending to zero.

Feedback fraction: $$\beta = \frac{R_i}{R_i + R_f} = \frac{200}{200+2000} = 0.09091$$

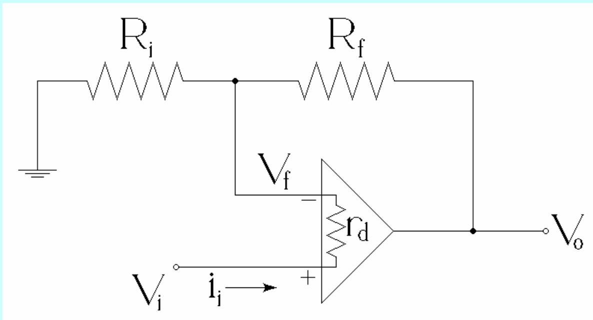
Loop gain = $A\beta = (1\times10^5)(0.09091) = 9090.91$

Exact gain = $$A_{CL} = \frac{-\frac{R_f}{R_i}}{1+\frac{1}{A\beta}} = \frac{-\frac{2000}{200}}{1+\frac{1}{(1\times10^5)(0.09091)}} = -9.9989$$

Approximate gain $A_{CL} = -\frac{R_f}{R_i} = -\frac{2000}{200} = -10$ $\quad V_o = -10 * V_i = -20$ mV

Voltage at amplifier: $$V^- = \frac{0.020V}{100000} = 2\times10^{-7} = 0.002 = 2 \text{ mV}$$

Effect on Input Resistance



$r_d \equiv$ open-loop differential input resistance

$A \equiv$ differential gain (of the operational amplifier)

Output resistance ≈ 0

V_i and i_i are assumed: $i_i = \dfrac{V_i - V_f}{r_d}$ $\qquad \beta = \dfrac{R_i}{R_i + R_f}$

V_f = (feedback voltage) = βV_0

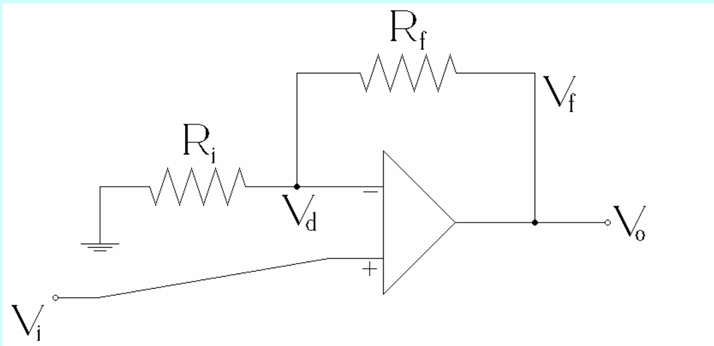
$$V_0 = \frac{A}{1+A\beta} V_i \qquad \text{(from previous sheet)}$$

$$\Rightarrow V_f = \frac{A\beta}{1+A\beta} V_i$$

$$\Rightarrow i_i = \frac{V_i - \left[\frac{A\beta}{(1+A\beta)}\right] V_i}{r_d} = \frac{1}{r_d(1+A\beta)} \times V_i$$

$$\Rightarrow R_{in} = \frac{V_i}{i_i} = (1+A\beta) r_d$$

Non-Inverting Circuit



V_i = input signal ≡ ground (non-inverting)

Feedback circuit (which consists of R_i and R_f) is arranged as a voltage divider between output and input.

$V_0 = AV_d = A(V_i - V_f)$

V_f = feedback voltage = fraction of output voltage appearing across R_i

At this point, the current in or out of the operational amplifier tends to zero.

$$\therefore V_f = \frac{R_i}{R_i + R_f} V_0 = \beta V_0$$

(where β = feedback factor)

$$A_{CL} = \frac{V_0}{V_i} = \frac{A(V_i - V_f)}{V_i} = \frac{A}{1 + A\beta} = \frac{A}{1 + AR_i / (R_i + R_f)}$$

$$= \frac{(R_i + R_f)/R_i}{1 + (R_i + R_f)/AR_i}$$

when $A \to \infty$ $\cong (R_i + R_f)/R_i$

Effect on Output Resistance

$$R_{out} = \frac{r_0}{1+A\beta}$$

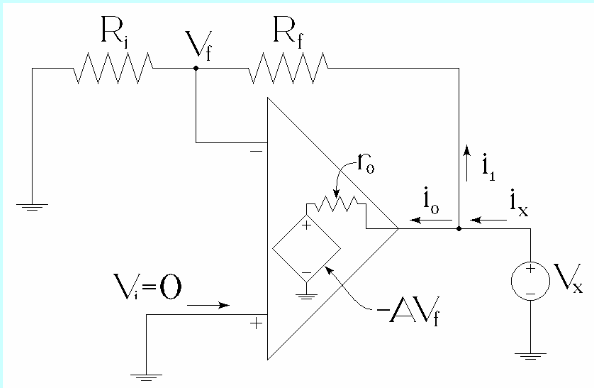
where r_0 = open loop operational amplifier output resistance

$$i_x \cong i_0 = \frac{V_x - (-AV_f)}{r_o} = \frac{V_x + AV_f}{r_o}$$

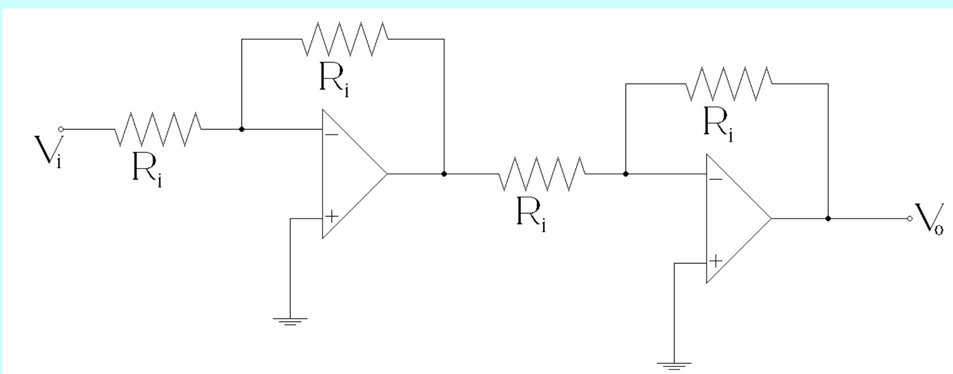
$$V_f = \beta V_0 = \beta V_x$$

$$i_0 = \frac{V_x + A\beta V_x}{r_o} = \frac{1+A\beta}{r_o} V_x$$

$$\therefore R_{out} = \frac{V_x}{i_x} = \frac{r_o}{1+A\beta} \qquad \text{(smaller)}$$



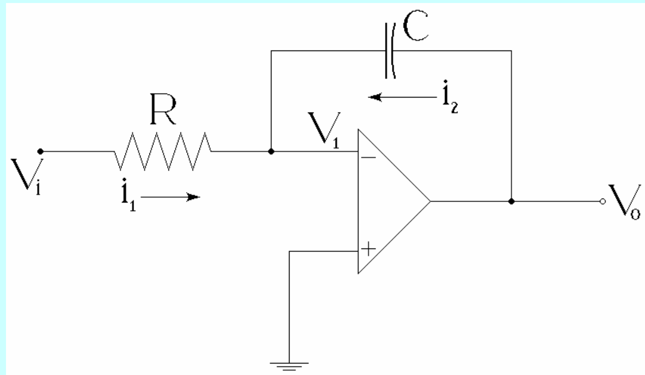
To obtain amplification α without a sign change, you can also use the following:



$$\frac{V_i}{V_0} = \frac{\alpha R_i}{R_i} = \alpha$$

Integration Circuit

$$V_1 = \frac{-V_0}{A}$$



$$i_1 = \frac{V_i - V_1}{R} \qquad i_2 = \frac{d(V_0 - V_1)}{dt} \cdot C$$

$$i_1 + i_2 = 0$$

$$\frac{V_i - V_1}{R} + \frac{d(V_0 - V_1)}{dt} \cdot C = 0 \qquad \frac{V_i + \frac{V_0}{A}}{R} + C \cdot \frac{d\left(V_0 + \frac{V_0}{A}\right)}{dt} = 0$$

$$\frac{V_i}{RC} + \cancel{\frac{V_0}{ARC}} + \frac{dV_0}{dt} + \cancel{\frac{d\left(\frac{V_0}{A}\right)}{dt}} = 0$$

as $A \rightarrow \infty$

$$\frac{V_i}{RC} + \frac{dV_0}{dt} = 0$$

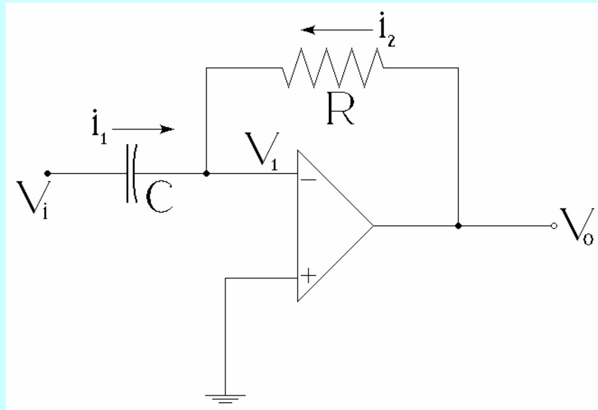
$$\boxed{V_0 = -\frac{1}{RC} \cdot \int_0^t V_i dt}$$

Differentiation Circuit

$$V_1 = \frac{-V_0}{A}$$

$$i_1 = C\frac{d(V_i - V_1)}{dt} \qquad i_2 = \frac{V_0 - V_1}{R}$$

$$i_1 + i_2 = 0$$



$$C\frac{d(V_i - V_1)}{dt} + \frac{V_0 - V_1}{R} = 0$$

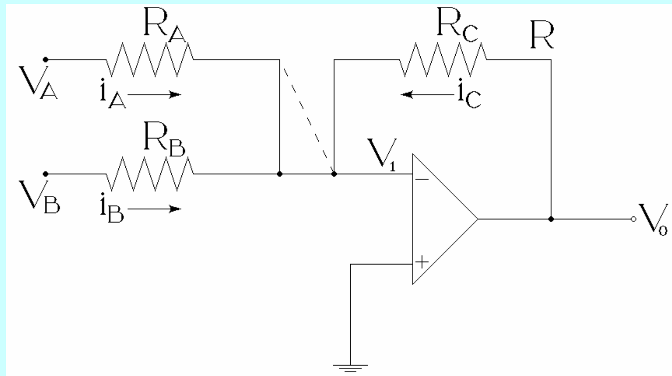
$$C\frac{d\left(V_i - \dfrac{V_0}{A}\right)}{dt} + \frac{V_0 - \dfrac{V_0}{A}}{R} = 0$$

$$\frac{dV_i}{dt} + \cancel{\frac{dV_0}{Adt}} + \frac{V_0}{RC} + \cancel{\frac{V_0}{ARC}} = 0$$

$$\boxed{\therefore V_0 = -\frac{1}{RC}\cdot\frac{dV_i}{dt}}$$

Summation Circuit

$$V_1 = \frac{-V_0}{A}$$



$$i_A = \frac{V_A - V_1}{R_A}$$

$$i_B = \frac{V_B - V_1}{R_B}$$

$$i_C = \frac{V_0 - V_1}{R_C}$$

$$i_A + i_B + i_C = 0$$

$$\frac{V_A + \frac{V_0}{A}}{R_A} + \frac{V_B + \frac{V_0}{A}}{R_B} + \frac{V_0 + \frac{V_0}{A}}{R_C} = 0$$

as $A \to \infty$

$$\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_0}{R_C} = 0$$

$$\boxed{V_0 = -\frac{R_C}{R_A} V_A - \frac{R_C}{R_B} V_B}$$

$$V_0 = C_A V_A - C_B V_B$$

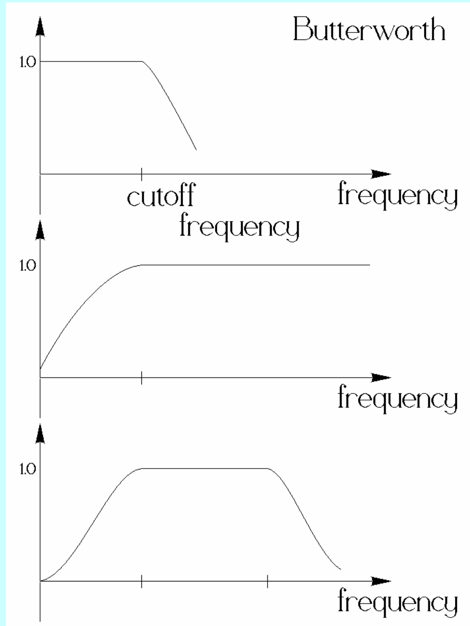
Design a circuit that gives as output the response V_x of the following equation…

$$\ddot{V}_x + a\dot{V}_x + bV_x = cV_i$$

(See 'Negative Feedback and Operational Amplifier Linear Circuits')

Filter Circuits
A filter circuit passes a specified range of frequencies

1. Low Pass
2. High Pass
3. Bandwidth (Notch)



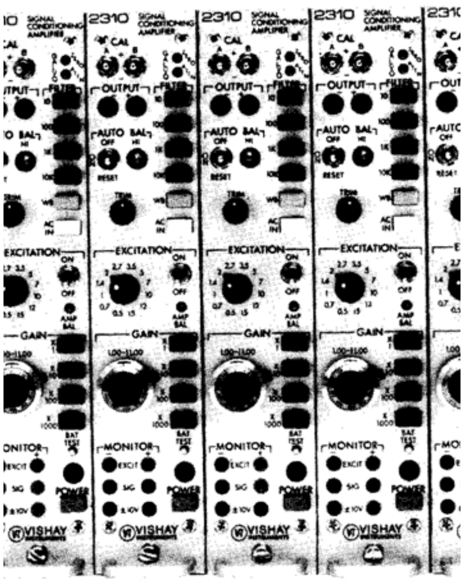
- Cutoff frequency: Response down 3dB
- These filters provide analogue noise reductions!!
- See following page for some circuits
- Attenuation = non-passing frequencies (reduced undesired output)
- Measurement of attenuation (Output/Input)

$$\text{Decibels} = \text{dB} = 20 \log (V_0/V_i)$$

V_0/V_i	dB
10.00	20
100.00	40
1,000.00	60
10,000.00	80
100,000.00	120

See: Background Information on Filter Circuits:

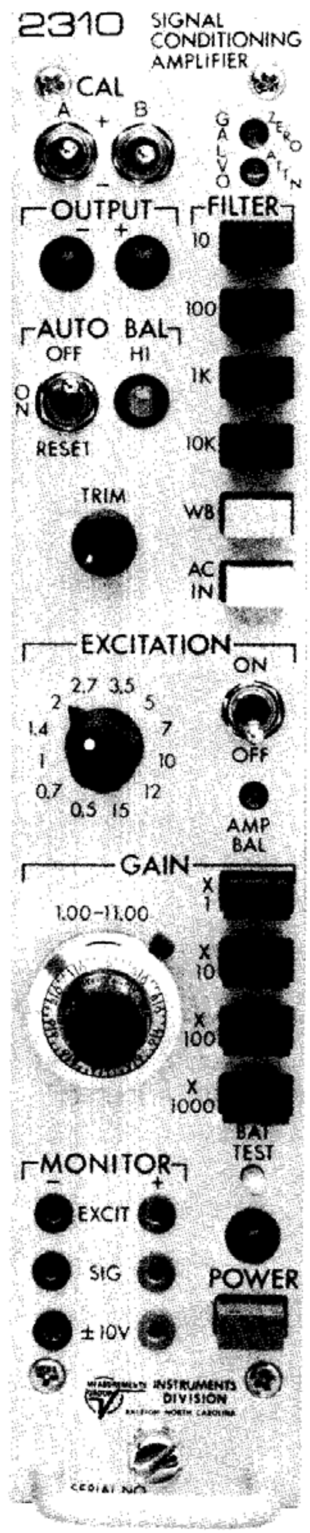
Commercial Conditioners Amplifiers



Complete 10-Channel 2300 System

Provides: Excitation
Balance
Bridge Completion
Amplification
Filtering

See: Vishay Conditioners Amplifiers



2310 Front Panel