Unified Force-Based
Real Time Dynamic Hybrid Simulation

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Outline

• Introduction
  – Currently used seismic testing methods
  – Proposed Real Time Dynamic Hybrid Simulation (RTDHS)

• Unified Formulation for RTDHS
• Unified Control Platform for RTDHS

• RTDHS Test Design
• A three story RTDHS example
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Introduction

– Quasi-static loading test method (QST)

– Shaking table testing method (STT)

– Effective force method (EFT)

– Pseudo-dynamic testing method (PDT)

– Real time pseudo-dynamic testing method (RTPDT)

– Real time dynamic hybrid testing method (RTDHT)
Quasi-static loading test method (QST)

- A test specimen is subjected to slowly changing prescribed forces or deformations by means of hydraulic actuators.

- Inertial forces within the structures are not considered in this method. Dynamic nature of earthquakes are not captured.

- Purpose is to observe the material behavior of structural elements, components, or junctions when they are subjected to cycles of loading and unloading.
Shaking table testing method (STT)

- Test structures may be subjected to actual earthquake acceleration records to investigate dynamic effects.
- Inertial effects and structure assembly issues are well represented.
- The size of the structures are limited or scaled by the size and capacity of the shake table.
Effective force testing method (EFT)

- Applying dynamic forces to a test specimen that is anchored rigidly to an immobile ground; perform real-time earthquake simulation

- The test specimen is fully assembled as shake table test. (mass, damping and stiffness)

- These forces are proportional to the prescribed ground acceleration and the local structural masses.

- Based on a force control algorithm
Pseudo-dynamic testing method (PSD)

- Applying slowly varying forces to a structural model
- Motions and deformations observed in the test specimens are used to infer the inertial forces that the model would have been exposed to during the actual earthquake
- Substructure techniques

RT-PSD

- Same as the PSD test except that it is conducted in the real time
- Introduce problem in control, such as delay caused by numerical simulation and actuator
Introduction

Experimental Methods for Seismic Structural Performance:

- **Quasi-Static Loading Testing (QST)**
- **Shaking Table Testing (STT)**
- **Effective Force Testing (EFT)**
- **Pesudo-dynamic Testing (PSD)**
Introduction

\[ M\ddot{x} + C\dot{x} + f(x) = -MR\ddot{u}_g \]

<table>
<thead>
<tr>
<th>Effective Force Test</th>
<th>Pseudo-dynamic Test</th>
<th>Shake Table Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P = -MR\ddot{u}_g )</td>
<td>( f(x) = -M\ddot{x} - C\dot{x} - MR\ddot{u}_g )</td>
<td>( \ddot{u}_g )</td>
</tr>
</tbody>
</table>

W1: Northridge Acceleration
Modern Seismic Simulation Techniques

– Substructure simulation
  • Simulation conducted for (either experimentally or numerically) only part of the structure to obtain the performance of the whole structure by extrapolation

– Pseudo-dynamic simulation
  • Simulation conducted with inertia effect of the structural system is numerically simulated in the computer and applied by hydraulic actuators or shake tables

– Dynamic simulation
  • Simulation conducted with structures’ inertia effect physically realized in the specimen and dynamical load is provided by actuators or shake tables

– Hybrid simulation
  • Simulation procedure combined both physical experiment and numerical computation to estimate/predict the structural seismic behavior

*PSD /substructure are by definition a hybrid simulation.*
Introduction

• Combination of the above simulation techniques render all kinds of different modern seismic simulation methods
  – Real Time (Substructure) Pseudo-dynamic Simulation
    • Simulated inertial effect applied by actuator (Nakashima, 1992, Darby et al. 1999 Blakeborough et al. 2001)
    • Simulated inertial effect applied by shake Table (Nield et al. 1995)
    • Simulated inertia effect applied by actuator while shake table introduce acceleration (Tamura and Kobayashi, 1998)
  – (Substructure ) Dynamic Simulation
    • Using both shake table and actuator in one dynamic simulation (Kausel, 1998)
    • Substructure dynamic simulation (Ito et al. 2000, 2004)
  – Quasi Dynamic Simulation
    • Hybrid simulation of dynamic/pseudo-dynamic (Cheng, 2006)

• Each test method are relatively independent requires a full development of the numerical algorithms and controllers for the corresponding loading system that cannot be used for other type of test.
Introduction

A force-based seismic simulation method and the corresponding controller platform that can UNIFY current seismic simulations methods

Real Time Dynamic Hybrid Simulation (RTDHS) – Combined use of earthquake simulators, actuators and computational engines for simulation.

Acceleration input: Table introduce inertia force

Has to operate in force control
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• **Unified Control Platform for RTDHS**

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Unified Formulation -- Substructure Partition

General Equation of Motion

\[
M\ddot{x} + C\dot{x} + Kx = -MR\ddot{u}_g
\]
Unified Formulation

\[
\begin{bmatrix}
\alpha, M_{i}\n\\
M_x
\\
\alpha, M_{j}
\end{bmatrix} \begin{bmatrix}
\ddot{X}_i
\\
\ddot{X}_j
\end{bmatrix} + \begin{bmatrix}
C^i_e, C_w
\\
C^i_l, C^i_w
\end{bmatrix} \begin{bmatrix}
\dot{X}_i
\\
\dot{X}_j
\end{bmatrix} + \begin{bmatrix}
K^i_u, K_{x}\n\\
K^j_u, K_{x}
\end{bmatrix} \begin{bmatrix}
X_i
\\
X_j
\end{bmatrix} = -\begin{bmatrix}
\alpha, M_{i}\n\\
M_x
\\
\alpha, M_{j}
\end{bmatrix} \begin{bmatrix}
\ddot{X}_i
\\
\ddot{X}_j
\end{bmatrix} + \begin{bmatrix}
P_{et}
\\
P_{eb}
\end{bmatrix}
\]

\[
M_{ep} \ddot{x}_{ep} + C_{ep} \dot{x}_{ep} + K_{ep} x_{ep} = -M_{ep} R_e \ddot{u}_g - T_{ep}
\]

**Mass splitting**

\[
M_{ep} = M^p_{ep} + M^v_{ep}
\]

\[
M^p_{ep} = (E - \alpha_m) M_{ep}
\]

\[
\alpha_m = \begin{bmatrix}
\alpha_i
\\
\alpha_e
\\
\alpha_j
\end{bmatrix}
\]

\[
M^p_{ep} \ddot{x}_{ep} + C_{ep} \dot{x}_{ep} + K_{ep} x_{ep} = -M^p_{ep} R_e \ddot{u}_g - T_{ep} - \alpha_m M_{ep} (R_e \dddot{u}_g + \ddot{x}_{ep}) = -M^p_{ep} R_e \dddot{u}_g - T''_{ep}
\]

**Load splitting**

\[
\alpha_i(s)
\]

\[
M^p_{ep} \ddot{x}_{ep} + C_{ep} \dot{x}_{ep} + K_{ep} x_{ep} = -M^p_{ep} R_e (E - \alpha_i(s)) \dddot{u}_g - (T''_{ep} + M^p_{ep} R_e \alpha_i(s) \dddot{u}_g)
\]
Unified Formulation

MDOF Experimental Substructure in Hybrid Testing (General Case)

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Test Structure Model</th>
<th>Table Acceleration</th>
<th>Actuators Forces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudo-Dynamic</td>
<td>$M^p_{ep} \ddot{x}<em>{ep} + C</em>{ep} \dot{x}<em>{ep} + K</em>{ep} x_{ep}$</td>
<td>$-M^p_{ep} R_e (E - \alpha_I(s)) \ddot{u}_g -$</td>
<td>$-T_m + M^p_{ep} R_e \alpha_I(s) \ddot{u}_g$</td>
</tr>
<tr>
<td>Dynamic Testing $\alpha_m = 0$</td>
<td>$M_{ep} \ddot{x}<em>{ep} + C</em>{ep} \dot{x}<em>{ep} + K</em>{ep} x_{ep}$</td>
<td>$\alpha(s)</td>
<td>_1 = E$</td>
</tr>
<tr>
<td>Quasi-Dynamic</td>
<td>$0 &lt; \alpha_m &lt; E$</td>
<td>$\alpha(s)</td>
<td>_1 = E$</td>
</tr>
</tbody>
</table>

Pseudo-dynamic Test

Effective Force Test
## Unified Formulation

### Single Story Structure in Hybrid Testing

<table>
<thead>
<tr>
<th>TEST STRUCTURE</th>
<th>FORCE BASED INPUT TOTAL DYNAMIC LOAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\left(1 - \alpha_m\right)M\dddot{x} + C\ddot{x} + Kx$</td>
<td>$-\left(1 - \alpha_m\right)M\left(\dddot{u}_g + \alpha_m\dddot{x}\right) - \alpha_p\left(\dddot{u}_g + \alpha_m\dddot{x}\right)T \left(1 - \alpha_m\right)M\left(\dddot{u}_g + \alpha_m\dddot{x}\right)\left(1 - \alpha_p\right)zT$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test Name</th>
<th>Test Structure</th>
<th>$\alpha(s)_t$</th>
<th>$\alpha(s)_p$</th>
<th>Table Acceleration Input</th>
<th>Actuator’s Applied Forces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudo-Dynamic Test $\alpha_m = 1$</td>
<td>$C\ddot{x} + Kx$</td>
<td>$\alpha(s) = 0$</td>
<td>$\alpha(s) = 1$</td>
<td>$\alpha(s) = 1$</td>
<td>$\alpha(s) = 1$</td>
</tr>
<tr>
<td>Quasi-Dynamic Test $0 &lt; \alpha_m &lt; 1$</td>
<td>$\left(1 - \alpha_m\right)M\dddot{x} + C\ddot{x} + Kx$</td>
<td>$\alpha(s)_t = 0$</td>
<td>$\alpha(s)_p = 0$</td>
<td>$\dddot{u}_eq = 0$</td>
<td>$T$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\alpha(s)_t = 1$</td>
<td>$\alpha(s)_p = 0$</td>
<td>$\dddot{u}_eq = \frac{T}{\left(1 - \alpha_m\right)}M$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\alpha(s)_t = 1$</td>
<td>$\alpha(s)_p = 1$</td>
<td>$\dddot{u}_eq - \frac{T}{\left(1 - \alpha_m\right)}M + \left(1 - \alpha_p\right)zT$</td>
<td></td>
</tr>
<tr>
<td>Dynamic Test $\alpha_m = 0$</td>
<td>$M\dddot{x} + C\ddot{x} + Kx$</td>
<td>$\alpha(s)_t = 0$</td>
<td>$\alpha(s)_p = 0$</td>
<td>$\dddot{u}_g = 0$</td>
<td>$T$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\alpha(s)_t = 0$</td>
<td>$\alpha(s)_p = 0$</td>
<td>$\dddot{u}_g = \alpha_p\dddot{x} + \frac{T}{\left(1 - \alpha_m\right)}M + \left(1 - \alpha_p\right)zT$</td>
<td></td>
</tr>
</tbody>
</table>

- **Pseudo-dynamic Test**
- **Effective Force Test**
- **Shake Table Test**
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• Unified Control Platform for RTDHS

• RTDHS Test Design
• A three story RTDHS example
Physical Platform

Physical Components and Connections

Data Acquisition and Information Streaming
- Internet
  - NTCP Server
    - NTCP to SCRAMNet Interface (Distributed Testing)
      - Linux
  - Pacific GUI
  - DAQ Host
- DAQ
  - SCRAMNet A/D & D/A Bridge
  - Proprietary OS
- xPC Target

Real Time Hybrid Simulation Controller
- Simulator
  - Structural Simulator
    - xPC Target
- Compensator
  - Compensation Controller

Structural and Seismic Testing Controllers
- Shake Table 1
  - GUI
  - Controller
    - PowerPC
- Shake Table 2
  - GUI
  - Controller
    - PowerPC
- STS
  - GUI
  - Controller
    - PowerPC
- FlexTest
  - GUI
  - Controller
    - PowerPC

Shared Common Random Access Memory Network

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Physical Platform

Real Time Dynamic Hybrid Simulation Controller

- **Real Time Structure Simulator**
  - Set up Equation of Motion of the Whole Structure
  - Solve the equation of motion using time history analysis

- **Interface Force Calculator**
  - Property matrix at the interface DOF
  - Calculate the Interface force
  - Map the state of interface DOF

- **Unified System Controller**
  - Ground Acceleration Input
  - Applied Acceleration
  - Interface Force
  - Applied interface force
  - Load Division Between Shake Tables and Structure Actuators

- **Compensation Controller**
  - Compensator For Shake Tables
  - Compensator For Structure Actuators
  - Acceleration Commands to Table Controllers
  - Displacement Commands to Actuator Controllers

- **System Compensation Controller**

- **Structure Simulator**

- **Compensation Controller**
Real Time Structure Simulator

Set up Equation of Motion of the Whole Structure

Solve the equation of motion using time history analysis

Property matrix at the interface DOF

Calculate the Interface force

Map the state of interface DOF

Real Time Structure Simulator

Interface Force Calculator

Input Data
1. Joint, Element
2. Material Library

Mass, Stiffness Matrix Assembly of the Computational Substructure (Static)*

Damping Matrix Assembly

Dynamic Matrix Partition Motion Vector Partition

Numerical Model Condensation

Numerical Integration

Responses of DOFs On Computational Substructure

Interface Force Calculation For Each Substructures

Go to SCC for Compensation

Responses of DOFs on Experimental Substructure

Start

* This procedure can be done using commercial finite element software such as ABAQUS, SAP2000, IDARC, ANSYS, etc.
System Compensation Controller

- **Unified System Controller (USC)**
  - Perform the unified formulation derived above for RTDHS
  - Redistribute the load between the shake tables and structure actuators

- **Compensation Controller**
  - Provide the compensation necessary for applying the desired load by hydraulic loading equipment

*Compensation for Structure Actuator*: Apply desired force to experimental substructure
- Dynamic Force Control
- Time Delay Compensation
- Effective Force Control

*Compensation for Shake Table*: Introduce desired acceleration input to experimental substructure
- When the applied acceleration is predetermined, no compensation is necessary for shake table.
- If quasi-dynamic test involved and using closed acceleration feedback to determine equivalent acceleration, then time delay compensation need to be applied in shake table compensator.

*Compensation for Hybrid Simulation*
- Compensate for different time delay between shake table and structure actuator.
Unified System Controller

- Input, output and predefined coefficients

**Structure acceleration feedback** can be implemented either in open loop or in closed loop

\[
M\dddot{x} + C\dot{x} + Kx = -M\dddot{u}_g + T
\]

\[
= -M^p \left( \alpha_1(s) \dddot{u}_{eq} - \alpha_1(s) \frac{T}{M^p} \right) \left( 1 - \alpha_p(s) \right) M\dddot{u}_{eq} + \left( 1 - \alpha_p(s) \right) T
\]

\[
= \left( \frac{T}{M^p} \right) M \left( \sum_{\alpha_{m}} \dddot{u}_g + \alpha_m \dddot{x} \right) + \left( 1 - \alpha_p(s) \right) M\dddot{u}_{eq} + \left( 1 - \alpha_p(s) \right) T
\]
Compensation Controller ---for Structure Actuator

Dynamic Force Control – Series Elasticity and Displacement Feedback
Compensation Controller  --- for Structure Actuator

Effective Force Control  -- Velocity Feedback

\[ m\ddot{x} + c\dot{x} + kx = -m\ddot{u}_g \]

\[ m\ddot{x} + (c + c_a)\dot{x} + kx = -m\ddot{u}_g + c_a\dot{x} \]
Compensation Controller ---for Structure Actuator

Time Delay Compensation – Smith’s Predictor

\[
G = \frac{Y}{X} = \frac{G_p T_p}{1 - G_p T_p} = \frac{G_p}{1 - G_m T_m - G_p T_p} T_p = \frac{G_p}{1 - G_p} T_p
\]
Compensation Controller --- for Hybrid Simulation

Shake Table and Structure Actuator
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RTDHS Test Design

Real Time Dynamic Hybrid Simulation Controller

User Preparation  UB-RTDHS Software Package
Real Time Structure Simulator

Input Data:

The input step is similar to most finite element program, which includes the coordinate of joints and their orientations; material properties; element definition; lumped mass at each joint as well as the ground acceleration time history.
Stiffness, Mass Matrix Assembly:

The stiffness and mass matrix are formed by following the same procedure as for static analysis based on the information input. Therefore the commercial finite element analysis programs, such as ABACUS, SAP2000, IDARC and ANSYS, are available to conduct this preprocessing.
**Real Time Structure Simulator**

**Damp Matrix Assembly:**

The damp matrix is then formed using the Rayleigh damp formula by considering the damping coefficients as a combination of mass-proportional damping and stiffness-proportional damping.
Real Time Structure Simulator

Substructure Partition:

The dynamic matrices of the whole structure model as well as the motion vector are partitioned to represent each substructure.
Real Time Structure Simulator

**Stiffness, Mass Matrix Condensation:**

The mass and stiffness matrix are condensed to reduce the total DOFs of the computational substructures’ model for real time simulation based on the assumption of their linear behavior during simulation. The condensation methods include static condensation and dynamic modal condensation.

Nonlinear Implementation
Real Time Structure Simulator

Numerical Integration:

One of the numerical integration methods can be used here to solve step by step the responses of the whole structure based the reduced order governing equation of motion of the structural modal.

Nonlinear Implementation
Real Time Structure Simulator

**Interface Force Calculation:**

With the available sub matrices defined and the simulated responses at the interface DOFs, one may use derive the interface forces necessary to be applied by the dynamic actuators.
Errors:

The errors between the measured response and the simulated response are used to adaptive the controller and update the numerical model in the structure simulation.

- Adaptive Control:
  - MCS (Stoten, 1993)
- Online Modal Estimation

Nonlinear Implementation
Real Time Structure Simulator

User preparation work list:
- Structural model assembly
  - Mass, Stiffness, Damp Matrix
- Determine ground acceleration input
- Perform substructure partition
- Perform model condensation of the computational substructure

User input to RTSS:
- Ground acceleration history ($\ddot{u}$)
- Reduced order structural model (ROM)
- Substructure partition information

* This procedure can be done using commercial finite element software such as ABACUS, SAP2000, IDARC, ANSYS, etc.
Unified System Controller (USC)

User input to USC:
- Ground acceleration history \( (\ddot{u}_g) \)
- Mass splitting coefficient \( \alpha_m \)
  - Physical mass in the specimen
  - Total mass necessary
- Load splitting coefficient \( \alpha_l \)

- Ground acceleration history \( (\dddot{u}_g) \)
- Mass splitting coefficient \( \alpha_m \)
  - Physical mass in the specimen
  - Total mass necessary
- Load splitting coefficient \( \alpha_l \), \( \alpha_p \)
Compensation Controller ---for Hybrid Simulation

From USC

Input to Compensation Controller (Research Engineer):
- Spring design and stiffness identification
- Additional damping identification
- Loading system identification and numerical model
- Physical specimen identification and numerical model
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Three Story Model

Inertial Reference Frame

Base Substructure

Experimental Substructure

Top Substructure

Ground
Step 1: Model Assembly

Step 2: Determine Ground Acceleration

**Equation of Motion**

\[
\begin{bmatrix}
m_3 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_1 \\
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_3 \\
\ddot{u}_2 \\
\ddot{u}_1 \\
\end{bmatrix}
+
\begin{bmatrix}
c_3 & -c_3 & 0 \\
-c_3 & c_3 + c_2 & -c_2 \\
0 & -c_2 & c_2 + c_1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{x}_3 \\
\dot{x}_2 \\
\dot{x}_1 \\
\end{bmatrix}
+
\begin{bmatrix}
k_3 & -k_3 & 0 \\
-k_3 & k_3 + k_2 & -k_2 \\
0 & -k_2 & k_2 + k_1 \\
\end{bmatrix}
\begin{bmatrix}
x_3 \\
x_2 \\
x_1 \\
\end{bmatrix}
= 0
\]

Where:

Absolute acceleration

\[
\begin{align*}
\begin{bmatrix}
\ddot{u}_3 \\
\ddot{u}_2 \\
\ddot{u}_1 \\
\end{bmatrix}
&= \begin{bmatrix}
\ddot{x}_3 \\
\ddot{x}_2 \\
\ddot{x}_1 \\
\end{bmatrix} + lii_g
\end{align*}
\]

\[
\begin{bmatrix}
m_3 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_1 \\
\end{bmatrix}
\begin{bmatrix}
\dddot{x}_3 \\
\dddot{x}_2 \\
\dddot{x}_1 \\
\end{bmatrix}
+
\begin{bmatrix}
c_3 & -c_3 & 0 \\
-c_3 & c_3 + c_2 & -c_2 \\
0 & -c_2 & c_2 + c_1 \\
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_3 \\
\ddot{x}_2 \\
\ddot{x}_1 \\
\end{bmatrix}
+
\begin{bmatrix}
k_3 & -k_3 & 0 \\
-k_3 & k_3 + k_2 & -k_2 \\
0 & -k_2 & k_2 + k_1 \\
\end{bmatrix}
\begin{bmatrix}
x_3 \\
x_2 \\
x_1 \\
\end{bmatrix}
= -\begin{bmatrix}
m_3 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_1 \\
\end{bmatrix}lii_g
\]
Step 3: Substructure partition

\[
\begin{bmatrix}
  m_3 & 0 & 0 \\
  0 & m_2 & 0 \\
  0 & 0 & m_1
\end{bmatrix}
\begin{bmatrix}
  \ddot{x}_1 \\
  \ddot{x}_2 \\
  \ddot{x}_3
\end{bmatrix} +
\begin{bmatrix}
  c_3 \\
  c_3 + c_2 \\
  c_2 + c_1
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2 \\
  \dot{x}_3
\end{bmatrix} +
\begin{bmatrix}
  -c_3 \\
  -c_2 \\
  -c_2
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3
\end{bmatrix} =
\begin{bmatrix}
  k_1 \\
  k_3 + k_2 \\
  k_2 + k_1
\end{bmatrix}
\begin{bmatrix}
  \dddot{x}_1 \\
  \dddot{x}_2 \\
  \dddot{x}_3
\end{bmatrix} +
\begin{bmatrix}
  0 \\
  0 \\
  0
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3
\end{bmatrix}
\]

Ground

**Top Substructure**

\[ m_3 \ddot{x}_3 + c_3 \dot{x}_3 + k_3 x_3 = -m_3 \dddot{x}_g + \]
\[ k_3 x_2 + c_3 \dot{x}_2 \]
\( (k_3 \text{ displacement} + c_3 \text{ velocity of experimental substructure}) \]

**Experimental Substructure**

\[ m_2 \dddot{x}_{21} + c_2 \dot{x}_{21} + k_2 x_{21} = -m_2 \dddot{x}_{11} + k_3 x_{32} + c_3 \dot{x}_{32} \]

**Base Substructure**

\[ m_1 \dddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 = -m_1 \dddot{x}_g + \]
\[ k_2 x_{21} + c_2 \dot{x}_{21} \]

\( \text{Force measured at the base of experimental substructure} \)
Step 4: Modal condensation

Motivation:
– Accelerate the online structure simulation to fit for real time testing.
– Behavior of the structure under investigation: computational substructure well known while experimental substructure less known.
– Adopt commercial FEM software in assembling the structural model

Methods:
Static condensation (Guyan Reduction)
Dynamic condensation (Reyleigh-Ritz Reduction)
Step 5: Prepare Data Input for RTSS

- Ground acceleration history
- Substructure partition information
- Reduced order structural model (ROM)

Inertial Reference Frame

Base Substructure

Experimental Substructure

Top Substructure

Ground

$\begin{align*}
\mathbf{m}_1 \ddot{\mathbf{x}}_1 + \mathbf{c}_1 \dot{\mathbf{x}}_1 + \mathbf{k}_1 \mathbf{x}_1 &= -\mathbf{m}_1 \ddot{\mathbf{u}}_1 + \mathbf{k}_2 \mathbf{x}_2 + \mathbf{c}_2 \dot{\mathbf{x}}_2 \\
\mathbf{m}_2 \ddot{\mathbf{x}}_2 + \mathbf{c}_2 \dot{\mathbf{x}}_2 + \mathbf{k}_2 \mathbf{x}_2 &= -\mathbf{m}_2 \ddot{\mathbf{u}}_2 + \mathbf{k}_3 \mathbf{x}_3 + \mathbf{c}_3 \dot{\mathbf{x}}_3 \\
\mathbf{m}_3 \ddot{\mathbf{x}}_3 + \mathbf{c}_3 \dot{\mathbf{x}}_3 + \mathbf{k}_3 \mathbf{x}_3 &= -\mathbf{m}_3 \ddot{\mathbf{u}}_3 + \mathbf{k}_4 \mathbf{x}_4 + \mathbf{c}_4 \dot{\mathbf{x}}_4
\end{align*}$

Force measured at the base of experimental substructure
Step 6: Prepare Data Input for USC

- Mass splitting coefficient \( \alpha_m \)
- Load splitting coefficient \( \alpha_l \) \( \alpha_p \)

Experimental Substructure EOM

\[
m_2 \ddot{x}_{21} + c_2 \dot{x}_{21} + k_2 x_{21} = -m_2 \dddot{x}_{1} + k_3 x_{32} + c_3 \dot{x}_{32}
\]

Physical Specimen

\[
\left[ 1 - \alpha_m \right] \left( m_2 \ddot{x}_{21} + c_2 \dot{x}_{21} \right) + k_2 \dddot{x}_{21}
\]

Apply Table Acceleration

\[
- m_2 \alpha_l(s) \dddot{x}_{1} - \alpha_p(s) \left( \frac{c_3}{m_2} \dddot{x}_{32} + \frac{k_3}{m_2} x_{32} \right)
\]

Apply Actuator Force

\[
- \left[ 1 - \alpha_l(s) \right] m_2 \dddot{x}_{1} - \alpha_m \left( m_2 \ddot{x}_{21} + c_2 \dot{x}_{21} \right) + \left[ 1 - \alpha_p(s) \right] \left( c_3 \dddot{x}_{32} + k_3 x_{32} \right)
\]
## Unified Formulation

### Single Story Structure in Hybrid Testing

<table>
<thead>
<tr>
<th>TEST STRUCTURE</th>
<th>FORCE BASED INPUT TOTAL DYNAMIC LOAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1-\alpha_m)M\ddot{x} + C\dot{x} + Kx)</td>
<td>(-\left(1-\alpha_m\right)M\left(\alpha_f(z)\frac{\ddot{u}_g + \alpha_m\ddot{x}}{1-\alpha_m} - \alpha_p(z)\right) + \frac{T}{1-\alpha_m} - \left(1-\alpha_f(z)\right)M(\ddot{u}_g + \alpha_m\ddot{x}) + \left(1-\alpha_p(z)\right)T)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test Name</th>
<th>Test Structure</th>
<th>(\alpha(s)_l)</th>
<th>(\alpha(s)_p)</th>
<th>Table Acceleration Input</th>
<th>Actuator’s Applied Forces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudo-Dynamic Test (\alpha_m = 1)</td>
<td>(C\ddot{x} + Kx)</td>
<td>(\alpha(s)_l = 0)</td>
<td>(\alpha(s)_p = 0)</td>
<td>0</td>
<td>(-M(\ddot{u}_g + \ddot{x}) + T)</td>
</tr>
<tr>
<td>Quasi-Dynamic Test (0 &lt; \alpha_m &lt; 1)</td>
<td>((1-\alpha_m)M\ddot{x} + C\dot{x} + Kx)</td>
<td>(\alpha(s)_l = 0)</td>
<td>(\alpha(s)_p = 0)</td>
<td>0</td>
<td>(-M(\ddot{u}_g + \alpha_m\ddot{x}) + T)</td>
</tr>
<tr>
<td></td>
<td>(\alpha(s)_l = 1)</td>
<td>(\alpha(s)_p = 0)</td>
<td>(\dddot{u}_{eq})</td>
<td>(\frac{T}{(1-\alpha_m)M})</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(\alpha(s)_l = 1)</td>
<td>(\alpha(s)_p = 1)</td>
<td>(\dddot{u}_{eq} + \frac{T}{(1-\alpha_m)M})</td>
<td>0</td>
<td>(-M(\dddot{u}_g + \alpha_m\dddot{x}) + (1-\alpha_f(z))M(\ddot{u}_g + \alpha_m\ddot{x}) + (1-\alpha_p(z))T)</td>
</tr>
<tr>
<td>Dynamic Test (\alpha_m = 0)</td>
<td>(M\ddot{x} + C\dot{x} + Kx)</td>
<td>(\alpha(s)_l = 0)</td>
<td>(\alpha(s)_p = 0)</td>
<td>0</td>
<td>(-M\dddot{u}_g + T)</td>
</tr>
<tr>
<td></td>
<td>(\alpha(s)_l = 1)</td>
<td>(\alpha(s)_p = 0)</td>
<td>(\dddot{u}_g)</td>
<td>(\frac{T}{M})</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(\alpha(s)_l = 1)</td>
<td>(\alpha(s)_p = 1)</td>
<td>(\dddot{u}_g + \frac{T}{M})</td>
<td>0</td>
<td>(-M(\dddot{u}_g + \alpha_m\dddot{x}) + (1-\alpha_f(z))M(\ddot{u}_g + \alpha_m\ddot{x}) + (1-\alpha_p(z))T)</td>
</tr>
</tbody>
</table>
Step 7: Other preparation

- **General physical test design**
  - Experimental substructure (physical specimen) design, fabrication drawings
  - Instrumentation design and configuration
  - Test protocol list
  - *Series spring and connection design (by RE)*

- **Physical test setup installation**
  - Specimen
  - Instrumentation
  - Series spring
  - Loading devices -- dynamic actuators / shake tables

- **Physical test setup identification**
  - *Design compensation controller (by RE)*

* Cooperation with research engineer, technicians, lab manager, etc.
Proof-of-Concept Test

Small Scale Pilot Experimental Setup

- Shake table is controlled in displacement
- Reduced mass model and full mass model

<table>
<thead>
<tr>
<th>Properties</th>
<th>Full Mass</th>
<th>Reduced Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>0.17 (kip)</td>
<td>0.039 (kip)</td>
</tr>
<tr>
<td>Stiffness</td>
<td>0.127 (kip/in)</td>
<td>0.129 (kip/in)</td>
</tr>
<tr>
<td>Damping Ratio</td>
<td>16.7%</td>
<td>27.2%</td>
</tr>
<tr>
<td>Damping Coefficient</td>
<td>2.49E-3</td>
<td>1.96E-3</td>
</tr>
<tr>
<td>Natural Frequency</td>
<td>2.7 (Hz)</td>
<td>5.7 (Hz)</td>
</tr>
<tr>
<td>Spring</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stiffness</td>
<td>0.111 (kip/in)</td>
<td>0.115 (kip/in)</td>
</tr>
<tr>
<td>Combined Natural Frequency</td>
<td>3.7 (Hz)</td>
<td>5.7 (Hz)</td>
</tr>
</tbody>
</table>
### Proof-of-Concept Test

#### Three Story Hybrid Simulation

<table>
<thead>
<tr>
<th>Implementation Case</th>
<th>Equations of motion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case 1</strong> For (\alpha_m = 0.77) and (\alpha_p(s) = 0):</td>
<td>(0.23(m_2\ddot{x}<em>{21} + c_2\dot{x}</em>{21}) + k_2x_{21} = )</td>
</tr>
<tr>
<td></td>
<td>(-m_2\ddot{u}<em>1 - 0.77(m_2\ddot{x}</em>{21} + c_2\ddot{x}<em>{21}) + (c_3\ddot{x}</em>{32} + k_3x_{32}))</td>
</tr>
<tr>
<td><strong>Top force</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Base acceleration</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Case 2</strong> For (\alpha_m = 0), (\alpha_i(s) = 0.5) and (\alpha_p(s) = 0.5):</td>
<td>(m_2\ddot{x}<em>{21} + c_2\ddot{x}</em>{21} + k_2x_{21} = )</td>
</tr>
<tr>
<td></td>
<td>(-m_2\ddot{u}<em>1 - 0.5\left(c_3\ddot{x}</em>{32} + k_3x_{32}\right))</td>
</tr>
<tr>
<td><strong>Top force</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Base acceleration</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Case 3</strong> For (\alpha_m = 0), (\alpha_i(s) = 1) and (\alpha_p(s) = 1):</td>
<td>(m_2\ddot{x}<em>{21} + c_2\ddot{x}</em>{21} + k_2x_{21} = )</td>
</tr>
<tr>
<td></td>
<td>(-m_2\ddot{u}<em>1 - \left(c_3\ddot{x}</em>{32} + k_3x_{32}\right))</td>
</tr>
<tr>
<td><strong>Top force</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Base acceleration</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Case 4</strong> For (\alpha_m = 0), (\alpha_i(s) = 0.0) and (\alpha_p(s) = 0.0):</td>
<td>(m_2\ddot{x}<em>{21} + c_2\ddot{x}</em>{21} + k_2x_{21} = )</td>
</tr>
<tr>
<td></td>
<td>(-m_2\ddot{u}<em>1 + (c_3\ddot{x}</em>{32} + k_3x_{32}))</td>
</tr>
<tr>
<td><strong>Top force</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Base acceleration</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Case 5</strong> For (\alpha_m = 0), (\alpha_i(s) = 1) and (\alpha_p(s) = 0.0):</td>
<td>(m_2\ddot{x}<em>{21} + c_2\ddot{x}</em>{21} + k_2x_{21} = )</td>
</tr>
<tr>
<td></td>
<td>(-m_2\ddot{u}<em>1 + \left(c_3\ddot{x}</em>{32} + k_3x_{32}\right))</td>
</tr>
<tr>
<td><strong>Top force</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Base acceleration</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Generalized Equation for All Cases**

\[
\begin{bmatrix}
1 - \alpha_m(m_1\ddot{x}_{21} + c_1\ddot{x}_{21}) + k_1x_{21} - m_1\ddot{u}_1 - \alpha_p(s)\left(c_3\ddot{x}_{32} + k_3x_{32}\right) \\
1 - \alpha_m(m_2\ddot{x}_{21} + c_2\ddot{x}_{21}) + k_2x_{21} - m_2\ddot{u}_1 - \alpha_m(m_1\ddot{x}_{21} + c_1\ddot{x}_{21}) - \alpha_p(s)(c_3\ddot{x}_{32} + k_3x_{32})
\end{bmatrix}
\]
Proof-of-Concept Test

Experimental Result of Three Story Hybrid Simulation -- Acceleration
Conclusions

• Reviewed the current seismic simulation methods
• Proposed force-based RTDHS combining
  – Shake table, dynamic actuators and numerical simulation
  – Substructure, Pseudo-dynamic, Dynamic and Hybrid simulation techniques
• Unified formulation
  – Substructure partition
  – Loading splitting coefficient (mass, dynamic load)
• Unified control platform
  – Real time structure simulator
  – System compensation controller

• Design a RTDHS
  – Procedure
  – Example – a three story hybrid simulation
Thank You! &
Questions?