

LOADING SYSTEMS: Dynamic Structural Testing

Loading Devices- Dynamic

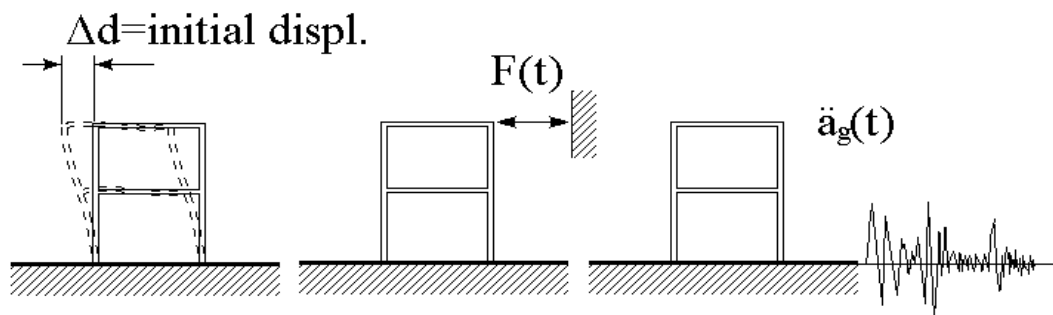
1. Objective of loading device

- Produce inertial load effects in structure
- Produce response in ‘real time’ or ‘deformed time’
- Determine high strain rate effects on member properties.

$$\left(\frac{d\varepsilon}{dt} = \dot{\varepsilon} = \text{strain rate} \right)$$

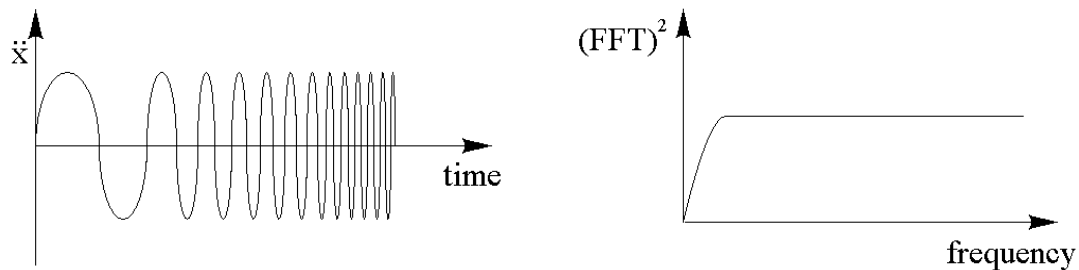
2. Testing for Inertial load effects

- Free vibrations – produced by changing initial conditions
- Forced vibrations – produced by time varying loads, or
- Base vibrations – produced by base movement



3. Loading Systems

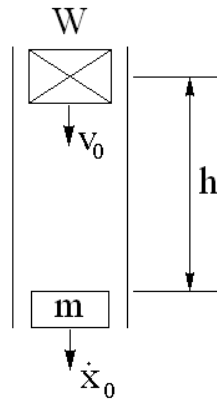
- **Impact devices** – for free vibration – changes initial velocity
- Cable snap-back (quick release) – free vibration – changes initial displacement
- Rotating motors – for forced vibration – sinusoidal sweep



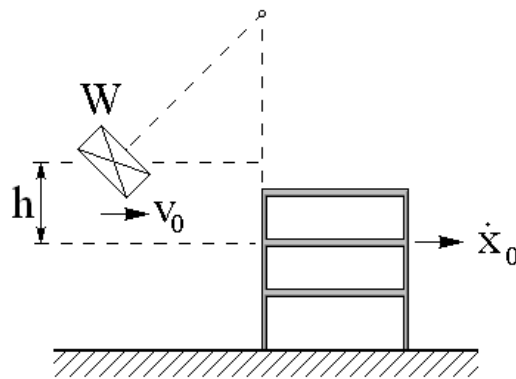
- Vibrating tables (forced vibration by base movement) – harmonic and random motion
 - Harmonic
 - **Sine Sweep** (like the rotating motors)
 - **White noise**
 - Random motions
 - Natural
 - Simulated
 - Spectrum compatible
 - Empirical
- Hydraulically driven actuators (forced vibration) – sinusoidal or random loading, and pseudo dynamic effects
- Hybrid systems

Impact**Falling weights**

$$V_0 = \sqrt{2gh} \quad \dot{X} = \frac{W}{g} \cdot \frac{V_0}{m}$$



(controlled impulse transfer)

Swinging weights

(controlled impulse transfer)

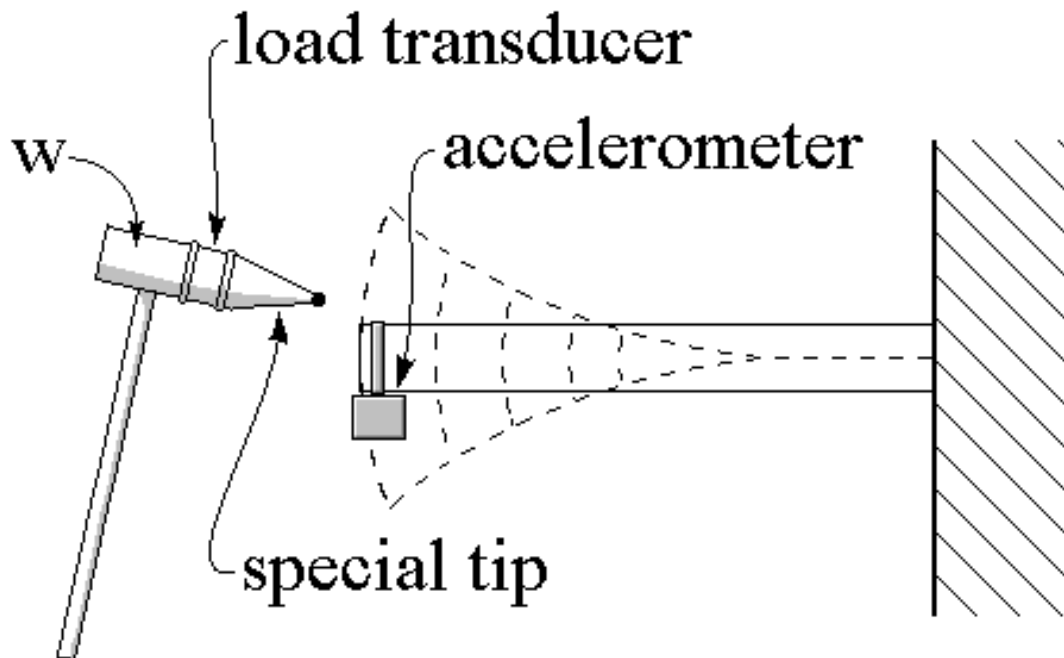
$$V_0 = \sqrt{2gh} \quad \dot{X} = \frac{W \cdot V_0}{g \cdot m}$$

This can produce local damage from plastic impulse transfer (on the same principle as a wrecking ball).

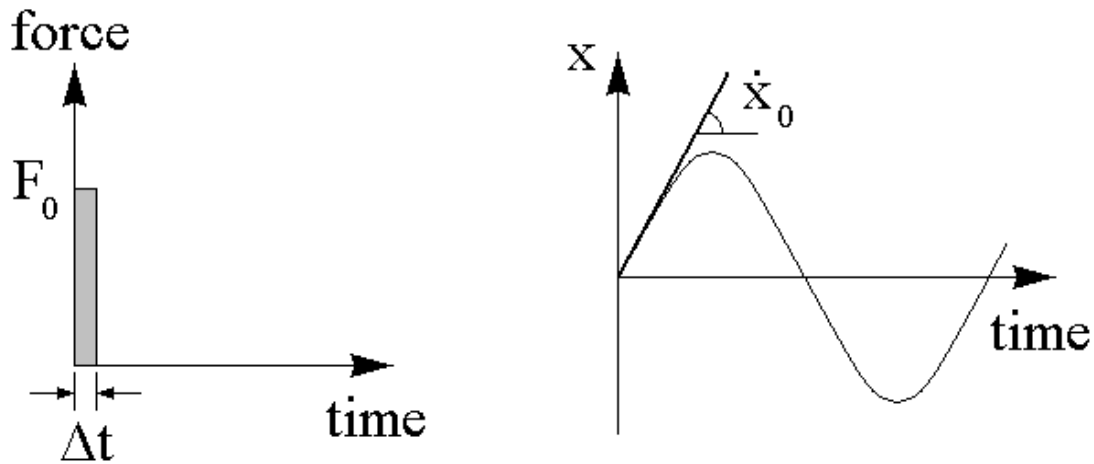
These devices can be used for strength qualifications.

'Impact hammers'

- Produce vibration effects without affecting strength.
- The vibrations are usually measured using accelerometers or velocity meters
- Built to measure the load (impact) history.



e.g. $m\ddot{x}+kx=0$ (an undamped free vibration system)



measured with a displacement transducer...

$$\text{Impulse} = \int F dt = m \Delta v$$

$$\Rightarrow F_0 \Delta t = m \dot{x}_0$$

$$\dot{x}_0 = \frac{F_0 \Delta t}{m}$$

$$x(t) \rightarrow x(\omega) \rightarrow \ddot{x}(\omega) \rightarrow \dot{x}(\omega)$$

$x(\omega)$ = frequency domain response function

In the frequency domain:

$$X(\omega) = H^{-1}(\omega) F(\omega) \quad H(\omega) F(\omega) = X(\omega)$$

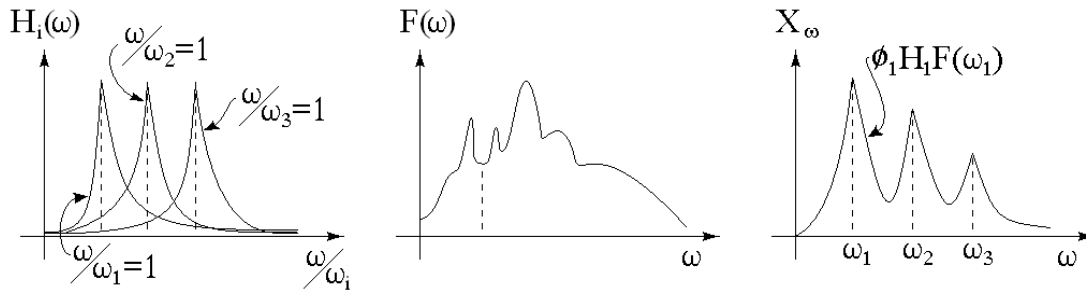
where $H^{-1}(\omega)$ is the frequency complex transverse function:

$$H(\omega) = \frac{\frac{\omega_0^2}{m}}{\left\{ \left[1 - \left(\frac{\omega}{\omega_0} \right)^2 \right] + \left[2\xi_i \frac{\omega}{\omega_0} \right] \right\}}$$

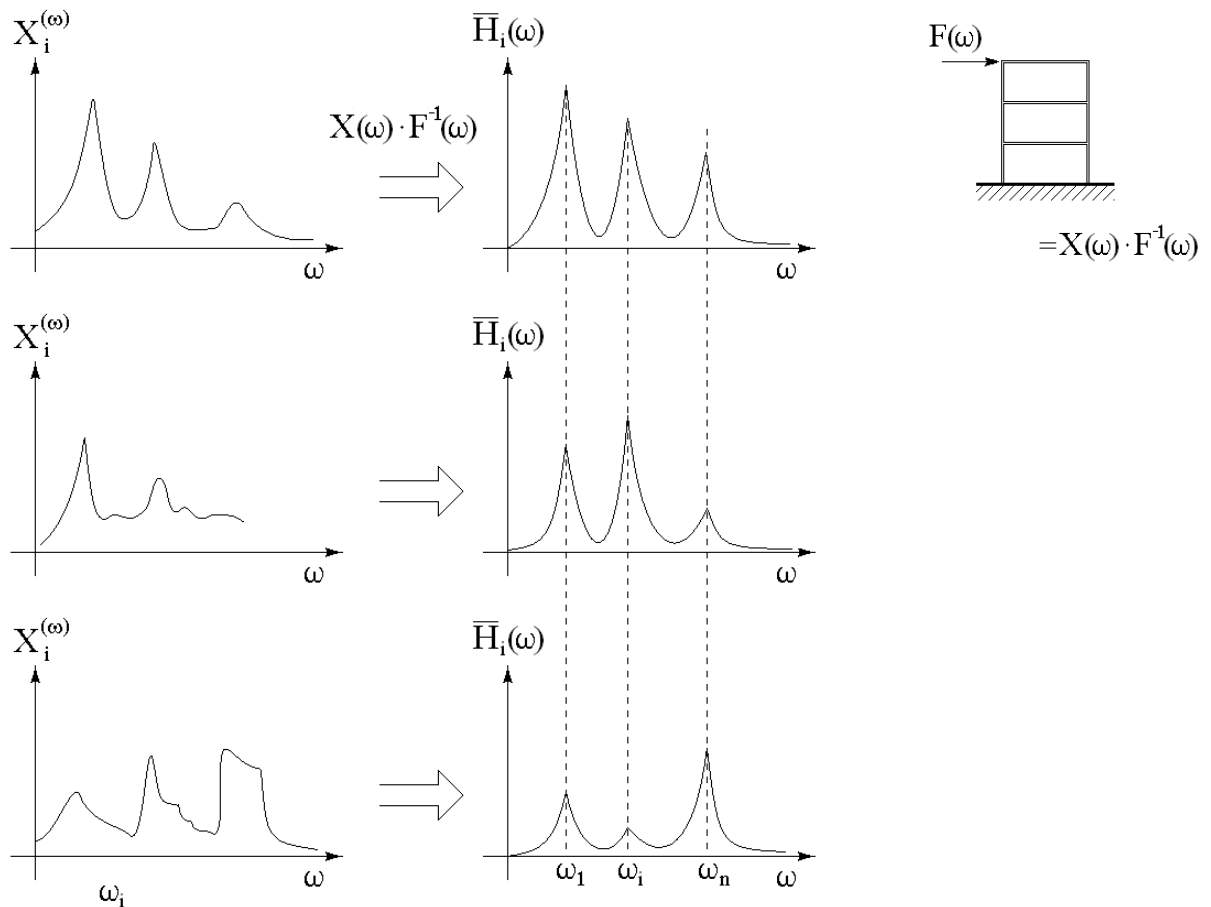
This function has information about frequencies.

If the structures have multiple degrees of freedom (N):

$$X(\omega) = \sum_1^N H_i(\omega) \cdot F(\omega) \cdot \phi_i(\omega_i) \Rightarrow$$



Modal identification with single impact source:



$$\bar{H}_n(\omega) = \sum_j H_j(\omega) \cdot \Gamma_j \cdot \varphi_{nj}(\omega_j)$$

for well separated peaks:

$$\bar{H}_n(\omega_j) = H_j(\omega_j) \cdot \Gamma_j \cdot \varphi_{nj}(\omega_j)$$

by studying $H_j(\omega_j)$; determine **damping ratio** using peak value, or half power band width.

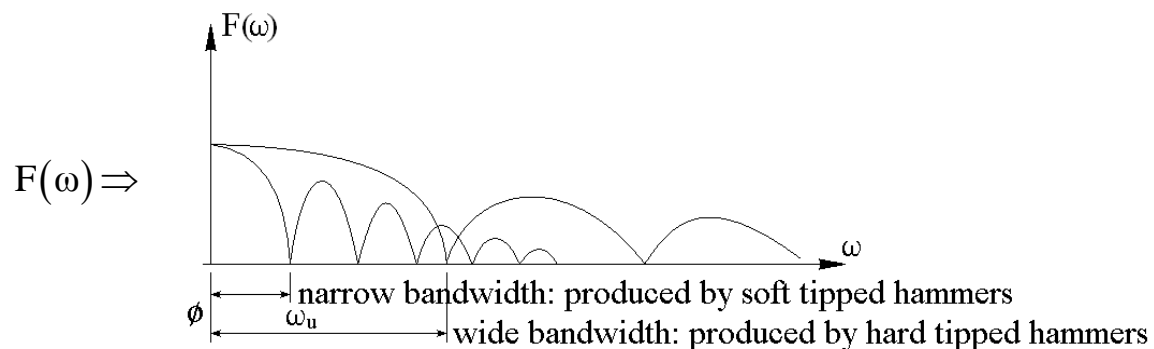
$$\text{Divide } \frac{\bar{H}_n(\omega_j)}{\bar{H}_1(\omega_j)} \Rightarrow \frac{\varphi_{nj}(\omega_j)}{\varphi_{1j}(\omega_j)} = \bar{\varphi}_{nj} \text{ (normalized)}$$

The impact hammer produces $F(\omega)$ and the accelerometer produces $X(\omega)$.

Structure identification: determine $H(\omega)$ and all dynamic characteristics...

$$H(\omega) = X(\omega) \cdot F^{-1}(\omega)$$

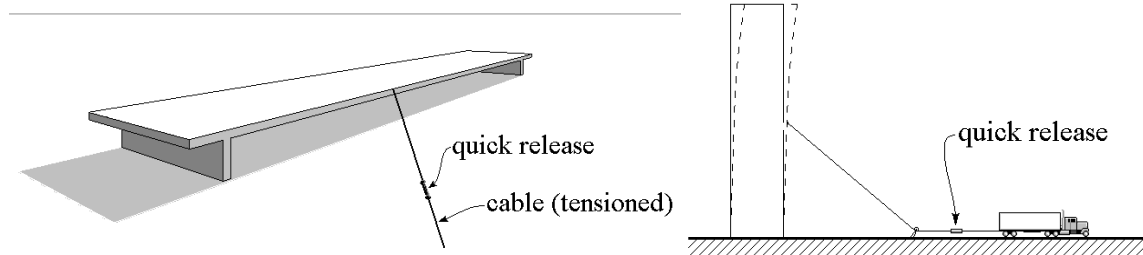
Usually



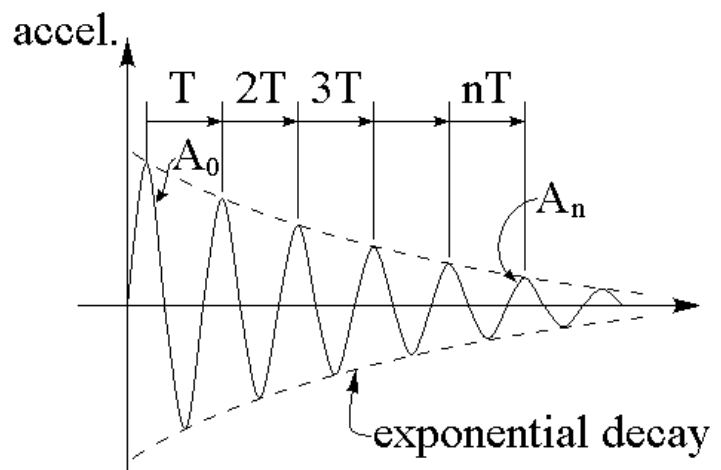
(Use hard tips for multimode identification)

(Use soft tips for high-resolution identification of lower modes)

a. Snap-back (cable) tests



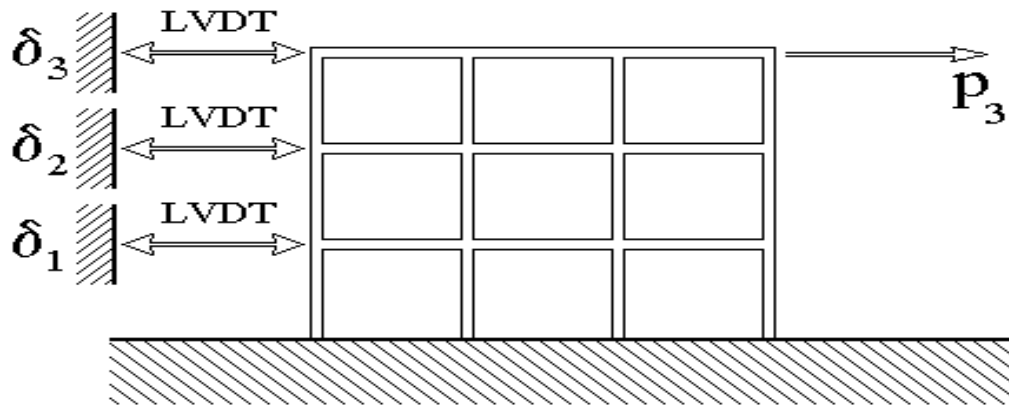
- The loading produces an initial static deformation at the loading level.
- Once ‘snapped’, free vibrations are produced in the structure, primarily in the lower modes.
- Loading can be applied symmetrically or asymmetrically (for torsional modes), based on the attachment location.
- Record time domain response with accelerometers
- Determine low (first) mode damping using the logarithmic displacement method:



$$\xi = \frac{1}{2n\pi} \ln \frac{A_n}{A_0}$$

- Record the transfer functions.

- The damping value is equivalent viscous damping created in response to a number of phenomena, such as cracking, slip, friction and the like...
- The damping value will therefore vary depending on the displacement amplitude.
- For snap back tests, displacement amplitudes should vary and an average equivalent viscous damping value should be determined.

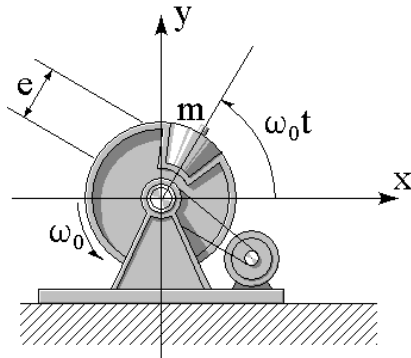


- During the static application of loading, deflections can be measured to find the structural stiffness matrix and inverted to find the structural stiffness matrix if the structure behaves as a shear building (i.e. with rigid floors). TEST ALSO NAMED “**PULL BACK**” TEST

$$f_{ij} = \begin{bmatrix} \delta_3/p_3 & \delta_2/p_3 & \delta_1/p_3 \\ \delta_3/p_2 & \delta_2/p_2 & \delta_1/p_2 \\ \delta_3/p_1 & \delta_2/p_1 & \delta_1/p_1 \end{bmatrix} \text{ in/kip}$$

$$k_{ij} = f_{ij}^{-1}$$

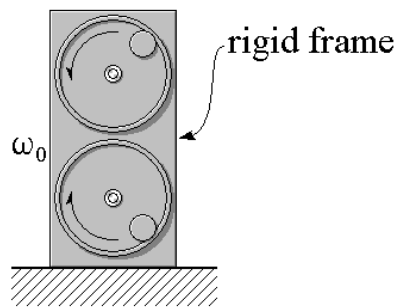
Rotating motors



Generated forces:

$$\rightarrow F_x = m e \omega_0^2 \cos \omega_0 t$$

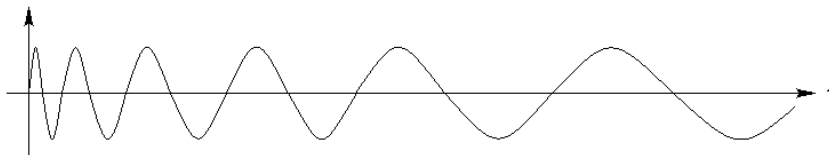
$$\downarrow F_y = m e \omega_0^2 \sin \omega_0 t$$



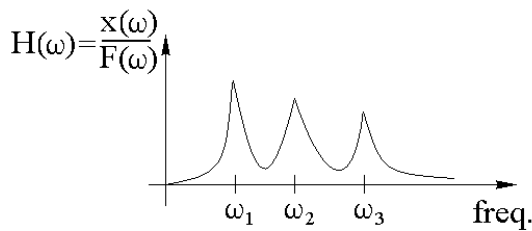
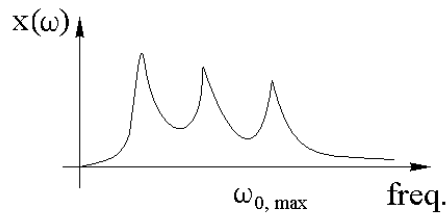
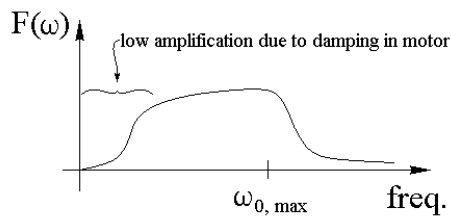
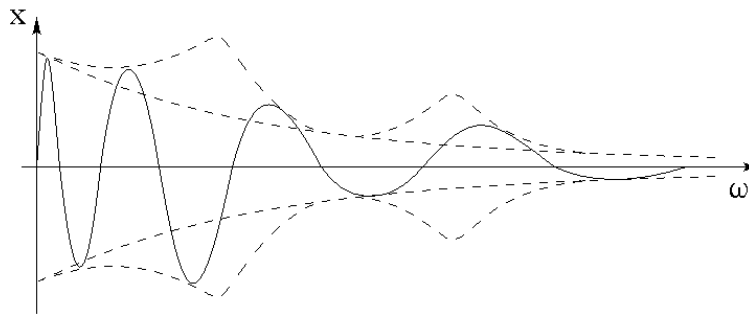
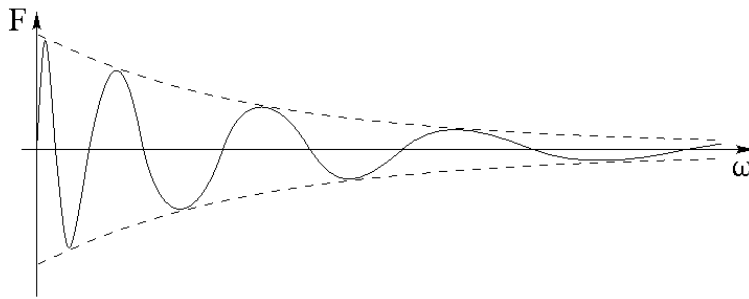
$$\rightarrow F_x = 2 m e \omega_0^2 \cos \omega_0 t$$

$$\downarrow F_y = 0$$

- Used in tall buildings, large bridges, dams, etc...
- Force depends on mass (m), and frequency of rotation ($\omega_{0\max}$) which is dependant on motor capacity.
- Produces a 'sine sweep history'.
 - bring motor to highest speed ($\omega_{0\max}$)
 - shut down motor excitation
 - motor slows down from $\omega_{0\max}$ to 0
 - motion produced is:



- the structural response will be:

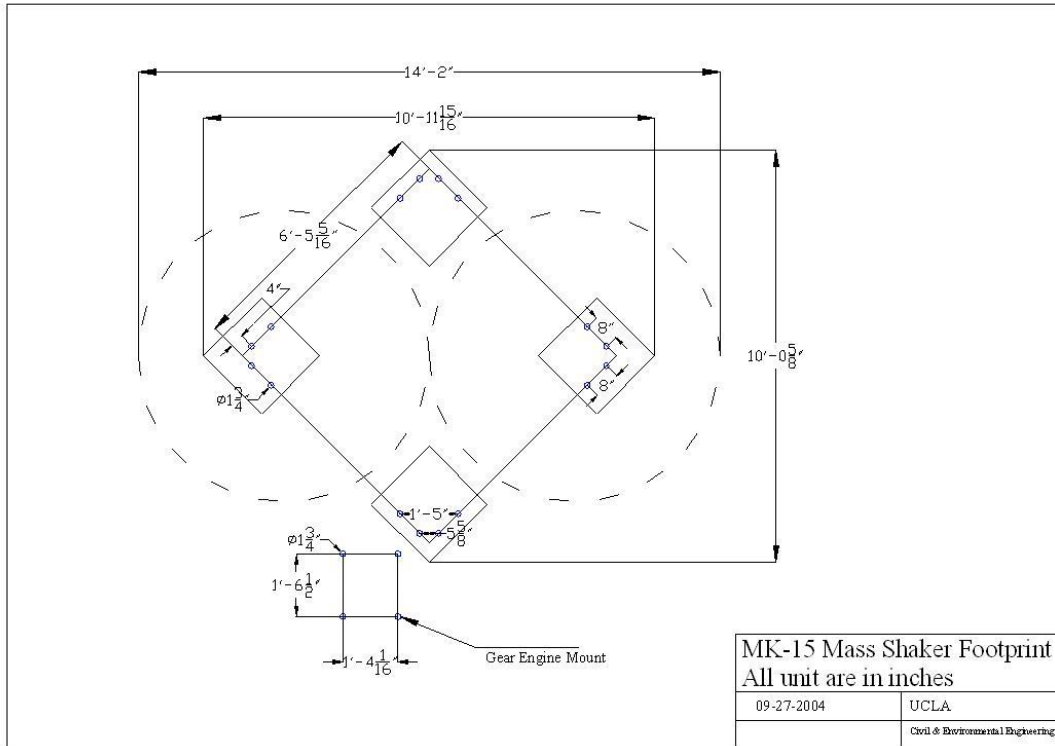


- The structural response is amplified at resonant frequencies between $\omega_{0\max}$ and 0.
- The resolution depends on the damping of the motor. For a low damping motor the resolution is better. The equipment needs to be good, well lubricated, and perfectly balanced and aligned.
- The measurements require time domain recording.
- It is suitable for large structures.

Shaker of NEES@UCLA



Rotational Shaker



Linear Shaker:



Sine Sweep (may use linear exciters or shake tables):

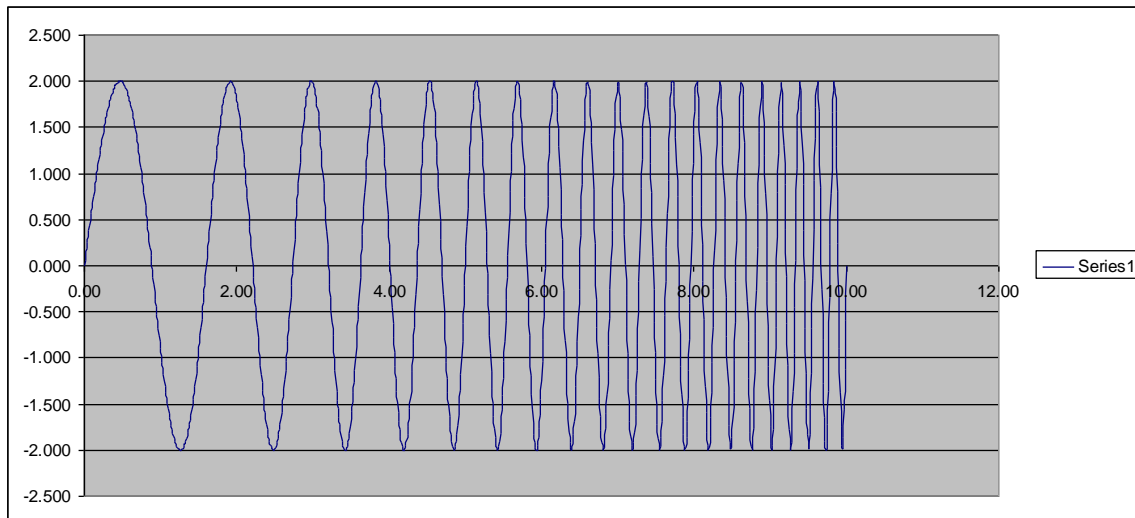
(a) First Formulation:

Based on octave: here double the frequency every t_o seconds

$$f(t_i) = f_o \cdot 2^{t_i/t_o}$$

$$x(t_i) = x_o \sin[2\pi t_i \cdot f(t_i)]$$

Example: for $t_o = 5$ sec, $f_o = 0.5$ Hz, $x_o = 2$ cm



Second formulation:

$$f(t_i) = f_o + (f_n - f_o) \cdot t_i / T_D$$

$$t_i = \Delta t \cdot (i - 1)$$

$$x(t) = x_o \sin[2\pi t_i \cdot f(t_i)]$$

$t(i) = dt \cdot (i - 1)$, $i = 0$ to $\text{Duration} / dt$

$\text{Frq}(i) = \text{IniFrq} + [(\text{FinFrq} - \text{IniFrq}) \cdot (t(i) / \text{Duration})]$

$\text{Ampl}(i) = \text{MaxAmpl} \cdot \text{sine}[2 \cdot \text{PI} \cdot \text{Frq}(i) \cdot t(i)]$

```
function u = isinesweep (dt,tend,delta,tdly,frq0,frq1,ampl)
// u = isinesweep (dt,tend,delta,tdly,frq0,frq1,ampl)
// Outputs:
//   u = sine sweep input signal
//
// Inputs:
//   dt = time step
//   tend = duration of signal in seconds
```

```
// delta = duration of sine sweep in seconds
// tdly = the start time of the sine sweep
// frq0 = the initial frequency (Hz)
// frq1 = the final frequency (Hz)
// ampl = amplitude of sine sweep
//

twopi = 2*4*atan(1);
frq0 = frq0 * twopi;
frq1 = frq1 * twopi;
nsw = ceil(delta/dt);
nst = ceil(tdly/dt);
dfrq = (frq1 - frq0)/delta;
tsw = dt*(0:nsw-1)';
frq = dfrq*tsw + frq0;
frs = ampl*sin(frq*tsw);

npts = ceil(tend/dt) - nsw - nst;
if (npts < 0)
    nsw=nsw +npts
end
u = [zeros(nst,1);frs;zeros(npts,1)];

end
```

Examples of Identification of properties using free and forced vibrations:

See reference: [Bracci et al. \(1992\) NCEER-Report-92-0027 – Section 4.0](#)

Special Topics

Identification of Damping

Half Power Bandwith

Logarithmic Decrement

Peak Transfer Function

Identification of Dynamic Characteristics

Identification of Structural Properties

Types of tests:

Impact Hammer Test

Snap Back Test

Pull Back Test

Shake Table – White noise Test.

Table 4-2 Dynamic Characteristics of the Unloaded Model

Test	f_i^u (Hz.)	Φ_{ij}^u	K_{ij}^u (kip/in)	k_i^u (kip/in)	ξ_i^u (%)	C_{ij}^u
HAMMER	$\begin{pmatrix} 3.40 \\ 11.00 \\ 17.60 \end{pmatrix}$	$\begin{pmatrix} 1.00 & -0.67 & -0.62 \\ 0.82 & 0.18 & 1.00 \\ 0.47 & 1.00 & -0.63 \end{pmatrix}$	$\begin{pmatrix} 70.1 & -72.1 & 10.3 \\ -72.1 & 115.5 & -59.2 \\ 10.3 & -59.2 & 97.3 \end{pmatrix}$	$\begin{pmatrix} 72.1 \\ 59.2 \\ 38.1 \end{pmatrix}$	$\begin{pmatrix} 2.7 \\ 1.5 \\ 1.0 \end{pmatrix}$	$\begin{pmatrix} 0.028 & -0.007 & -0.003 \\ -0.007 & 0.028 & -0.005 \\ -0.003 & -0.005 & 0.033 \end{pmatrix}$
STAAD (0.565 EI _y)	$\begin{pmatrix} 3.70 \\ 10.81 \\ 16.50 \end{pmatrix}$	$\begin{pmatrix} 1.00 & -0.81 & -0.43 \\ 0.78 & 0.53 & 1.00 \\ 0.40 & 1.00 & -0.88 \end{pmatrix}$	$\begin{pmatrix} 46.7 & -50.9 & 4.4 \\ -50.9 & 102.7 & -56.2 \\ 4.4 & -56.2 & 108.2 \end{pmatrix}$	$\begin{pmatrix} 50.9 \\ 56.2 \\ 52.0 \end{pmatrix}$	-	-







Table 4-3 Dynamic Characteristics of the Loaded Model

Test	f_i (Hz.)	Φ_{ij}	K_{ij} (kip/in)	k_i (kip/in)	ξ_i (%)	C_{ij}
PULL	$\begin{pmatrix} 1.76 \\ 5.34 \\ 8.15 \end{pmatrix}$	$\begin{pmatrix} 1.00 & -0.82 & -0.41 \\ 0.76 & 0.55 & 1.00 \\ 0.40 & 1.00 & -0.88 \end{pmatrix}$	$\begin{pmatrix} 47.4 & -52.7 & 3.2 \\ -52.7 & 109.3 & -59.8 \\ 3.2 & -59.8 & 113.9 \end{pmatrix}$	$\begin{pmatrix} 52.7 \\ 59.8 \\ 54.1 \end{pmatrix}$	-	-
SNAP	$\begin{pmatrix} 1.86 \\ 5.66 \\ 8.40 \end{pmatrix}$	-	-	-	$\begin{pmatrix} 2.5 \\ 4.8 \\ 4.0 \end{pmatrix}$	-
WHN_B (Eq.(4.20))	$\begin{pmatrix} 1.78 \\ 5.32 \\ 7.89 \end{pmatrix}$	$\begin{pmatrix} 1.00 & -0.82 & -0.46 \\ 0.80 & 0.46 & 1.00 \\ 0.42 & 1.00 & -0.83 \end{pmatrix}$	$\begin{pmatrix} 51.9 & -53.4 & 2.5 \\ -53.4 & 102.4 & -54.4 \\ 2.5 & -54.4 & 104.7 \end{pmatrix}$	$\begin{pmatrix} 53.4 \\ 54.4 \\ 50.3 \end{pmatrix}$	$\begin{pmatrix} 2.0 \\ 2.4 \\ 2.0 \end{pmatrix}$	$\begin{pmatrix} 0.072 & -0.042 & -0.012 \\ -0.042 & 0.097 & -0.029 \\ -0.012 & -0.029 & 0.112 \end{pmatrix}$
WHN_B (Eq.(4.19))	-	-	-	-	$\begin{pmatrix} 1.7 \\ 1.6 \\ 1.4 \end{pmatrix}$	-
STAAD (0.565 EI _y)	$\begin{pmatrix} 1.78 \\ 5.20 \\ 7.94 \end{pmatrix}$	$\begin{pmatrix} 1.00 & -0.81 & -0.43 \\ 0.78 & 0.53 & 1.00 \\ 0.40 & 1.00 & -0.88 \end{pmatrix}$	$\begin{pmatrix} 46.7 & -50.9 & 4.4 \\ -50.9 & 102.8 & -56.3 \\ 4.4 & -56.3 & 108.2 \end{pmatrix}$	$\begin{pmatrix} 50.9 \\ 56.3 \\ 51.9 \end{pmatrix}$	-	-

Testing Procedures - Standard

Static

Dynamic

Name ▲	Date Modified	Size	Type
 1-STATIC	10/12/2010 10:2...		File Folder
 2-DYNAMIC-FREQUENCY SEARCH IDENT...	10/12/2010 10:2...		File Folder
 3-DYNAMIC PERFORMANCE	6/18/2010 2:56 PM		File Folder
 99-DOCUMENTS	10/12/2010 10:2...		File Folder
 DYNAMIC-FREQUENCY SEARCH IDENTIF...	6/1/2010 3:25 PM	1,815 KB	IZArc RAR Archive
 STATIC TESTING.rar	6/1/2010 3:25 PM	4,014 KB	IZArc RAR Archive

Suggested Test Protocols

Static

Name ^	Date Modified	Size	Type
ASTM - Misc	8/29/2010 10:43 PM		File Folder
ATC58	10/20/2010 1:06 ...		File Folder
CUREE-SAC-STEEL-WOOD	10/20/2010 1:06 ...		File Folder
QUASI-Static	8/29/2010 10:43 PM		File Folder

Dynamic

Name ^	Date Modified	Size	Type
AC156	8/29/2010 10:43 PM		File Folder
ASME-Nuclear,	10/20/2010 1:10 ...		File Folder
ATC58	10/20/2010 1:10 ...		File Folder
IBC-Spectra	8/29/2010 10:43 PM		File Folder
IEC-Testing-Protocols	8/29/2010 10:43 PM		File Folder
IEEE-693	8/29/2010 10:43 PM		File Folder
MIL-STD-167-1 (1974)-Shipboard Equipm...	8/29/2010 10:43 PM		File Folder
NISTIR-Base-Isolation	8/29/2010 10:43 PM		File Folder
Telcordia-Bellcore	8/29/2010 10:43 PM		File Folder

Suggested lab exercise:

Part 1: Impact Hammer

- a. Use a small impact hammer with a larger model (from lab #1).
- b. Use a soft tip and determine transfer function (identify frequencies and mode shapes).
- c. Use a hard tip and repeat b.
- d. Record motion at first floor
- e. Identify the frequencies of structure (as many as you can).

Part 2: Perform a sine sweep test using 10 seconds per octave from 0.5 Hz up to 16 Hz in acceleration (.05g) control using the shake table:

- b. Record response of structure at top and at first floor
- c. Identify frequencies from time domain trace
- d. Calculate the FFT of the input and output
- e. Identify as many frequencies as you can

Part 3: Perform a white noise test using 0.05 rms for 30 seconds in displacement control

- f. Record response at first and top floor
- g. Calculate transfer functions
- h. Determine as many frequencies as you can.