Abdelhalim AIROUCHE, Hassan AKNOUCHE, Andrei Reinhorn

The information in this notes is based on the references listed at the end of this SECTION

1. Introduction

The servovalve shown in figure (1) provides the final control element in a closed loop servo-hydraulic system. The servovalve ports the fluid provided by the hydraulic power system, into the appropriate side of the actuator's chambers. This causes the actuator's piston to move the actuator's arm in the desired direction

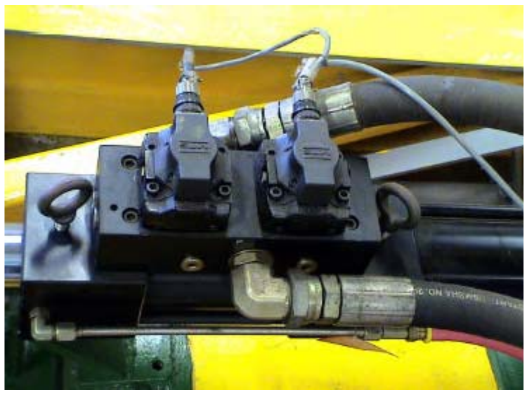


Figure (1): Servovalve

In figure (2) it is schematically represented the functioning of the servovalve. The servovalve command υ_c is sent to the torque motor armature that controls the rotation of the pilot flapper. The rotation of the pilot flapper generates a differential pressure in the pilot stage ΔP_p that controls the flow of hydraulic fluid in the second stage. The flow of hydraulic fluid in the second stage controls the position of the second stage spool that in turn controls the flow of hydraulic fluid in the third stage and the position of the third stage spool x_{3s}. The position of the third stage spool x_{3s} controls the flow of the high pressure hydraulic fluid in the actuator load chamber q_s. A mathematical model for this system will be derived in the following paragraph.

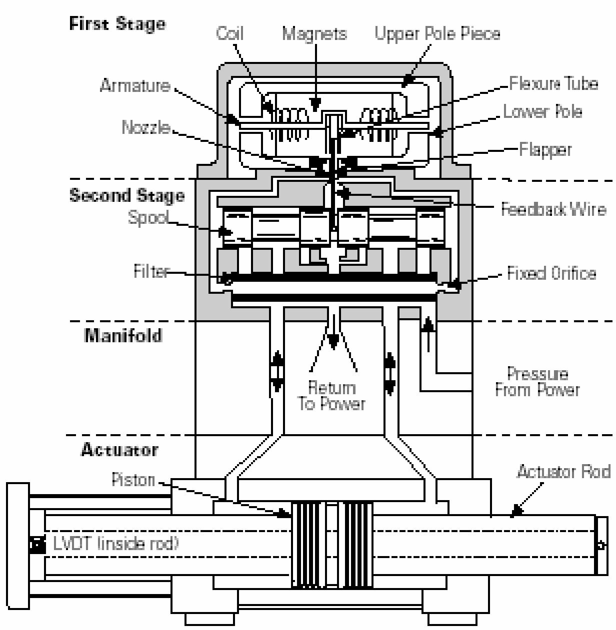


Figure (2): Cross Section of Three Stage Servovalve

2. Three stage servovalve transfer function (adapted from Conte et al.)

The servovalve signal command before it is sent to the first stage is processed by the controller through the so called “Inner loop”. The inner loop uses as input the so-called “Inner loop” $e_i(t)$ error signal that is obtained subtracting the magnitude of the inner loop feedback signal $x_{3s}(t)$ from the servo valve command signal $\upsilon_c(t)$, as described by the equation.

$$\boxed{e_i(t) = \upsilon_c(t) - x_{3sa}(t)} \quad (1)$$

In Laplace notation:

$$\boxed{e_i(s) = \upsilon_c(s) - x_{3sa}(s)} \quad (2)$$

Where:

$e_i(t)$ Inner loop error signal,
$\upsilon_c(t)$ Servo valve command signal,
$x_{3sa}(t)$ Conditioning feedback signal.

The relationship between the electrical signal representing the position of the third stage spool $x_{3s}(t)$ and the feedback signal $x_{3sa}(t)$ is given by

$$\boxed{x_{3sa}(t) = k_{df}\, x_{3s}(t)} \tag{3}$$

where k_{df} is the Displacement feedback gain

In Laplace transform:

$$\boxed{x_{3sa}(s) = k_{df}\, x_{3s}(s)} \tag{4}$$

Substituting (4) in equation (2), we obtain a new expression for the inner loop error signal:

$$\boxed{e_i(s) = \upsilon_c(s) - k_{df}\, x_{3s}(s)} \tag{5}$$

The output signal of the inner loop, $\upsilon_{ci}(t)$, is obtained through the sum of two contribution as expressed in the following equation

$$\boxed{\upsilon_{ci}(t) = \varepsilon_{pi}(e_i(t)) + \varepsilon_{Di}(\dot{e}_i(t))} \tag{6}$$

where $\varepsilon_{pi}(e_i(t)); \varepsilon_{Di}(e_i(t))$ are respectively the inner loop proportional gain electrical component and derivative gain electrical component.

Or, in Laplace notation:

$$\boxed{\upsilon_{ci}(s) = \varepsilon_{pi}(e_i(s)) + \varepsilon_{Di} s(e_i(s)) = (\varepsilon_{pi} + s\varepsilon_{Di})(e_i(s))} \tag{7}$$

The inner loop error signal is processed by the PD controller, a proportional, derivative controller has a transfer function of the form:

$$\boxed{PD(s) = \frac{\upsilon_{ci}(s)}{e_i(s)} = k^i_{pro} + s k^i_{der}} \tag{8}$$

in which:

$e(s)$ inner loop error signal,
$\upsilon_{ci}(s)$ output signal, or PD adjusted error signal,
k^i_{pro} proportional gain,
k^i_{der} derivative gain.

Substituting the expression (notation) of the transfer function Eq(8), and the expression of the inner loop error signal Eq(5) into the Eq(7) of the output signal of the inner loop, we obtain:

$$\boxed{\upsilon_{ci}(s) = (k^i_{pro} + sk^i_{der})(\upsilon_c(s) - A_i(s)x_{3s}(s))} \quad (9)$$

Where $A_i(s)$ is the Laplace transform of k_{df} in Eq(5)

The relationship between the conditioned servovalve command, $\upsilon_{ci}(t)$, and the pilot stage differential pressure can be assumed to be linear and expressed as follows:

$$\boxed{\Delta P_p(t) = k_1\upsilon_{ci}(t)} \quad (10)$$

ΔP_p is the differential pressure in the pilot stage,

k_1 is the flapper gain,

Also, it can be assumed a linear relationship between the pressure drops induced across the pilot stage and the displacement of the main stage spool. This relation can be expressed as follows:

$$\boxed{x_{3s}(t) = k_2\Delta P_P(t)} \quad (11)$$

where :

x_{3s} is the third stage spool displacement,

k_2 is the second stage gain factor,

ΔP_p is the pressure drop across pilot stage spool.

Combining equation (11) and (10), we obtain the relationship between the conditioned servovalve command and the piston of the third stage spool.

$$\boxed{x_{3s}(t) = k_2k_1\upsilon_{ci}(t)} \quad (12)$$

or in Laplace notation

$$\boxed{x_{3s}(s) = k_2k_1\upsilon_{ci}(s)} \quad (13)$$

Assuming a linear relationship between the fluid flow to the actuator and the third stage spool displacement for small valve opening, the fluid flow from the main stage to the actuator can be expressed as

$$\boxed{q_s(t) = k_{xm}x_{3s}(t)} \quad (14)$$

or in Laplace notation : $q_s(s) = k_{xm}x_{3s}(s)$ (15)

k_{xm} flow-gain coefficient,

q_s hydraulic high pressure fluid flow.

By substituting of equation (13) in equation (15), it can be obtain the relation between the servovalve conditioned command and the servovalve flow to the actuator

$$\boxed{q_s(t) = k_{xm}k_1k_2\upsilon_{ci}(t)} \tag{16}$$

or in Laplace notation:

$$\boxed{q_s(s) = k_{xm}k_1k_2\upsilon_{ci}(s)} \tag{17}$$

By substituting of Eq(13) in Eq(9), we obtain:

$$\boxed{\upsilon_{ci}(s) = (k^i_{pro} + sk^i_{der})(\upsilon_c(s) - k_{df}(k_1k_2\upsilon_{ci}(s))} \tag{18}$$

By rearranging Eq(18), we obtain:

$$\boxed{\upsilon_{ci}(s) = \frac{(k^i_{pro} + sk^i_{der})}{1 + k_{df}k_1k_2(k^i_{pro} + sk^i_{der})}\upsilon_c(s)} \tag{19}$$

From the servo valve flow Eq(17), we obtain :

$$\boxed{q_s(s) = k_{xm}\frac{k_1k_2(k^i_{pro} + sk^i_{der})}{1 + k_{df}k_1k_2(k^i_{pro} + sk^i_{der})}\upsilon_c(s)} \tag{20}$$

Eq (20) can be expressed as follows

$$\boxed{q_s(s) = H(s)\upsilon_c(s)} \tag{21}$$

where : $H(s)$ is the three stage servovalve transfer function.

In improvement of the three stages servovalve model can be obtained improving the relationship between the differential pressure and the position of the main stage spool of the third spool.

$$\boxed{x_{3s}(t) = k_2\Delta P_P(t)} \tag{22}$$

Introducing a time offset τ between the moment in which a differential pressure occurs and the moment in which the third stage spool moves.

$$\boxed{x_{3s}(t) = k_2\Delta P_P(t-\tau)} \tag{23}$$

$$\boxed{\Delta P_p(t-\tau) = k_1\upsilon_{ci}(t-\tau)} \tag{24}$$

$$\boxed{x_{3s}(t) = k_1 k_2 \upsilon_{ci}(t-\tau)} \tag{25}$$

or in Laplace notation :

$$\boxed{x_{3s}(s) = k_1 k_2 e^{-s\tau} \upsilon_{ci}(s)} \tag{26}$$

By substituting the new expression for the third stage spool position given by Eq(26) in the equation of the fluid flow from the servovalve Eq(17). It can be obtained the following expression that gives the relationship between the servovalve conditioned command and the servovalve flow to the actuator:

$$q_s(t) = k_{xm} k_1 k_2 \upsilon_{ci}(s) e^{-s\tau} \tag{27}$$

$$q_s(s) = k_{xm} k_1 k_2 \frac{(k^i_{pro} + sk^i_{der})}{1 + k_{df} k_1 k_2 (k^i_{pro} + sk^i_{der})} \upsilon_c(s) e^{-s\tau} \tag{28}$$

$$H(s) = k_{xm} \frac{k_1 k_2 (k^i_{pro} + k^i_{der})}{1 + k_{df} k_1 k_2 (k^i_{pro} + sk^i_{der})} e^{-s\tau} \tag{29}$$

In the case when the three stage servovalve transfer function is considered to be linear ($k_{df} = 0; k^i_{pro} = 1$ and $k^i_{der} = 0$). The presence of a time delay” τ ” in the third stage spool motion leads to the following expression of the transfer function

$$\boxed{H(s) = k_{xm} k_1 k_2 e^{-s\tau} = k_t e^{-s\tau}} \tag{30}$$

where

k_t is the three stage servovalve transfer function

3. The oil flow in the actuator

The differential equation governing the actuator piston can be expressed as follows.

$$\boxed{q_s(t) = A\dot{x}_t(t) + \frac{V}{4\beta A}\dot{F}_a(t) + k_{le}F_a(t)} \qquad (31)$$

where :

- $\dot{x}_t(t)$ Velocity of the table;
- A The effective piston area.
- $F_a(t)$ Force into the actuator.
- V Volume of the actuator load chamber;
- β Is the bulk modulus of the oil;
- $\dot{F}_a(t)$ Is the time derivative of the force in the actuator.
- Kle Is the proportionality factor for leakage (in seals, etc)

The above equation indicates that the flow of high pressure hydraulic provided by the servovalve can be decomposed in three components: a flow generated by the displacement of the piston, a flow induced by the compression of the oil in the actuator chamber, and a flow due to leakage in the actuator.

In Laplace transform Eq (31) can be expressed as follows:

$$\boxed{q_s(s) = H(s)\,x_c(s) = sAx_t(s) + k_{le}F_a(s) + s\frac{V}{4\beta A}F_a(s)} \qquad (32)$$

The actuator force-mass acceleration relationship, neglecting the damping effects, is expressed by

$$\boxed{F_a(t) = m\ddot{x}_t(t)} \qquad (33)$$

In Laplace notation

$$\boxed{F_a(s) = s^2 m\ddot{x}_t(s)} \qquad (34)$$

Where : m Coupled mass including the actuator piston mass

Substituting equation (34) in equation (32) we can finally obtain an expression that links the servovalve command to the actuator displacement.

$$\boxed{H(s)\upsilon_c(s) = sAx_t(s) + k_{le}s^2 mx_t(s) + s\frac{V}{4\beta A}s^2 mx_t(s)} \qquad (35)$$

Eq (35) provides an expression that gives a relation between the servovalve command $\upsilon_c(s)$ and the actuator displacement x_t and the force in the actuator F_a.

From Eq(35) it can be derived the following expression for the actuator displacement.

$$x_t(s) = G(s)\upsilon_c(s) = \frac{H(s)}{s^3(\frac{Vm}{4\beta A}) + s^2 mk_{le} + sA}\upsilon_c(s) \tag{36}$$

where: $G(s)$ The transfer function of the servo-hydraulic system.

The amplitude and phase of the transfer function may be obtained by substituting ($i\omega$) for (s) in the above equation and separating it into real and imaginary part

Often, it is more convenient to express the transfer function in terms of its open loop parameters, such as the open loop frequency and open loop damping, this leads to simplifications of the above transfer function expressions. The open loop transfer function G_{Open} is obtained by setting the feedback parameters, $k_{df} = 0$, $k^i_{pro} = 1$ and $k^i_{der} = 0$ in Eq(36) to get

$$G_{Open}(s) = (\frac{k_t}{A})\frac{1}{s\left[s^2(\frac{Vm}{4\beta A^2}) + s\frac{mk_{le}}{A} + 1\right]} \tag{37}$$

where

$$H(s) = k_1 k_2 k_{xm} = k_t \tag{38}$$

This may be expressed in a more familiar form as:

$$G(s) = \frac{1}{s\left[s^2(1/\omega_0^2)s^2 + (2\xi_0/\omega_0)s + 1\right]} \tag{39}$$

where :

$$\omega_0^2 = \frac{4\beta A^2}{mV}$$

$$\xi_0 = \frac{mk_{le}\omega_0}{2A}$$

$$k_0 = \frac{k_t}{A}$$

ω_0, ξ_0 amd k_0 are the open loop frequency, damping and gain respectively. As is clear from the above expression, the open loop frequency is inversely proportional to the square root of the mass, and the open loop damping is directly proportional to the square root of the mass.

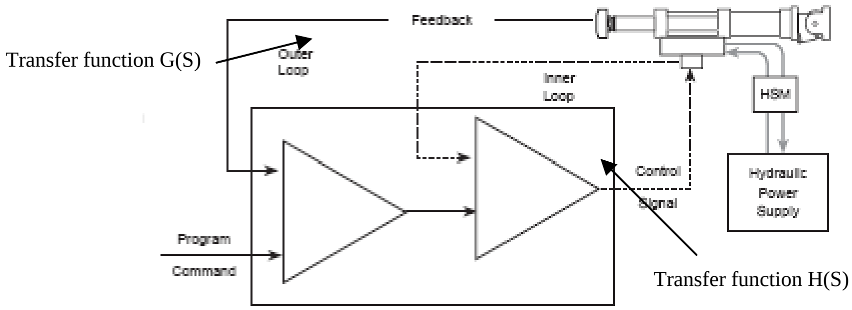


Figure (3): Basic closed loop-control

In time domain the differential equation of the system relating the command signal to the output piston displacement can be written as follows.

$$\left(\frac{Vm}{4\beta A}\right)\frac{d^3x}{dt} + mk_{le}\frac{d^2x}{dt} + A\frac{dx}{dt} = k_t\upsilon_c(t) \tag{40}$$

which takes the form

$$a_3\frac{d^3x}{dt} + a_2\frac{d^2x}{dt} + a_1\frac{dx}{dt} = b_1\upsilon_c(t) \tag{41}$$

This equation is a third order linear differential equation with constant coefficient, and can be solved numerically by writing it in the state equation form

$$\dot{X} = AX + F \tag{42}$$

Or

$$\frac{d}{dt}\begin{Bmatrix} x_t \\ \dot{x}_t \\ \ddot{x}_t \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\frac{a_1}{a_3} & -\frac{a_2}{a_3} \end{bmatrix}\begin{Bmatrix} x_t \\ \dot{x}_t \\ \ddot{x}_t \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ b_0/a_3 \end{Bmatrix}\upsilon_c \tag{43}$$

where $X = \begin{bmatrix} x_t & \dot{x}_t & \ddot{x}_t \end{bmatrix}^T$, A and F are the square matrix and vector in Eq(43).

4. Controller Model

The servovalve command can be written as the sum of three components, as shown in Figure(4).

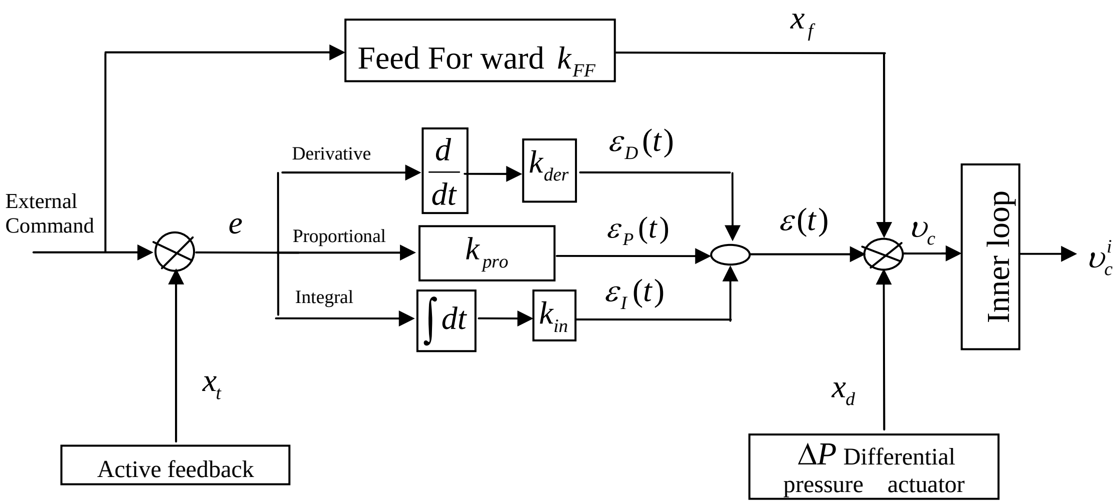


Figure (4) : Block diagram of servovalve with PIDF controller and conditioned displacement and feedback

$$x_c(t) = \varepsilon(t) + x_f(t) + x_d(t) \tag{44}$$

$\varepsilon(t)$ Is the component due to the PID gains;

$x_f(t)$ Is the component due to the feed forward gain;

$x_d(t)$ Is the component due to the Delta pressure gain

4.1 PID Gain Component; $\varepsilon(t)$

The command signal $\overline{\upsilon}_c$, minus the feedback signal results in an error signal, e, known as the DC error. This error is processed by the PID controller as shown in Figure (5). The output signal of the PID gain $\varepsilon(t)$ is obtained through the sun of three contributions. A proportional, integral, derivative (PID) controller has a transfer function of the form.

$$H_{PID}(s) = \frac{\varepsilon(s)}{e(s)} = k_{pro} + \frac{1}{s}k_{\text{int}} + sk_{der} \quad (45)$$

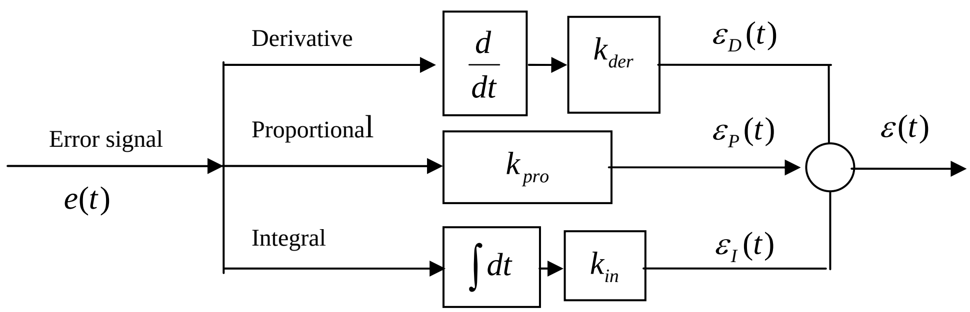


Figure (5): Block diagram of PID Controller

4.2 Feed forward gain component, $x_f(t)$

The feedforward transfer function has the form

$$H_{FF}(t) = \frac{x_F(s)}{\overline{\upsilon}_C(s)} = sk_{FF} \quad (46)$$

The derivative of the command signal is multiplied by the feedforward gain k_{FF}, and added directly to the servovalve command signal

4.3 Delta pressure gain component, $x_d(t)$

This differential pressure uses as input the differential pressure $\Delta P(t)$ that exists across the actuator. The differential pressure is converted to electrical signal by delta-P-cell mounted on the actuator. The electrical signal provided by the Delta pressure cell is then multiplied via the DC conditioner by the constant k_{dp} in order to obtain the differential pressure component $x_d(t)$ of the controller command as follows:

$$\boxed{x_d(t) = \frac{s^2 k_{dp} m_t}{A} x_t(s) = H_{Dp} x_t(s)} \qquad (47)$$

where H_{Dp} is the delta pressure transfer function

Finally the relationship between the command signal $\overline{\upsilon}_c(t)$ and the adjusted command signal due to PIDF adjustment and delta-P feedback is :

$$\boxed{\upsilon_c(s) = H_{PID}(s)\left[\overline{\upsilon}_c(s) - x_t(s)\right] + H_{FF}(s)\overline{\upsilon}_c(s) + H_{Dp}(s)x_t(s)} \qquad (48)$$

Combining Eq (35) and Eq (48), the transfer function between external command and the piston displacement can be written as follows:

$$\boxed{H(s) = \frac{G(s)\left[s.k_f + k_{pro} + \frac{1}{s}k_{\text{int}} + sk_{der}\right]}{1 + G(s)\left[k_{pro} + \frac{1}{s}k_{\text{int}} + sk_{der} - (k_{dp}\frac{s^2 m_t}{A})\right]}} \qquad (49)$$

The effect of the different factor gain defined above on the stability and the response of the servo-control loop is discussed on the MTS manual titled “Model 793.00 System Software” and the following discussion is done by MTS **FlexTest** GT Software.

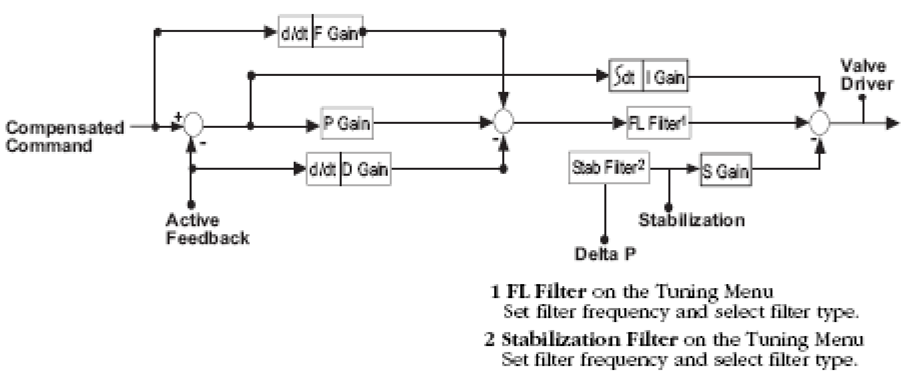


Figure (6): PIDF controller used in MTS Flextest Software

Flextest software uses a group of gain controls proportional, integral, derivative and feedforward gain. The PIDF controller uses also stabilization gain and an adjustable

Forward loop filter, as shown in Figure (6). Each control mode has different tuning characteristics.

The purpose of a tuning program is to improve the performances of the system by reducing error and phase lag as illustrates in a Figure (7)

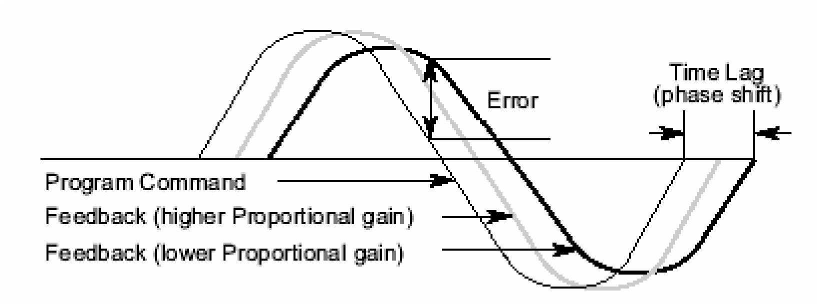
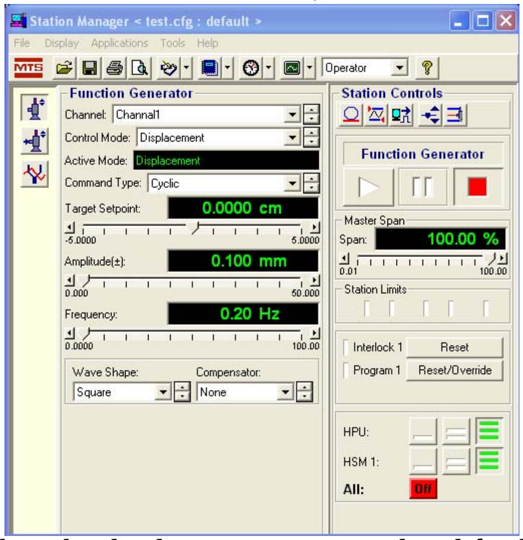


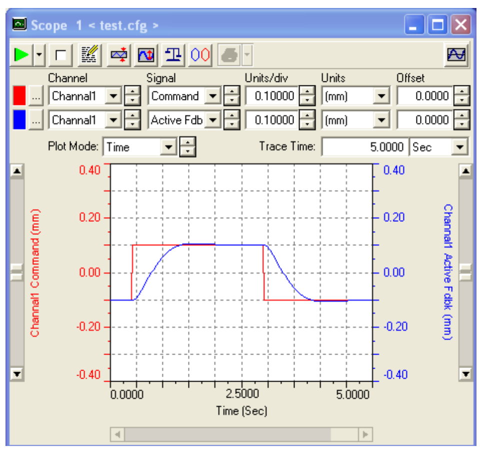
Figure (7) : Definition of Error and Phase Lag

Square waveform is preferred for displacement tuning due to the fact that this waveform have abrupt changes and excite the system frequencies likely to be unstable with excessive gain.

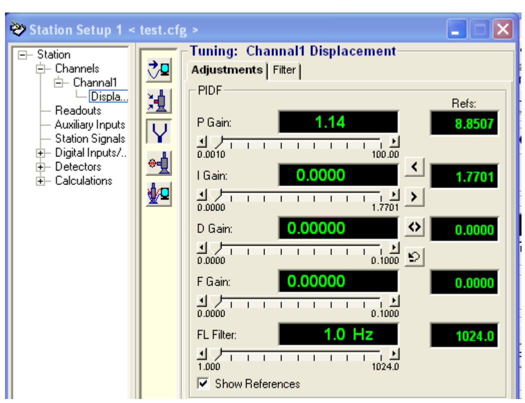
In the **Station Manager** window's in MTS FlexTest GT Software, select an access level of tuning. Select the channel that uses the displacement signal you want to tune. On the control mode selection list, select a displacement control mode, set the **Target Set-point** to zero, set the **Amplitude** to about 0.1**mm** of full scale, set the **Frequency** to 1 Hz, select **Square** under **Wave Shape,** turn on hydraulic pressure for appropriate HSM, and click **Program Run** on the **Station Controls** to start the function generator.



On the **Scope** dialog, select the displacement command and feedback signals for channel desired and set the **Trace Time** to 5 seconds.



In the **Station Setup** window navigation pane, select the channel that uses the displacement signal you intend to tune. Click **Channel Tuning** to display the **Tuning** panel.



5.1 Proportional gain:

The error signal is multiplied by the proportional gain, k_{pro}, and this corrected signal is sent to the servovalve as shown above. The following windows show the effect of the P Gain on the square waveform.

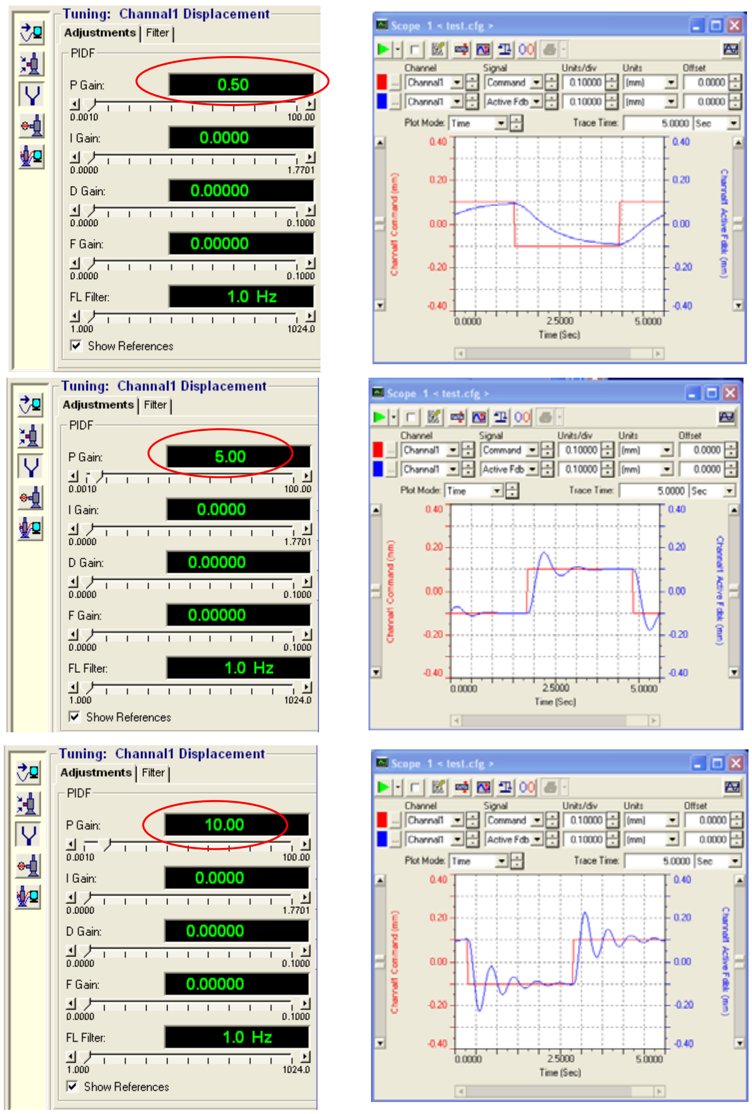


Figure (8): Effect of P Gain on a square waveform

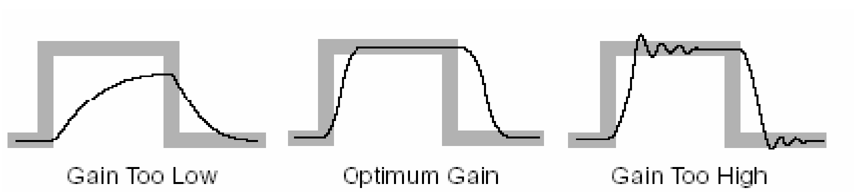


Figure (9): Effects of proportional gain on sensor feedback

As seen from the above applications when the proportional gain increases the error decreases and the feedback signal tracks the command signal more closely. However, too much proportional gain can cause overshooting and possibly drive the system into an unstable state. In the other direction, too little proportional gain can cause the system to become sluggish. Figure (9) resume the effect of the P.Gain on sensor feedback.

5.2 Derivative gain

The time derivative of the error is multiplied by the derivative gain, k_{der}, and the resulting signal is sent to the servovalve, as shown above. The following windows and Figure (10) show the effect of D Gian using MTS FlexTest software.

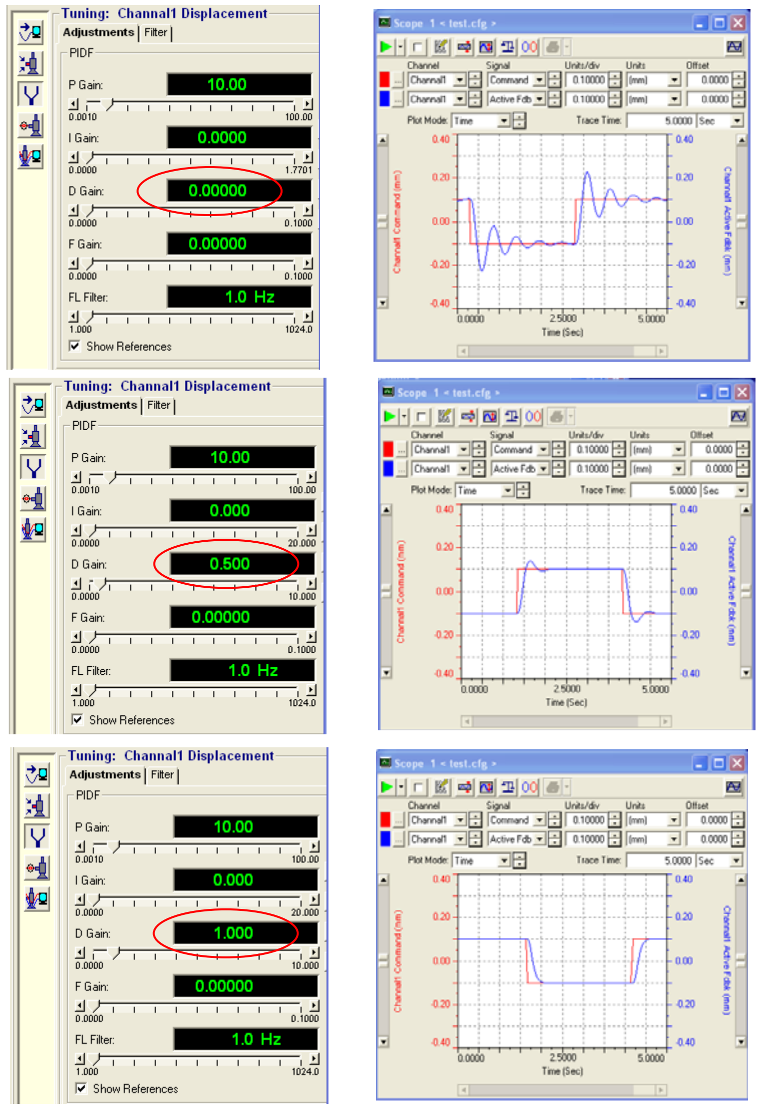


Figure (10): Effect of D Gain on a square waveform

It can be seen that the effect of derivative control is to reduce the overshoot caused by the proportional gain. Figure (11) shows the effect of derivative gain.

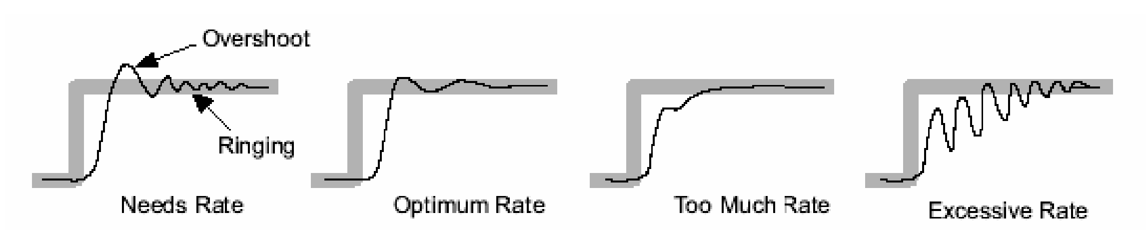


Figure (11) : Effects of derivative gain on sensor feedback

5.3 Integral gain

The integral of the error signal over time interval is multiplied by the integral gain, k_{int}, and the resulting signal is sent to the servo valve. The effect is to reduce the Accumulated error. The following windows show the effect of I Gain using MTS FlexTest software.

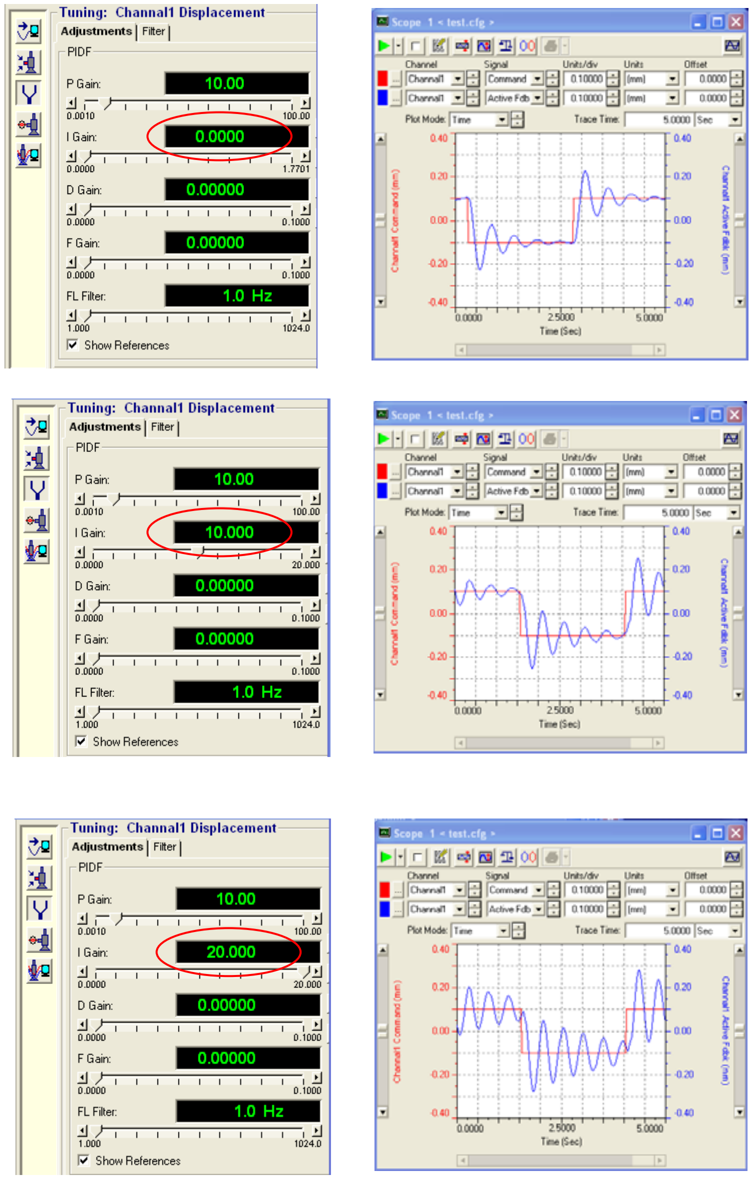


Figure (12): Effect of I Gain on a square waveform

Higher integral gain settings increase system response; too much integral gain can cause a slow oscillation or hunting. The MTS rule is to set the integral gain equal to 10% of the proportional gain.

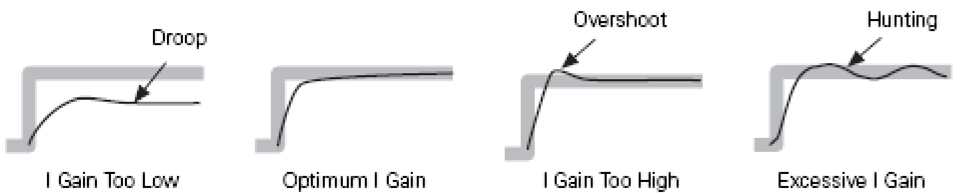


Figure (13) : Effects of integral gain on sensor feedback

5.4 Feedforward gain

F gain should be used like D gain. However, F gain applies to the test command signal while D gain applies to feedback signal**.** The following windows show the effect of integral gain using MTS FlexTest software.

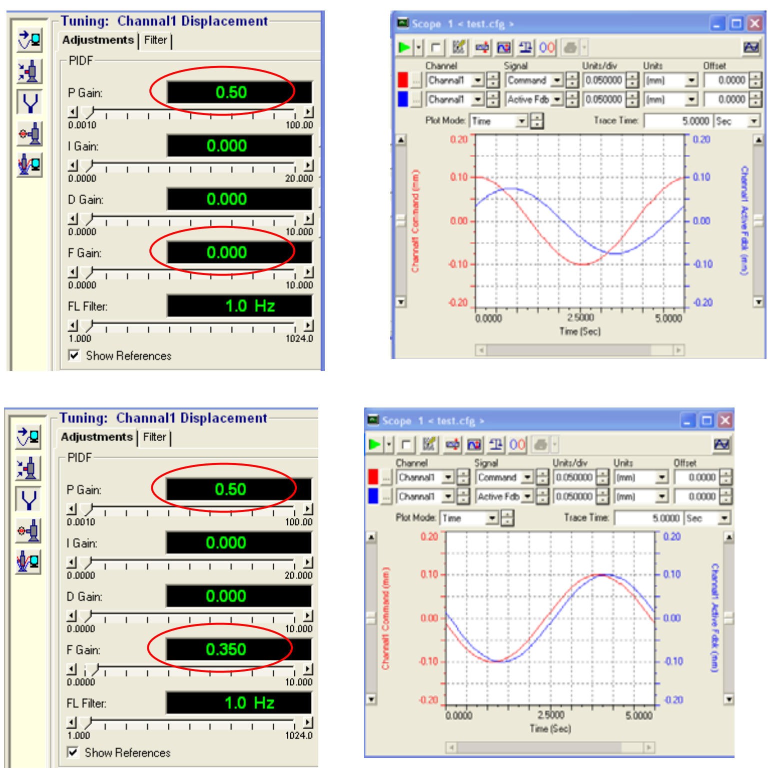


Figure (14): Effect of FF.Gain on a square waveform

The F gain is employed when more gain is needed, but the proportional gain cannot be increased without causing system instability. Figure(15) resume the effect of feed forward gain.

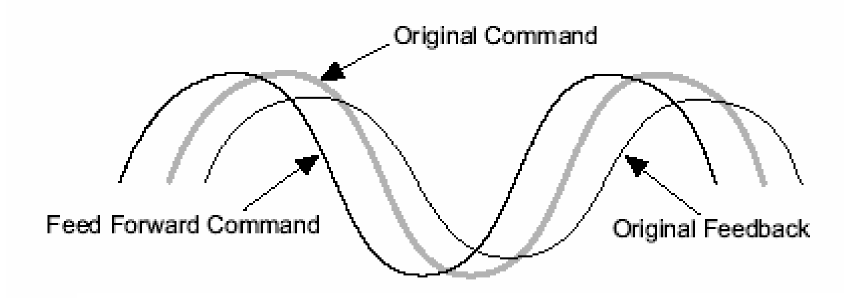


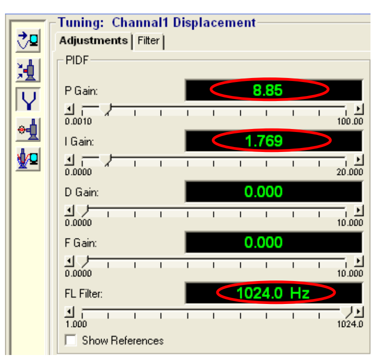
Figure (15) : Effect of Feed forward gain on sensor feedback

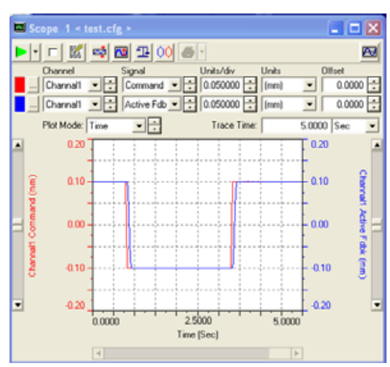
5.5 Delta-P gain

The delta-P gain is applied to the feedback signal form the differential pressure sensor mounted on the actuator. Delta-P improves displacement control of heavy mass loaded systems. This adjustment is usually needed when the natural frequency of the actuator is less than the 90^0 phase lag of frequency of the servovalve.

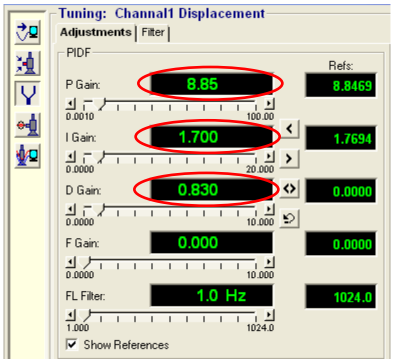
It is best to auto-tune your PIDF Control modes first. If the result from auto-tuning is not satisfactory, you should manually tune each control mode.

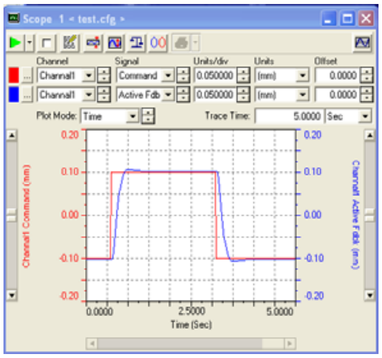
When we run basic auto-tuning we obtained the following PIDF values.





For the next application we propose the following values





References

- **J.P.Conte and T.L Trombetti** "Linear Dynamic Modeling of a Uni-axial-servo Hydraulic Shaking Table System" Earthquake Engineering and Structural Dynamics 2000,29, 1375-1404.
- **Tomaso Trombetti** "Experimental/analytical Approaches to Modeling Calibrating and Optimizing Shaking Table Dynamics for Structural Dynamic Applications" PHD Thesis, Rice University, Huston, Texas 1998.
- MTS Model 793 System Software, FlexTest GT.