Lecture 2 - Modeling of Structures and Similitude

Introduction:

Modeling –

Reduces a complicated or complex structure (building, bridge, etc…) called a PROTOTYPE to a significantly simpler structure without losing important characteristics in the behavior of the prototype.

Scale Modeling –

Reduces the size of a structural model without losing important characteristics in the behavior of the prototype.

Objectives of Scale Modeling –

1. Develop experimental data to verify the accuracy of analytical models
2. Study the behavior of complex structural systems for which analytical models are too complex. (i.e. shells, lattice domes, etc…)
3. Study the behavior of structural systems which combine elements with different kinds of response behavior.
4. Develop a database for macro-models using variable parameters

Process of Modeling

1. Define the scope of the experimental testing
   -desired response
   -failure mechanism

2. Select scale
   -normally governed by the resources of the laboratory (floor space, loading devices etc…)

3. Select model material
   -based on availability

4. Select similitude requirements
   -may not conform to a ‘true’ model

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5. Design model and plan fabrication

6. Select instrumentation (sensors) to measure the important response behavior.

7. Record and retrieve response (data acquisition)

8. Observe response and analyze data
   -maintain a log book of all experimental activities
   -extrapolate date and compare to prototype

9. Report findings

Advantages and Limitations of Modeling

Advantages include:

1. Low cost
2. Testing capabilities improved with reduced scales
3. Better understanding of local and global response behavior

Limitations include:

1. Availability of laboratory resources
2. Accuracy in elastic region
3. Difficult to model all structural and non-structural details

Accuracies of Structural Models

- Dependant on materials, scaling, similitude

≤ 20% in elastic region for concrete

≤ 10% in elastic region for other materials (such as steel, aluminium, etc…)

≤ 30% in dynamic measurements

Types of Structural Models

1. Components
   -columns, beams, joints, etc…

2. Subassemblages
   -beam–column–slab cruciforms

3. Substructure
   -frames, roofs, foundations, etc…

4. Structures
   -buildings, bridges, space structures, etc…

Geometric Scales (typically used)

	Elastic Models (stiffness similitude)	Strength Models (strength similitude)
Buildings	1 : 25	1 : 10 - 1 : 3
Shells, roofs	1 : 200 - 1 : 50	1 : 30 - 1 : 10
Bridges	1 : 25	1 : 20 - 1 : 4
Reactor vessels	1 : 100 - 1 : 50	1 : 20 - 1 : 4
Slabs	1 : 25	1 : 10 - 1 : 4
Dams	1 : 400	1 : 75
Wind effects	1 : 300 - 1 : 50	-

The smaller the scale factor the better the similitude, the bigger the size and closer the reality.

Reference: **Harris and Sabnis (1999)**

Model Studies of Structures

History of some model structures' studies at SEESL

Similitude and Dimensional Analysis

Theory of similitude:

Principles which underlie the proper design, construction, and interpretation of test results of models

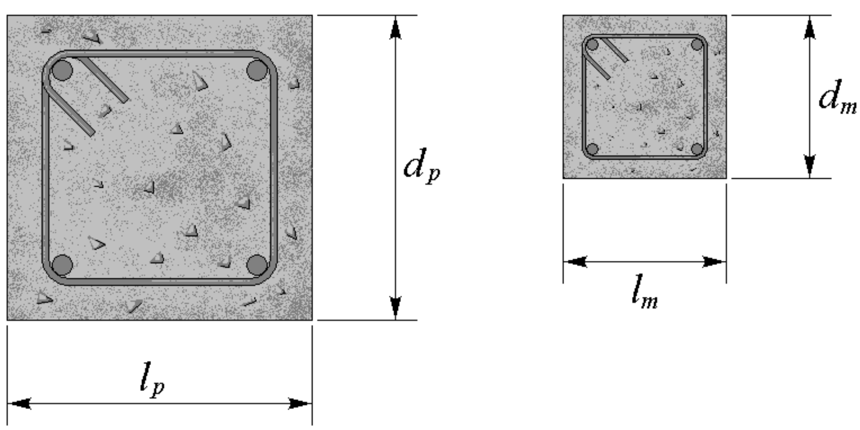
Dimensional Analysis:

Analytical tool to develop similitude between the model and the prototype

Types of Similitude

Geometric Similarity

Model and prototype have same shape and behavior i.e.



$$\frac{l_p}{d_p} = \frac{l_m}{d_m} \quad \text{so} \quad \frac{A_p}{A_m} = \left(\frac{d_p}{d_m}\right)^2 = \left(\frac{l_p}{l_m}\right)^2$$

Distorted Similitude

The model is a reproduction of the prototype, but two or more different scales are used.

E.g. One scale for depth and breadth, and another for height.

Dissimilar Similitude

No physical resemblance between model and prototype.

E.G. Vibrating mechanical systems may be predicted from observations made on an electrical circuit.

Characteristics of Observations

1. **Qualitative:** Characteristics of behavior identified so that the phenomenon may be accurately described.

   This is described in terms of standardized operations which identify classes of quantities such as length (L), force (F) and time (T).

   Note that L, F, and T are measurable quantities.

2. **Quantitative:** Involves both a number and a standard of comparison. i.e.

   3ft , 9lbs , 13.2 minutes These are called UNITS

   For example: Velocity has dimensions of LT^{-1}, and units such as mph, ft/sec, and knots.

For scientific measurements,

M , L , and T are regarded as basic,

But for engineering purposes,

F , L , and T are more convenient.

Note that they are interrelated through Newton's Second Law of Motion

$$F=ma$$

or

$$\boxed{F=M\left(\frac{L}{T^2}\right)=MLT^{-2}}$$

By equating the proposed basic quantities to the F , L , and T equivalents:

$$\text{Work} = W = FL$$
$$\text{Impulse} = I = FT$$
$$\text{Area} = A = L^2 \quad \text{(rectangle)}$$

Any quantity may be expressed in terms of the basic quantities as follows:

$$A = F^{C1} L^{C2} T^{C3}$$

Where:

A = dependent variables

F, L, T = primary quantities

C_1, C_2, C_3 = constants

Dimensional Analysis:

Based on relationships that exist among variables due to the dimensions.

Rule #1 :

Absolute numerical equality of quantities may exist only **when the quantities are similar qualitatively** (dimensionally)

i.e. Quantity measured in F $\neq$ quantity of L, T, M, V, etc…

Rule #2 :

The ratio of the magnitudes of **two like quantities** is independent of the units used in their measurement, provided that the same units are used for evaluating each.

i.e. l/d = constant regardless of the units

Form of dimensional equation

$$A \doteq F^{C_1} L^{C_2} T^{C_3}$$

where

A = dependent variables

F, L, T = primary quantities

C_1, C_2, C_3 = constants

From dimensional analysis,

$$\alpha = C_\alpha a_1^{C_1} a_2^{C_2} a_3^{C_3} \ldots a_n^{C_n}$$

where

α = secondary quantity

a_i = primary quantities

C_i = constants

C_α = constant

Example: general equation for distance of a freely falling object…

$$S=f\left(g,t\right)$$

or

$$S=C_\alpha g^{C_1}t^{C_2}$$

or dimensionally

$$L=C_\alpha\left(LT^{-2}\right)^{C_1}\left(T\right)^{C_2}$$

$\rightarrow$ 1 = C1 (from the powers of L)
$0 = -2C_1 + C_2$ or $C_2 = 2$ (from the powers of T)

$\therefore$ $\boxed{S=C_\alpha gt^2}$ C_α can be determined experimentally

Extension of Procedure for Secondary Quantities > Primary Quantities

If the number of ‘a’ variables is more than three, then there will not be enough equations to solve for all unknowns. But we can solve for three and determine the rest experimentally.

i.e. Determine the equation for the distance a sphere will fall in time t if it starts with an initial velocity and fluid resistance is considered.

$$s=f\left(g,v,t,m,d,\rho,\mu\right)$$

or

$$s=Cg^{C_1}v^{C_2}t^{C_3}m^{C_4}d^{C_5}\rho^{C_6}\mu^{C_7}$$

Note that there are eight unknowns, but we can find three from dimensional analysis

$$L\doteq\left(LT^{-2}\right)^{C_1}\left(LT^{-1}\right)^{C_2}T^{C_3}M^{C_4}L^{C_5}\left(ML^{-3}\right)^{C_6}\left(ML^{-1}T^{-1}\right)^{C_7}$$

or

$$C_7=-C_4-C_6$$
$$C_1=C_3+2C_4+C_5-C_6-1$$
$$C_2=-C_3-3C_4-2C_5+3C_6+2$$

$$\therefore s=C\left(g^{C_3+2C_4+C_5-C_6-1}\right)\left(v^{-C_3-3C_4-2C_5+3C_6+2}\right)t^{C_3}m^{C_4}d^{C_5}\rho^{C_6}\left(\mu^{-C_4-C_6}\right)$$

or $$s=C_\alpha\left(\frac{v^2}{g}\right)\left(\frac{gt}{v}\right)^{C_3}\left(\frac{g^2m}{v^3\mu}\right)^{C_4}\left(\frac{gd}{v^2}\right)^{C_5}\left(\frac{\rho v^3}{g\mu}\right)^{C_6}$$

Note that there are 5 unknowns!

Other forms of the above equation also exist…(from multiplications of the above ratios) …..

$$s=C_\beta d\left(\frac{gd}{v^2}\right)^{C_1}\left(\frac{vt}{d}\right)^{C_3}\left(\frac{d^3\rho}{m}\right)^{C_6}\left(\frac{d^2\mu}{vm}\right)^{C_7}$$

$$\frac{s}{vt}=C_\alpha\left(\frac{gt}{v}\right)^{C_1}\left(\frac{d}{vt}\right)^{C_5}\left(\frac{\rho v^3t^3}{m}\right)^{C_6}\left(\frac{\mu t^2v}{m}\right)^{C_7}$$

or $$\frac{s}{vt}=f\left(\frac{gt}{v},\frac{d}{vt},\frac{\rho v^3t^3}{m},\frac{\mu t^2v}{m}\right)$$

These dimensionless groups are called Pi Terms (π)

$$\Rightarrow \pi_1=f\left(\pi_2,\pi_3,\pi_4,\pi_5\right)$$

Buckingham Pi Theorem

The number of required no dimensional ratios (π terms) is limited by the following relation:

$$s = n - b$$

where

s = required number of π terms (5)
n = total number of quantities involved (8)
b = number of basic dimensions (3)

Five π terms are required!

$$\pi_1 = f\left(\pi_2, \pi_3, \pi_4, \ldots\ldots, \pi_s\right)$$

Note that π terms are dimensionless and independent. b = 3 for L, F, T.

Determination of π Terms

An infinite number of possibilities exist!! – See above.

The solutions of the remaining unknowns are found experimentally.

Theory of Models

General equation of prototype

$$\pi_1 = f\left(\pi_2, \pi_3, \pi_4, \ldots\ldots., \pi_s\right)$$

Since π_i are general, non-dimensional, and independent, it also applies to any system. For the model system:

$$\pi_{1m} = f\left(\pi_{2m}, \pi_{3m}, \pi_{4m}, \ldots\ldots., \pi_{sm}\right)$$

Predicting π_1 from π_{1m} as follows:

$$\frac{\pi_1}{\pi_{1m}} = \frac{f\left(\pi_2, \pi_3, \pi_4, \ldots\ldots., \pi_s\right)}{f\left(\pi_{2m}, \pi_{3m}, \pi_{4m}, \ldots\ldots., \pi_{sm}\right)}$$

Now if the model is designed so that

$$\pi_{2m} = \pi_2 \qquad \pi_{4m} = \pi_4$$

$$\pi_{3m} = \pi_3 \qquad \pi_{5m} = \pi_5$$

Then

$$f\left(\pi_2, \pi_3, \pi_4, \ldots\ldots., \pi_s\right) = f\left(\pi_{2m}, \pi_{3m}, \pi_{4m}, \ldots\ldots., \pi_{sm}\right)$$

$$\Rightarrow \qquad \pi_1 = \pi_{1m}$$

If all design conditions are satisfied ($\pi_i = \pi_{im}$), then it can be considered a ‘true’ model. If not, the model may be distorted.

Types of Models

1. **True** → All significant characteristics are reproduced.
2. **Adequate** → Accurate predictions of one characteristic of the prototype may be made, but possibly not others. (i.e. select beam with proper area, but not moment of inertia.)
3. **Distorted** → Some design conditions are violated.
4. **Dissimilar** → Model bears no resemblance to prototype

Scales

$$\text{Length scale} = \frac{\text{distance in prototype}}{\text{distance in model}}$$

Other scales can also be used, such as force scale, mass scale, area…….

Scale Model Rules for Vibration Studies

Non-dimensional ratio must be preserved

$$\pi = \frac{Ft^2}{ML} = \frac{F}{Ma}$$

Equilibrium and physical laws must be preserved

$$\Sigma F = 0$$

Assume scaling factors λ_i = prototype's quantity / model quantity

$$\lambda_l = \frac{L_p}{L_m} \qquad \lambda_f = \frac{F_p}{F_m} \qquad \lambda_t = \frac{T_p}{T_m}$$

Leads to:

$$\pi_{prot} = \pi_{model} \quad \Rightarrow \quad \lambda_f = \lambda_M * \lambda_a = \lambda_M * \lambda_l / \lambda_t^2$$

The prototype is the original system to be tested, while the model is a representation of the original, possible at a reduced scale.

$$\pi_1 = \frac{Ft^2}{ML}$$

L=l (upper and lower case indicate length dimension)

$$\pi_2 = \frac{F}{l^2} \cdot \frac{t^2 l}{M} = \frac{E}{\rho_m}$$

$$\rho_m = \left[\frac{M}{l^3}\right] \qquad a = \left[\frac{l}{t^2}\right] \qquad E = \left[\frac{F}{l^2}\right]$$

Determining minimum π numbers:

$$\psi=\left[u,E,l,t,w,\gamma_f,g,a\right] \qquad \text{w=distributed load}$$

$$u=\left[K_\alpha E^a l^b t^c w^d \gamma_f^e g^f a^g\right]$$

$$L=K\left(FL^{-2}\right)^a L^b T^c \left(FL^{-1}\right)^d \left(FL^{-3}\right)^e \left(LT^{-2}\right)^f \left(LT^{-2}\right)^g$$

…may neglect (g) since $\left(\dfrac{lt^{-2}}{g}\right)=\pi$

$$\begin{cases} 0=a+d+e \\ 1=-2a+b-d-3e+f\ (+g) \\ 0=c-2f\ (-2g) \end{cases}$$

Choose expected independent variables: $E_{(material)}$; $T_{(time)}$; $w_{(load)}$;

a,c,d ← our choice

$$\begin{cases} e=-a-d \\ b=2a+d+3(-a-d)-c/2=-a-2d-c/2 \\ f=c/2 \end{cases}$$

$$u=K\left(E^a l^{-a-2d-c/2} t^c w^d \gamma^{-a-d} g^{c/2}\right)$$

$$\frac{u}{l}=\psi\left[\left(\frac{E}{l\cdot\gamma}\right)\cdot\left(\frac{w}{\gamma\cdot l}\right)\cdot\left(\frac{gt^2}{l}\right)\right] \qquad \text{basic } \pi \text{ for structural systems}$$

Other:

$$\left(\frac{a}{l\cdot t^{-2}}\right)=\pi_a \qquad \left(\frac{F\cdot t^2}{M\cdot l}\right)=\pi_b$$

1 Use different materials and same acceleration scale, i.e.: Model a concrete structure in plexiglass.

$E_p = 2 \times 10^5$ kg/cm^2 $E_m = 2 \times 10^4$ kg/cm^2

$\rho_p = 2.4$ tons/m^3 $\rho_m = 1.6$ tons/m^3

Required length scale factor:

$$\lambda_l = \frac{\lambda_E}{\lambda_{\rho_{(f)}}} = \left(\frac{2\times10^5}{2\times10^4}\right)\left(\frac{2.4}{1.6}\right) = 6.66$$

Note that it can only be made for elastically responding structures!!!

$$\boxed{\lambda_l = \frac{E}{l\,\rho_{mass}[a]} = \frac{E}{l\,\rho_F}} \quad => \quad \boxed{\lambda_{\rho_{(mass)}} = \frac{\lambda_E}{\lambda_l}\cdot\lambda_a}$$

2 Use same materials and same acceleration scale ,
i.e.: Model a steel structure in steel:

$\lambda_E=1$ $\lambda_\rho=1$ $\lambda_a=1$

Required length scale factor:

$\lambda_l=1$

This is a trivial solution that does not satisfy our objective.

$$\left[\left[[E] = \frac{[\rho]}{[l]^{-1}} = \frac{ML^{-3}}{L^{-1}}\right] \quad \begin{array}{l} [E]=[\rho]\times[l]=FL^{-3}L=FL^{-2} \\ [E]=[\rho]\times[l]=MLT^{-2}L^{-3}L=ML^{-1}T^{-2} \end{array}\right]$$

$$\boxed{\lambda_{\rho_{(mass)}} = \frac{\lambda_E}{\lambda_l}\cdot\lambda_a}$$

Modeling / scaling in same acceleration field

$$\lambda_a = \frac{a_p}{a_m}$$

$$\pi_p = \pi_m$$

$$\frac{E_p}{l_p \gamma_p} = \frac{E_m}{l_m \gamma_m}$$

$$\frac{E_p / E_m}{\left(l_p / l_m\right)\left(\gamma_p / \gamma_m\right)} = 1$$

$$\boxed{\frac{\lambda_E}{\lambda_l \lambda_\gamma} = 1}$$

Examples: Select $\lambda_L=3$, $\lambda_E=1$, $\lambda_a=1$,

Now; $\lambda_L=3$ $\therefore$ $width_p = 3 \times width_m$

Cross sectional area:

$$A=L^2 \qquad \Rightarrow \lambda_A = \lambda_l^{\,2}$$

Moment of inertia:

$$I=L^4 \qquad \Rightarrow \lambda_I = \lambda_l^{\,4}$$

Volume V:

$$V=L^3 \qquad \Rightarrow \lambda_V = \lambda_l^{\,3}$$

Density ρ:

$$\rho = \frac{m}{V} = \frac{F}{aV} = \frac{E}{aL} \qquad \Rightarrow \lambda_\rho = \frac{\lambda_E}{\lambda_a \lambda_l}$$

Note that $\lambda_\rho = \frac{1}{1 * 3} = 0.33$ (model density required is not that of prototype).

Time (Period) t:

$$a = \frac{L}{t^2} \qquad t = \sqrt{\frac{L}{a}} \qquad \Rightarrow \lambda_t = \left[\frac{\lambda_l}{\lambda_a}\right]^{1/2}$$

Velocity v:

$$v = at \qquad \Rightarrow \lambda_v = \lambda_a \cdot \sqrt{\lambda_l / \lambda_a} = \sqrt{\lambda_l \cdot \lambda_a}$$

Mass m:

$$m = \rho V \qquad \Rightarrow \lambda_m = \lambda_\rho \lambda_V = \lambda_\rho \lambda_L^3$$

Force F:

$$F = ma \qquad \Rightarrow \lambda_f = \lambda_\rho \lambda_L^3 \lambda_a = \frac{\lambda_E}{\lambda_a \lambda_L} \lambda_L^3 \lambda_a = \lambda_E \lambda_L^2$$

Stress:

$$\Rightarrow \lambda_\sigma = \lambda_E$$

Strain:

$$\varepsilon = \frac{\sigma}{E} \qquad \Rightarrow \lambda_\varepsilon = \frac{\lambda_E}{\lambda_E} = 1$$

Impulse:

$$I = Ft \qquad \Rightarrow \lambda_i = \lambda_E \lambda_L^2 \cdot \sqrt{\frac{\lambda_L}{\lambda_a}}$$

$$= \lambda_E \lambda_L^2 \cdot \sqrt{\frac{\lambda_L}{\left({}^{1}\!/_{\lambda_l} \cdot {}^{\lambda_E}\!/_{\lambda_\rho} \right)}}$$

$$= \lambda_E \lambda_L^2 \cdot \sqrt{\frac{\lambda_L^2 \lambda_\rho}{\lambda_E}}$$

$$= \lambda_L^3 \cdot \sqrt{\lambda_E \lambda_\rho}$$

Frequency:

$$\omega = \sqrt{\frac{K}{m}} \qquad \Rightarrow \lambda_\omega = \sqrt{\frac{\left({}^{\lambda_F}\!/_{\lambda_l} \right)}{\left({}^{\lambda_\rho}\!/_{\lambda_l^3} \right)}}$$

$$= \sqrt{\frac{\left({}^{\lambda_E \lambda_L^2}\!/_{\lambda_L} \right)}{\lambda_\rho \lambda_l^3}}$$

$$= \sqrt{\frac{\lambda_E \lambda_L}{\lambda_\rho \lambda_L^3}}$$

$$= \frac{1}{\lambda_L} \sqrt{\frac{\lambda_E}{\lambda_\rho}}$$

Strain Rate

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{\Delta\varepsilon}{\Delta t} \quad \text{as} \quad \Delta t \to 0$$

$$\lambda_{\dot{\varepsilon}} = \frac{\lambda_{\varepsilon}}{\lambda_t} = \frac{1}{\sqrt{\lambda_l / \lambda_a}}$$

Note that $\lambda_{\dot{\varepsilon}} \neq 0$ for reduced scales

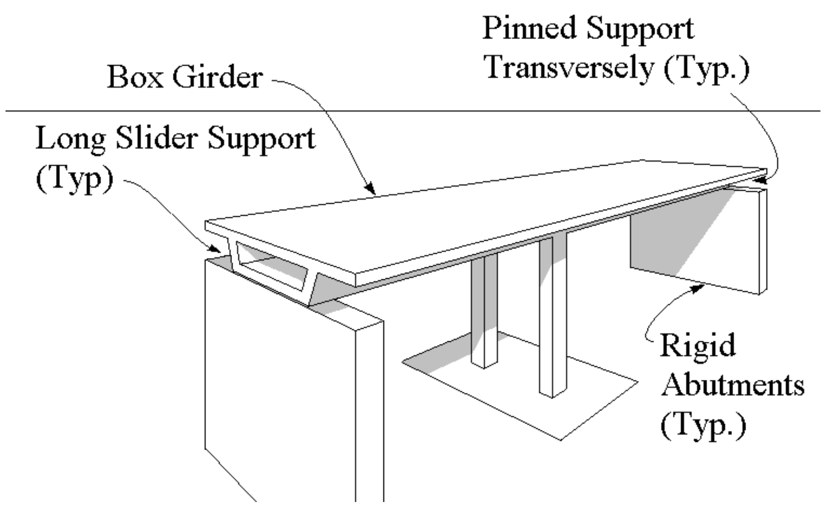
See page 'Scaling Factors for Modeling of Dynamic Behavior'

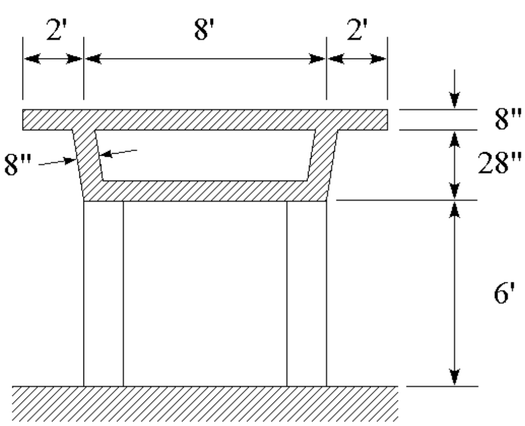
Remarks

- Scale model studies are the only practical way to experimentally test civil engineering structures which are extremely large.
- The results of model studies can be extrapolated for elastic and for ideal plastic behavior (although this is difficult).
- Model studies serve to validate analytical tools, provide data for parametric studies, explore behavior of complex systems, and validate pilot implementation.
- Design of models and their construction requires extreme care. Small variations can be critical for interpretation of results.
- Non-linear structures can be studied using analytical models that are validated by scaled models.

EXAMPLE:

For a prototype bridge, develop a half scale model for cyclic testing to evaluate the pier flexural strength in the lateral direction.





Columns: 16” × 16” with 8 #8 bars

$A_s = 0.79 in^2$

Girder spans = 60 ft (120 ft total)

Use normal concrete and rebar

$\gamma_c = 0.15\ K/ft^3$

Use same material and same acceleration

$$\lambda_l = 2$$
$$\lambda_A = \lambda_l^2 = 4$$
$$\lambda_I = \lambda_l^4 = 16$$
$$\lambda_w = \lambda_l^2 = 4$$

Columns → use 8” × 8” × 3’ length

Rebar?

For moment capacity, force in rebar important.
→ Scale cross section area, use 8 #4 bars

Beam → Same factor applies (dimensions divided by 2)

See page ‘Scaling Factors for Modeling of Dynamic Behavior’

Weight:

Dead weight on prototype column: (use tributary area)

$$= 0.15(60)\left(\frac{1}{2}\right) \times \left(12 \times \frac{8}{12} + \frac{28}{12} \times \frac{8}{12} \times 2 + 8 \times \frac{8}{12}\right)$$

$= 74$ Kips

Dead weight on model column:

$$= 0.15\left(30 \times \frac{1}{2}\right) \times \left(6 \times \frac{4}{12} + \frac{14}{12} \times \frac{4}{12} \times 2 + 4 \times \frac{4}{12}\right)$$

$= 9.25$ Kips

Required scale of weight

$$w_m = w_p / \lambda_w = w_p / \lambda_l^2 = 18.5 \text{ Kips}$$

∴ Need to add an additional 9.25 Kips onto slab!

Use same material and same velocity

i.e. $\lambda_E = \lambda_\rho = \lambda_v = 1$

Then the following rules must be satisfied:

Acceleration	$\lambda_a = \lambda_l^{-1}$
Gravity loads	$\lambda_{fg} = \lambda_l^{3}$
All other forces	$\lambda_f = \lambda_l^{2}$

Note that the discrepancy in gravity loads requires special devices in the tests to simulate gravity loads without altering inertial effects.

See page 'Scaling Factors for Modeling of Dynamic Behavior'

Use same material, same acceleration scale and…!!!

i.e. $\lambda_E = \lambda_a = 1 \Rightarrow \lambda_\rho = \lambda_E / \lambda_l = \lambda_l^{-1}$

Note that this requires therefore a different density scale for the modeling material. The density of the model should be larger than the density of the prototype. This can be overcome by adding mass to the elements of the structure. This is usually added in concentrated locations without substantially altering the structural behavior. Not suitable for continuous medium studies.

Artificial Mass Simulation

Make model of same material $\lambda'_\rho = 1$ while the required $\lambda''_\rho = \lambda_l^{-1}$ and make adjustments where required:

i.e. Mass similitude is altered while all other quantities are preserved.

$$\lambda_m = \lambda_\rho \lambda_l^{-3} = \lambda_l^{-3} \qquad \text{for} \qquad \lambda'_\rho = 1 \qquad \text{(provided)}$$

$$= \lambda_l^{-2} \qquad \text{for} \qquad \lambda''_\rho = \lambda_l^{-1} \qquad \text{(required)}$$

Added mass is required and can be determined:

$$m_m = m_\rho \lambda_l^{-3} + m_\rho \left(\lambda_l^{-2} - \lambda_l^{-3}\right) = m'_m + \Delta m$$

$$\boxed{\Delta m = m_\rho \left(\lambda_l - 1\right) / \lambda_l^{3}}$$ as function of the mass of the prototype

or

$$\boxed{\Delta m = m_m \left(\lambda_l - 1\right)}$$ as function of the mass for the model

Note that body forces are not preserved since the additional mass is usually placed in discrete locations.

See ‘Example of Mass Similitude’ in Bracci et al, 1992

Model Studies for Inelastic Structures

The similitude relations shown do not satisfy inelastic behavior. The rules for inelastic behavior are extremely complex.

It is therefore suggested to use the prototype material in the model and hold the same stress-strain rates. In spite of such modeling not all inelastic modeling is similar to the prototype.

See 'Modeling' in Bracci et al, 1992' pg: 3-23

The extrapolation from the model to the prototype cannot be made directly. Model studies are only used to validate analytical tools. The analytical tools are then used to predict the behavior of the prototype. Note that this rule also applies to operating the shaking table-model interaction.

See "Experimental results" in Bracci et al, 1992

Suitable Scales for Modeling Specific Building Types and Purposes

Type of Structure	Elastic Models	Strength Models
Shell roof	1/200 to 1/50	1/30 to 1/10
Highway bridge	1/25	1/20 to 1/4
Reactor vessel	1/100 to 1/50	1/20 to 1/4
Slab structures	1/25	1/10 to 1/4
Dams	1/400	1/75
Wind effects	1/300 to 1/50	Not applicable

Harris and Sabnis "Theory of Structural Models" Chapter 2 , pg 42-83

Model Studies of Steel Structures

All studies use mass simulation

Examples of studies include:

1. A 1:2.5 scale model of a five storey structure in Beijing, P.R. China. The model was 6.5 metres high, and the prototype was around 15.5 metres high. It was tested for scaling validation, fabrication techniques, structural identification, and protective systems.

2. A 1:2 scale model of a gable frame in Buffalo, USA. The model was of 16' span and was 10' high. It was tested to validate analytical techniques, provide data for structural behavior in the inelastic range, and provide information on parameters required for design.

3. A 1:5 scale model of a gable frame with tapered members in Buffalo, U.S.A. Model of 16' span 3' to 6' high. Tested for validation of analytical methods, to identify behavior in the inelastic range, and prepare information for code recommendations.

4. 1:4 scale models with active control, between 6' and 20' high. Tested to validate control techniques, analytical models solve implementation problems. The results were directly scalable to the prototype.

5. 1:2 scale models of frames with semi-rigid connections. Models were of 8' span and 16' high. They were tested for validation of analytical techniques, identification of structural behavior and energy absorption capacity. The results were not scalable.

6. 1:3 scale model of "zipper frames" with geometric nonlinearities up to fracture. Model includes three stories frame and a gravity load resisting system independent of the lateral. The results are not directly scalable but help validate computational model

Model Studies of Reinforced Concrete

1 The most complex test to date is a 1:5 scale model of a 7 storey building tested at U.C. Berkeley in the U.S.-Japan program coordinated by V.V. Bertero. This was tested for understanding of complex structural behavior, and the validation and development of analytical techniques.

2 Recent tests include 1:6 scale models of inelastic flexible floor diaphragms at S.U.N.Y at Buffalo. These models were tested for the validation of analytical techniques. The results were extrapolated to realistic prototypes using analytical models. The results were processed for code recommendations.

Note that during the tests input motion is altered by structure-table interaction while the response is in the inelastic range. Initial predictions are not verifiable directly. Recorded motion should be used for validations.

Model Studies of Water Tanks

Note that the similitude parameters are different for bending and membrane behavior.

Similitude rules:

$$\frac{\lambda_E \lambda_h}{\lambda_\rho \lambda_R^2 \lambda_g} = 1 \rightarrow \text{membrane - axial} \qquad \frac{\lambda_E \lambda_h^3}{\lambda_\rho \lambda_R^2 \lambda_y \lambda_l^2} = 1 \rightarrow \text{bending}$$

Required scaling factors (geometry):

$$\lambda_l = \lambda_h = \lambda_R = \lambda_E \quad \left(= \tfrac{1}{2} \text{ for study}\right)$$

Time scale requirements:

$$\lambda_t = \lambda_{f\omega}^{-1} = \lambda_L^{1/2} \qquad \left(= \sqrt{\tfrac{1}{2}} \text{ for study}\right)$$

Scaling rules are then different than for frame structures.

Examples of shaking table studies include:

1 1:2 scale models of cylindrical plastic tanks. The models were 4’ in diameter and 6’ high. They were tested to verify scaling rules for quasi-linear materials, verify the performance of plastic tanks, and identify inelastic behaviour.

2 1:10 scale models of cylindrical metal tanks. The models were 3’ in diameter and 4’ high. They were tested to reproduce elephant foot buckling failure. The results are not directly scalable due to scaling difficulties.

Scale Model Design for the Shaking Table

The required information includes:

The shaking table capacity (table weight W_T, actuator size R_s, maximum accelerations a_{max}, frequency ranges, maximum velocity, etc…)

Information about the prototype (weight W_p, damping ratios, expected amplification factors α_{dyn}, etc…)

Required scaling rules

The equilibrium relation must be satisfied at maximum requirements. We assume:

$$R_a = \frac{W_p}{g\lambda_F} \cdot \frac{a_{max}}{\lambda_a} \cdot \alpha_{dyn} + \frac{W_T}{g} \cdot \frac{a_{max}}{\lambda_a}$$

for:

$$\lambda_a = \lambda_E = 1 \qquad \lambda_F = \lambda_l^2$$

then:

$$R_a = \left(\frac{W_p \alpha_{dyn}}{\lambda_l^2} + W_T \right) \cdot \frac{a_{max}}{g}$$

The maximum acceleration that can be applied to the model if the scale is known:

$$\frac{a_{max}}{g} = \frac{R_a \lambda_l^2}{W_p \alpha_{dyn} + W_T \lambda_l^2}$$

The minimum geometric scale that can be used for a given maximum acceleration:

$$\lambda_{l,min} = \left[\frac{W_p \cdot \alpha_{dyn} \cdot a_{max}}{R_a \cdot g + W_T \cdot a_{max}} \right]^{\frac{1}{2}}$$

Concluding Remarks

Scale models are the only way to test civil engineering structures that are extremely large in size and weight.

The results of model studies can be extrapolated to prototypes for elastic behavior (with considerable effort)

Model studies serve in validating analytical tools, providing data for parametric studies, exploration of the structural behavior of complex systems, validating pilot implementations and such…

The design and construction of models requires extreme care. Small variations can be critical for the interpretation of results.

Non-linear structures can be validated using analytical models validated by scale models.

References:

Harris and Sabnis "Theory of Structural Models" Chapter 2 , pg 42-83