TABLE 16.8 PERFORMANCE SPECIFICATIONS OF SERVOVALVES

<table>
<thead>
<tr>
<th>Valve Type</th>
<th>Maximum Working Pressure (psi)</th>
<th>Maximum Flow at 1000 psi Pressure Drop (gpm)</th>
<th>Frequency at 90° Lag (Hz)</th>
<th>Hysteresis (%)</th>
<th>Resolution (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spool</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One stage</td>
<td>5000</td>
<td>3500</td>
<td>200</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>Two stage</td>
<td>7000</td>
<td>1000</td>
<td>200</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>Three stage</td>
<td>4500</td>
<td>300</td>
<td>200</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>Flapper-nozzle/spool</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One and two stage</td>
<td>5000</td>
<td>1000</td>
<td>500</td>
<td>0.2</td>
<td>0.01</td>
</tr>
<tr>
<td>Three stage</td>
<td>5000</td>
<td>1000</td>
<td>500</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>Jet pipe/spool</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One and two stage</td>
<td>4500</td>
<td>300</td>
<td>500</td>
<td>2</td>
<td>0.1</td>
</tr>
<tr>
<td>Three stage</td>
<td>4500</td>
<td>300</td>
<td>200</td>
<td>2</td>
<td>0.1</td>
</tr>
<tr>
<td>Sliding plate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One and two stage</td>
<td>300</td>
<td>40</td>
<td>150</td>
<td>3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Source: From Ref. 25.

16.8.2 Mathematical Model of a Spool-Type Valve

Spool-type valves are the most popular due to their ease of construction. They are also easier to analyze than other types of valves. Figure 16.33 shows a typical spool-valve configuration and defines the important variables and parameters. The valve has three “energy ports” where energy or power flows from or to the environment of the valve. Correspondingly, the valve can be modeled using the three mathematical equations given in functional form as follows:

\[ Q_m = f(x, P_m, P_s) \]  \hspace{1cm} (16.53)

\[ Q_s = f(x, P_m, P_s) \]  \hspace{1cm} (16.54)

\[ F_v = f(x, P_m, P_s) \]  \hspace{1cm} (16.55)

Equation 16.53 gives the pressure-flow-displacement characteristics of the valve. These characteristics are needed in the dynamic analysis of a servovalve which employs the valve. Equation 16.54 is used to compute the required flow rate from the source and will not be considered further here. Equation 16.55 is used to calculate the force required to move the spool (e.g., force output requirement of the torque motor in the case of an electrohydraulic servovalve).

The steady-state pressure-flow-displacement characteristics of the spool valve are characterized by the nonlinear orifice equation (Refs. 30 and 37):

\[ Q_m = C_d w(x + U) \sqrt{\frac{P_s - P_m}{\rho}} \]  \hspace{1cm} (16.56)
\[ \Delta Q_m = K_1 \Delta x - C_1 \Delta P_m \]  

(16.57)

where

\[ K_1 = \frac{\partial Q_m}{\partial x} \bigg|_{P_{m0}, x_0} = C_d w \sqrt{\frac{P_s - P_{m0}}{\rho}} \]  

(16.58)

\[ C_1 = -\frac{\partial Q_m}{\partial P_m} \bigg|_{x_0, P_{m0}} = \frac{C_d w x_0}{2 \sqrt{\rho(P_s - P_{m0})}} \]  

(16.59)

The terms \( \Delta Q_m, \Delta x, \) and \( \Delta P_m \) represent small changes of the corresponding variables about the steady-state operating point \( x_0, P_{m0} \). The constants \( K_1 \) and \( C_1 \) are evaluated at the operating point. These expressions assume that the valve port shape does not vary with displacement.

The static and dynamic behavior of the valve spool (Eq. 16.55) can be modeled by considering the forces which act on the spool (Ref. 30). These forces include the externally imposed force (input) as well as steady and unsteady flow forces resulting from flow through the orifices. Additional forces that may be present include the viscous damping between the spool and the sleeve and any mechanical spring forces acting on the spool (not shown in Fig. 16.33). The force balance equation for the spool shown in Fig. 16.33 is

\[ F_v = m_s \ddot{x} + \frac{m A_s}{h} \dot{x} + \left[ C_d w \sqrt{2 \rho \left( \frac{P_s - P_m}{2} \right)} (L_1 - L_2) \right] \dot{x} + \left[ 2 C_d C_v w \left( \frac{P_s - P_m}{2} \right) \cos \theta_j \right] x \]  

(16.60)

where

\[ A_s = \text{Net shear area of spool lands} \]
\[ C_d = \text{Metering orifice discharge coefficient} \]
\[ C_v = \text{Metering orifice velocity coefficient (see Ref. 30)} \]
\[ F_v = \text{External force on spool (e.g., imposed by torque motor)} \]
\[ h = \text{Radial clearance between spool and sleeve (valve body)} \]
\[ L_1 = \text{Length of fluid column to be accelerated at inlet (see Fig. 16.33)} \]
\[ L_2 = \text{Length of fluid column to be accelerated at outlet (see Fig. 16.33)} \]
\[ m_s = \text{Mass of spool} \]
\[ P_m = \text{Pressure drop across servomotor} \]
\[ P_s = \text{Supply pressure} \]
\[ w = \text{Circumferential width of metering ports in valve} \]
\[ x = \text{Displacement of spool} \]
\[ \dot{x} = \text{Velocity of spool} \]
\[ \ddot{x} = \text{Acceleration of spool} \]
\[ \rho = \text{Mass density of fluid} \]
\[ \mu = \text{Absolute viscosity of fluid} \]
\[ \theta_j = \text{Effective angle of fluid jet (see Ref. 30)} \]

The fourth term on the right-hand side of Eq. 16.60 is the steady flow-induced force and the third term on the right-hand side is the unsteady flow-induced force. The steady flow force is a "spring-
like" force that always opposes the motion of the spool, and hence is a stabilizing force. The unsteady flow force is a "damping-like" force that changes its direction of action depending on the flow direction, and hence it can be a stabilizing or destabilizing force. The valve is dynamically stable if \((L_1 - L_2) > 0\). A more complete discussion of the dynamic modeling of the valve spool is given in Ref. 30.

### 16.8.3 Mathematical Models for an Electrohydraulic Servovalve

The steady-state pressure-flow characteristics for an electrohydraulic servovalve of the type shown in Fig. 16.30 are identical to those of the spool-type valve in the previous section except the input \(x\) is replaced by the current \(I\). That is, in the steady state, the motion of the spool in the electrohydraulic servovalve is directly proportional to the current input to the valve. The steady-state pressure-flow-current characteristics for the "idealized" electrohydraulic servovalve (e.g., Fig. 16.30; spool matched perfectly with sleeve such that effective underlap \(U = 0\)) are given by the equation

\[
Q_m = K_v I \sqrt{\frac{P_s - P_m}{\rho}}
\]  

(16.61)

where

- \(K_v\) = A size factor
- \(I\) = Current input to servovalve
- \(P_s\) = Supply pressure
- \(P_m\) = Pressure drop across the servomotor
- \(Q_m\) = Control flow rate to the servomotor
- \(\rho\) = Mass density of fluid

This model assumes that the exhaust pressure \(P_e = 0\). Equation 16.61 may be linearized for operation about an initial steady-state operating point with the result:

\[
\Delta Q_m = K_1 \Delta I - C_1 \Delta P_m
\]  

(16.62)

where

\[
K_1 = \left. \frac{\partial Q_m}{\partial I} \right|_{P_m, I_0} \quad \text{(Flow sensitivity)}
\]  

(16.63)

\[
C_1 = -\left. \frac{\partial Q_m}{\partial P_m} \right|_{I_0, P_m} \quad \text{(Flow-pressure sensitivity)}
\]  

(16.64)

Equation 16.62 is valid for cases when \(U \neq 0\) as well. The terms \(\Delta Q_m\), \(\Delta I\), and \(\Delta P_m\) represent small changes of the corresponding variables about the initial steady-state operating point \(I_0, P_m, P_m\). The constants \(K_1\) and \(C_1\) are evaluated at the operating point.

Typical steady-state pressure-flow-current characteristics for an "idealized" electrohydraulic servovalve that employs a spool-valve second stage (governed by Eq. 16.61) are shown in Fig. 16.34. Characteristics for other types of electrohydraulic servovalves are given in Refs. 34 and 35.

Another important characteristic of an electrohydraulic servovalve is its hysteresis due to the characteristics of the permanent magnets in the torque motor. The hysteresis characteristic is determined from a measurement of the output flow rate as a function of the input current for a constant (usually zero) pressure drop across the valve (load pressure drop). A typical hysteresis characteristic is shown in Fig. 16.35. The slope of the flow-current curve is the "flow sensitivity" of the valve (i.e., \(K_1\) in Eq. 16.63).

It is often convenient in the dynamic analysis of servoactuators to have an approximate dynamic model for the servovalve. Experience has shown that linearized transfer functions based on empirical approximations from measured servovalve responses are adequate for most system designs. Reference 39 outlines considerations underlying the determination of approximate transfer function models for electrohydraulic servovalves. Figure 16.36 shows typical frequency-response plots for an electrohydraulic flow-control servovalve, along with approximate transfer functions. For a frequency range of 0-50 Hz, the following first-order expression has been found to be adequate for two-stage electrohydraulic servovalves:
Fig. 16.34 Typical steady-state pressure-flow characteristics of an electrohydraulic servovalve. (Courtesy of Moog Inc., East Aurora, NY)

1 GPM = $6.3 \times 10^{-5}$ m$^3$/s

Fig. 16.35 Typical steady-state flow-current characteristics of an electrohydraulic servovalve. (Courtesy of Moog, Inc., East Aurora, NY)
Fig. 16.36 Typical dynamic behavior of electrohydraulic servovalves (taken from Reference 39).

DASHED CURVES FOR
\[
\frac{\Delta Q_m}{\Delta t}(s) = \frac{K}{1 + \tau s}
\]
WHERE
\[
\tau = \frac{1}{2\pi 120} = 0.0013 \text{ SEC}
\]

DASHED CURVES FOR
\[
\frac{\Delta Q_m}{\Delta t}(s) = \frac{K}{1 + \left(\frac{25}{\omega_n}\right) s + \left(\frac{s}{\omega_n}\right)^2}
\]
WHERE
\[
\omega_n = 2\pi 185 = 1160 \text{ RAD/SEC}
\]
\[
\frac{s}{\omega_n} = 0.7
\]
TABLE 16.9  TYPICAL DYNAMIC CHARACTERISTICS OF TWO-STAGE ELECTROHYDRAULIC FLOW-CONTROL SERVOVALVES

<table>
<thead>
<tr>
<th>Flow-control servovalve</th>
<th>Max. flow capacity at 3000 psi gpm</th>
<th>Approximate dynamics 3000 psi 100°F</th>
<th>Peak-to-Peak Input at 50% rated current</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1st order</td>
<td>2nd order</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \tau )</td>
<td>( f_n )</td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>0.0013</td>
<td>240</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td>0.0015</td>
<td>200</td>
</tr>
<tr>
<td>C</td>
<td>12</td>
<td>0.0020</td>
<td>160</td>
</tr>
<tr>
<td>D</td>
<td>18</td>
<td>0.0023</td>
<td>140</td>
</tr>
<tr>
<td>E</td>
<td>30</td>
<td>0.0029</td>
<td>110</td>
</tr>
</tbody>
</table>

Source: From Reference 39.

\[
\frac{\Delta Q_m(s)}{\Delta I(s)} = \frac{K_1}{\tau s + 1} \tag{16.65}
\]

where

\( Q_m(s) \) = Laplace transform of control flow rate to the servomotor

\( I(s) \) = Laplace transform of the current input to the servovalve

\( K_1 \) = Servovalve flow sensitivity at \( P_m = 0, I = 0 \)

\( \tau \) = Apparent servovalve time constant (s)

Typical time constants for electrohydraulic flow-control servovalves are given in Table 16.9.

If a good approximation is desired over a wider frequency range, the following second-order model may be preferred:

\[
\frac{\Delta Q_m(s)}{\Delta I(s)} = \frac{K_1}{(s/\omega_n)^2 + (2\zeta/\omega_n)s + 1} \tag{16.66}
\]

where

\( \omega_n = 2\pi f_n \) = Apparent natural frequency (rad/s)

\( \zeta \) = Apparent damping ratio (dimensionless)

Typical values of \( f_n \) and \( \zeta \) for two-stage electrohydraulic flow control servovalves are given in Table 16.9.

16.9 ELECTROMECHANICAL AND ELECTROHYDRAULIC SERVOSYSTEMS

16.9.1 Typical Configurations of Electromechanical Servosystems

An electrical servomotor may be combined with an electrical or electronic modulator to form an electromechanical servoactuator. The addition of a feedback transducer forms a servosystem. Figure 16.2 shows an electromechanical linear-motion servosystem which incorporates a rotary brushless DC servomotor and tachometer feedback; the electronic modulator is not shown.

16.9.2 Typical Configurations of Electrohydraulic Servosystems

A servoactuator comprising an electrohydraulic servovalve and a servomotor may be combined with an electronic servoamplifier (or modulator) and an appropriate feedback transducer to form a high-performance servosystem. Schematic diagrams of three typical electrohydraulic servosystems are shown in Fig. 16.37.

Figure 16.38 shows a photograph and a cross section of a high-performance servosystem which incorporates an electrohydraulic servovalve, an axial-piston servomotor, and a tachometer. The serv-