Name: Solution

# Midterm Examination

Duration one hour and fifteen minutes (75 minutes)
Open Book, Notes & Other stuff.
No Neighborly Participation

**"The fine print": Notes on the whole examination:**

1) *The total points on the exam are 120. You are given an extra 20 points to account for computational and inadvertent errors. However your grade will not exceed 100 even if your score is more than 100. If you score 93, you will get a 93. If you score 110 your score will be 100.*
2) *Numbers in parentheses indicate the tentative grading for each requirement.*
3) ***PLEASE BUDGET YOUR TIME**. YOU MAY WANT TO ANSWER THE SHORT (OR BEST KNOWN ANSWER) QUESTIONS FIRST.*

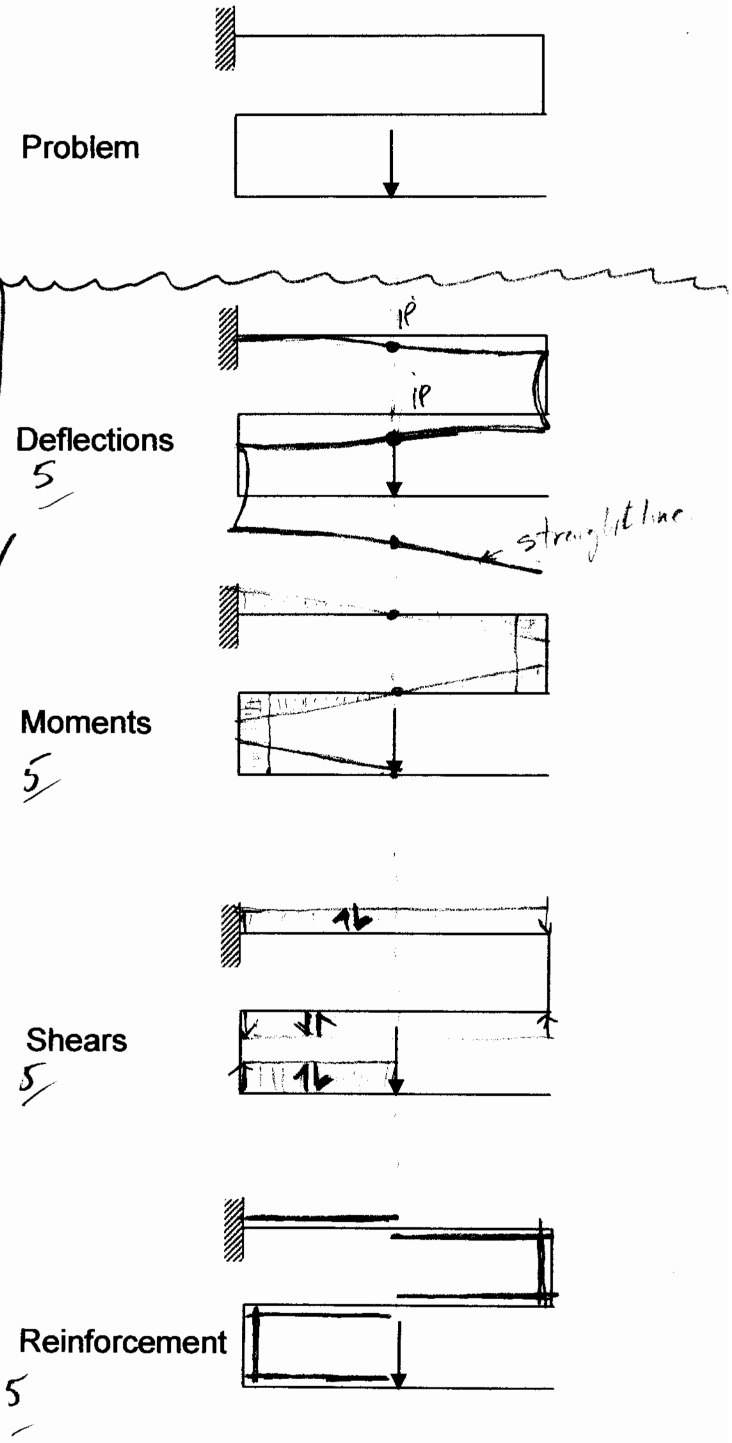
## Problem #1: (20 points)

For the cantilever shown in the sketch determine:

a) The deflected shape (5)
b) The moment diagram (5)
c) The shear diagram (5)
d) Show the location of reinforcement if made of reinforced concrete (no details required) (5)

**Notes:** *Show the answer graphically. Use the exam sheet to answer your question. Indicate the critical points. Indicate moments on the tension side and indicate the shear force pair (sign) for orientation.*

answer



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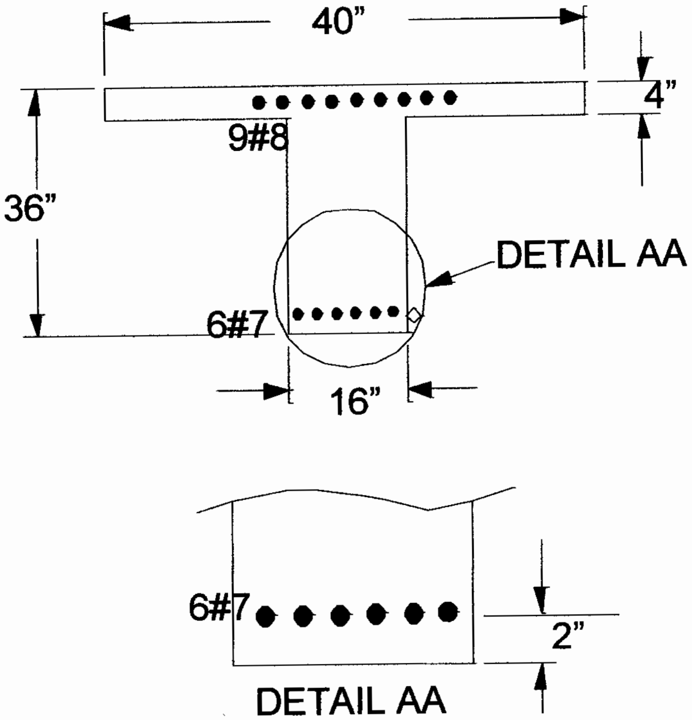
**Problem #2: (40 points)**

Determine the design moment capacity $M_{cap}$ of the T section shown in the figure on the right, if the compression in concrete is at the bottom side of the section.

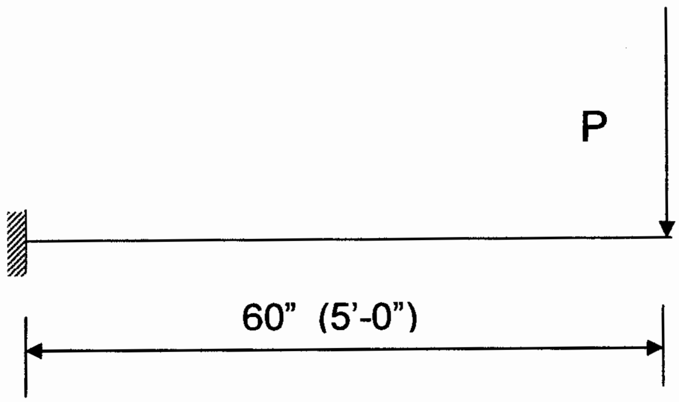
1) Verify adequacy of placement of reinforcement in section (5)
2) Determine the nominal moment capacity (30)
3) Determine the design moment capacity (5)
4) For 20 extrapoints, determine the moment capacity of same section if the compression was at the top side of the section and the reinforcement would be reversed (the larger amount at the bottom) with 12#8 bars instead of 9#8 in two layers.

Parameters: $f_c' = 4$ ksi, $f_y = 60$ ksi




**Problem #3 (40 points)**

Using the same cross section as in Problem#2 with same reinforcement as shown in the figure, determine the size of a concentrated live force (P) - in terms of service load values – which can loaded at the end of the cantilever if the dead load component of the design load is assumed to be 30% and the reinforcement arrangement:




Required:

1. Show the approximated moment diagram and calculate the value of the concentrated force. (10)
2. Show the correct arrangement of the cross section in a proper sketch (20)
3. Show the reinforcement along the beam. Indicate approximately how much of the reinforcement can be reduced at the mid length of the beam (10)

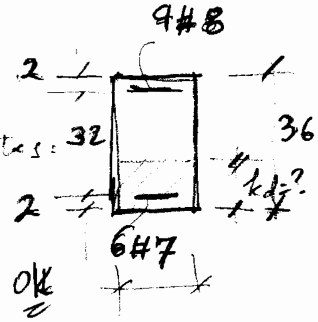
[If you did not calculate the moment in Problem #2 you may use 1200 k-ft for the design moment – for a deduction of 5 points from this problem]

**Problem #4 (20 points)**

For same cross section and the same beam as in Problem#3, determine the shear reinforcement in the critical section near the application of the concentrated load. [Assume that the only contributor to the shear demand in the critical section is the live load]. Show cross section with suggested reinforcement.

PROBLEM 2

Section works like a rectangular shape 16 × 36; doubly reinforced




(5) 1) The placement of reinforcement of 6#7 in 16"?: By table from notes:
6#7 are ok in 16" for 3/4" aggregates. OK.
No problem placing the 9#8 bars in a 40" wide flange. Reinf OK

(30) 2). Areas. (6#7) = $A_s' = 3.60 in^2$

(9#8) = $A_{s_{TOT}} = 7.11 in^2$

$A_s = A_{s_{TOT}} - A_s' = 3.51 in^2$ $\quad k = \left(\frac{3.51 \times 60}{16 \times 34 \times 0.85 \times 4}\right) = 0.114$

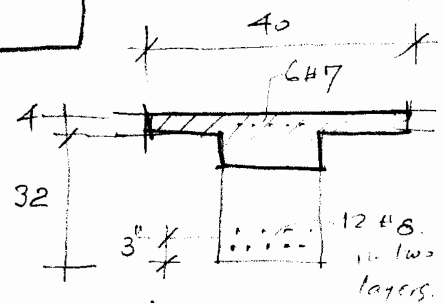
$$M_n = \underbrace{3.60 \times 60 \times 32}_{6912} + \underbrace{3.51 \times 60 \times 34\left(1 - 0.114/2\right)}_{6752} = 13664 \text{ k-in} = 1139 \text{ k-ft}$$

<may check if negative reinforcement yield>

(5) 3) $M_d = \underset{\phi}{0.9} \times 1138 = 1025$ K-ft.

MAIN QUESTION

Section acts as T; doubly reinforced




$d = 33.$

$t = 4.$

$\bar{t} = \frac{4}{33} = 0.123.$

20 4) $A_s = 12.64 in^2$ $\quad A_s' = 3.60$

flange.

$M_f = (40-16) \times 33^2 \times 0.85 \times 4.0 \times 0.123 (1 - 0.123) =$

$= 10258$ K-in.

$A_{sf} = \frac{M_f}{f_y d(1 - \bar{t}/2)} = \frac{}{60 \times 33 \times (1 - 0.123/2)} = 5.52 in^2$

~~web~~ doubly reinforced.

$A_{sw} = A_{s_{TOT}} - A_{sf} = 12.64 - 5.52 = 7.12 in^2$

$A_s' = 3.60 in^2 \Rightarrow M_w' = \overset{A_s' \cdot f_y \cdot d(1-d')}{3.6 \times 60 \times 31} = 6696$

$A_s^+ = \overset{(A_{sw} - A_s')}{7.12 - 3.6 in^2} = 3.52$ $\quad k = \frac{3.52}{16 \times 33} \times \frac{60}{0.85 \cdot 4} = 0.126 \Rightarrow M_w^+ = 3.52 \times 60 \times 33 \left(1 - \frac{0.126}{2}\right)$

$\bar{k} > \bar{t}$ $\quad (0.126 > 0.123).$ $\quad = 6531$

$M_{TOT} = M_f + M_w' + M_w = 10258 + 6696 + 6531 = 23484$ K-in $\cong 1957$ kft

$M_d = 0.9 M_{TOT} = 1761$ K-ft.

BONUS

## Problem #3

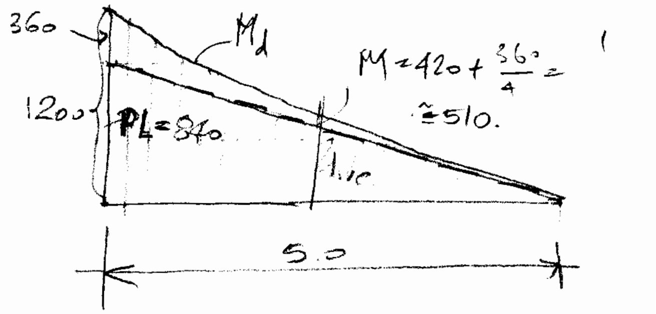
10 (1) See sketch.

$M_d = 1200$ k-ft.

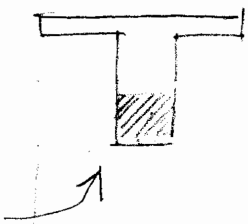
$M_L^D = 0.70 \times 1200 = 840$ k-ft

$P = \frac{M_L^D}{1.7 \times L} = \frac{840}{1.7 \times 5.0} = 98.8$ kips.

(use $\gamma_f = 1.7$; maybe use $\gamma_f = 1.6$)




(2)



20 (2) The moment produces tension at the top, therefore the cross section with the flanges at top and web at bottom fit the loading conditions for this problem.

<More efficient arrangement is obtained when section is reinforced like in 2(d) with flanges at bottom ⊥>

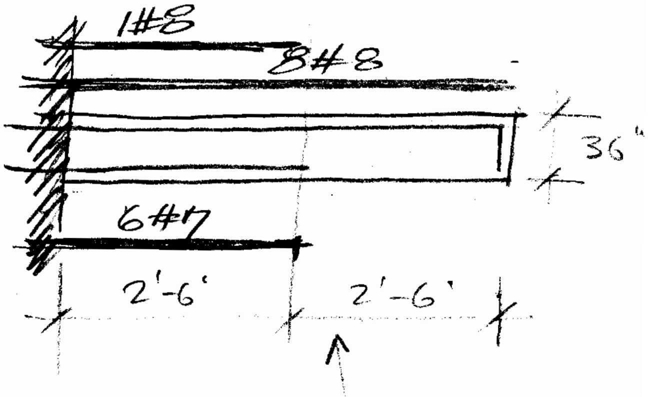
10 (3) The reinforcement arrangement:

At center beam:

$M = \frac{840}{2} + \frac{360}{4} = 510$ k-ft

$M_u = \frac{510}{0.9} = 567$ k-ft

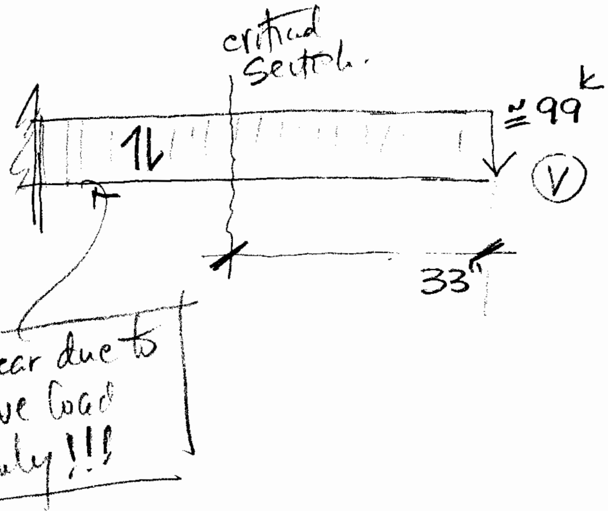
$M_u^{max} = \frac{1200}{0.9} = 1333$ k-ft.




- Compressive reinforcement share ≈ 50% of total ≅ $\frac{1333}{9} \approx 667$. k-in
- Tension reinforcement needed (out of 9 bars) $= \frac{567}{667} \times 9 = 7.65 \Rightarrow 8.$

Therefore one can remove all compressive reinforcement (6#7) and one tension rod (1#8). [See sketch above]

Problem #4




- Critical section is at "d" distance from load application 33" from end i.e. 2'-9". (see sketch)

- Shear stress for shear force in critical section:

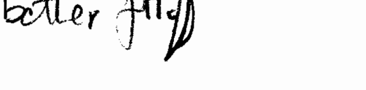
$$f_v = \frac{99 \times 1000}{33 \times 16 \times 0.85} = 221 \text{ psi}$$

$$f_{vc} = 2\sqrt{f'_c} = 2\sqrt{4000} = 126 \text{ psi}$$

$$f_{vs,req} = 221. - 126 = 95 \text{ psi}$$ ⇒ more than 50psi (minimum) ⇒ reinf req'd.

Choose #3 @ 6" closed stirrups  (n=2. (allowed by code here)

$$f_s = \frac{n \cdot A_{sv} \cdot f_y}{s \cdot d} = \frac{2 \times 0.11 \times 60 \times 10^3}{6 \times 33} = 66 \text{ psi} < 95 \text{ psi (required.)}$$

NOT enough; only 2 arms; Possibly two stirrups per section.  with four arms.

Therefore shear reinforcement can be 2#3@6".

$$f_{s\,total} = \frac{4 \times 0.11 \times 60 \times 10^3}{6 \times 33} = 132 \text{ psi} > 95 \text{ psi (required) OK.}$$

(May refine this to 2#3@8" if one wants better fit)